Details of Ion-Temperature-Gradient-Driven Instability Saturation

by

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Abstract

Heat and particle transport due to plasma microturbulence represents a challenge in the development of commercial fusion power. Microturbulence is caused by gyroradius-scale instabilities, of which the Ion-Temperature-Gradient-Driven (ITG) instability is one. This thesis uses gyrokinetic simulations to characterize how the linear ITG instability saturates, as well as to study the turbulent state that arises.

In all of the ITG parameter cases investigated here, zonal flows, which are flows constant along a flux surface, are prominent. Zonal flows are linearly damped and thus must be driven nonlinearly. Some nonlinear energy transfer from the instability sustains the flows, but the majority of the energy injected by the instability is balanced by nonlinear transfer to smaller radial-scale stable and unstable eigenmodes. Unlike Kolmogorov turbulence, the dissipation scale overlaps with the injection scale, so there is no inertial range.

This thesis investigates two phenomena relating to ITG in particular, the nonlinear critical temperature-gradient upshift (the Dimits shift) and the nonlinearly enhanced transport reduction which occurs when the normalized plasma pressure β is increased. It is found that, while stable modes are the primary saturation mechanism, the Dimits shift cannot be attributed to a direct reduction of the heat flux by stable modes, or to their effects on the energy injection rate. It has been suggested that energy transfer out of zonal flows, in a manner like tertiary instability, could limit their amplitude and set the nonlinear critical gradient. We find that this is not possible directly, as transfer of flow energy is into the zonal flows for all wavenumber couplings.

We apply these same techniques to the nonlinearly enhanced reduction of transport with β . Qualitatively, the saturation process is similar throughout the range of β . Stable mode effects only slightly increased with β . Instead, the majority of the reduction is explained by better frequency matching between interacting modes, which causes energy transfer to be more efficient. This effect can be included in quasilinear mixing-length transport models, and we show that it explains 50% to 100% of the enhanced transport reduction with β in five out

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of six test cases.

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1.1 Motivation

Magnetic Confinement Fusion

Development of controlled nuclear fusion, for eventual use in power plants, has been ongoing since the 1950s. Fusion requires ion collisions with enough energy to overcome the electrostatic repulsion between like charges. Achieving some fusion is straightforward; for example, beamtarget fusion has been used for decades to measure nuclear cross-sections. However, this technique is not a candidate for power plants, because directly accelerating ions to the required energies is inefficient. The primary area of investigation for fusion energy involves magnetic confinement of plasmas, where some of the energy released by fusion will couple back into the plasma and will be responsible for a large fraction of the heating. This requires power produced by fusion, which is measured in comparison to the heating power by

$$Q = (\text{fusion power})/(\text{heating power}), \tag{1.1}$$

and a current goal in the field is to achieve $Q \approx 5$. Presently, in 2019, the world record is Q = 0.67 from JET, set in 1997 [1].

To roughly understand magnetic confinement, consider single particle motion in a background magnetic field. Being charged, ions feel a force ($\mathbf{F} \propto \mathbf{v} \times \mathbf{B}$) perpendicular to their velocity (\mathbf{v}) and the magnetic field (\mathbf{B}), which causes them to orbit field lines and allows them free motion along the direction parallel to the field. In an oversimplification, a number of plasma confinement devices are toroidal so that field lines stay inside of the device. Usually, field lines lie in surfaces called flux surfaces, along which the parallel transport sets roughly constant pressures.

One major class of toroidal design is known as the tokamak (see Ref. [2] for a reference text). The magnetic fields in tokamaks are generated by the combination of external coils, which produce multiple Tesla toroidal fields on modern experiments, and by the plasma current. Figure 1.1 shows a schematic of flux surfaces in a tokamak, which are circular in the illustration and correspond to toroids with minor radius ρ . Transport across flux surfaces is classified into classical (collisional), neoclassical (collisions and drifts with geometric effects), and anomalous transport (due to turbulence). This thesis studies the last effect.



Figure 1.1: Three flux surfaces in a tokamak geometry, with a magnetic field line marked in green. The pitch of the magnetic field lines can change with the flux surface. Figure taken from Ref. [3].

As the sun has existed for ~ 5 billion years, it can be inferred to have a low rate of fusion. To achieve higher reaction rates, current plans for power plants involve deuterium-tritium fusion, which has a much higher reaction cross-section than the proton-proton chain active in the sun. Modern experiments also have temperatures in the tens to around a hundred million degrees Kelvin. For comparison, the core of the sun is only fifteen million Kelvin.

Confining the pressures sufficient for a significant fusion yield is a challenge. The plasma edge is always cooler than the core, causing there to be strong radial temperature and density gradients. The plasma core is heated by induced current, various beams, and in a power plant, fusion. In steady state, this heating is balanced by unavoidable heat transport, which among other things, depends on temperature and gradients. Particle densities have similar physics in terms of fueling and transport. The self consistent balance sets temperature and density profiles, which determines core temperature and fusion yield. Temperature and density profiles are also important because plasmas are subject to a number of profile-dependent macroscopic fluid instabilities, which can disrupt the plasma. For illustration, Figure 1.2 shows a typical temperature profile for an existing tokamak as a function of normalized minor radius. Temperature peaks in the center and decreases with radius. Towards the edge, there is a region of stronger normalized temperature gradients.



Figure 1.2: Temperature profile for JET, and projections for ITER and DEMO. Taken from Ref. [4].

There are multiple contributors to transport. One of the first considered from a theoretical standpoint was collisions, which move particles and energy between flux surfaces, leading to heat diffusion. Collisionality decreases strongly with temperature. This is because with higher temperatures come higher particle velocities and thus shorter interaction times. From the perspective of trying to acheive fusion this is a good scaling, however, it is misleading because it is only one of several heat transport mechanisms with different scalings.

In reality, heat transport is much larger than what is given by classical and neoclassical transport. The difference is referred to as anomalous transport [5], and is now understood to be largely the result of gyroradius-scale turbulence, called microturbulence. Pressure gradients provide a source of free energy which microinstabilities can feed off of. Unlike device-scale instabilities, microinstabilities do not cause disruptions and instead saturate at a finite amplitude where the resulting turbulence causes heat and particle transport. This thesis focuses on a type of microinstability called Ion-Temperature-Gradient-Driven (ITG) instability, which is responsible in many experiments for heat transport.

1.2 Thesis Statement and Organization

This thesis is about ion-temperature-gradient-driven instability and how it saturates. It applies a number of diagnostics to the turbulent distribution function, some of them novel, to reveal new details of the saturation process. The knowledge gained here can be applied to reduced models, and an application is included as Chapter 6. There are two main focuses for physics investigation, the nonlinear critical gradient upshift in ITG (Dimits shift), and the effect of normalized plasma pressure β on ITG saturation. (Quasi-)Linear transport models¹ fail at nonzero β , and this thesis both identifies a large contributor and tests a modification to the saturation rule which improves their accuracy at nonzero β .

This thesis is organized as follows: Chapter 2 consists of a conceptual justification of the Vlasov equation and gyrokinetics, followed by the actual gyrokinetic equation and definitions of the measures used here to study ITG saturation. In Chapter 3, these measures are applied to a standard ITG parameter case, to establish a 'default' view of saturation which can then be compared to what happens at different temperature gradients or in parameter cases with electromagnetic effects. The measures are used in Chapter 4 to test possible causes of the nonlinear critical gradient upshift. We find evidence against several plausible causes, but can provide no predictive explanation for the shift. Chapter 5 applies the same measures to ITG turbulence at nonzero β , in an attempt to discover why quasilinear transport models overpredict transport as β is increased. This reveals that while the saturation process is very similar, the tiplet correlation time $\tau_{k,k'}$, which acts as an efficiency of nonlinear energy transfer, greatly increases with β . Chapter 6 discusses some difficulties in quasilinear modeling with nonzero β , and then tests a quasilinear transport model modified to include the nonlinear energy transfer efficiency. In five out of six cases, this modification reduces the overprediction by 50% to 100%. The failure of the sixth can be explained, as a recent paper [6] revealed that toroidal Alfvén eigenmodes affect saturation in that parameter case. The last chapter summarizes the results, as well as collects ideas for research continuations mentioned throughout the body of the text. This is followed by an appendix providing technical detail on the various parameter cases.

¹Quasilinear mixing-length transport models are a class of reduced model which predict transport from linear instability properties and a saturation rule.

2.1 Kinetics (conceptual)

Vlasov-Maxwell Equations

2

Collisions thermalize particle velocity distributions. In other words, collisions make the distributions more Maxwellian ($\sim e^{-mv^2/2}$). It is because of this that fluid equations which lack resolved velocity-space physics, such as Navier-Stokes, can accurately describe water and air under normal conditions.

As a side effect of having low collisionality, *fusion plasmas are not Maxwellian*, so they exhibit behaviors which require velocity-space physics to model. When describing plasmas, it is useful to have a particle density with both spatial and velocity components, called the distribution function:

$$f \equiv f_{\alpha}(\mathbf{r}, \mathbf{p}, t), \tag{2.1}$$

where α is a species label such as electron or proton, **r** is the spatial coordinate vector, **p** is the momentum coordinate vector, and t is time. The choice of coordinates is arbitrary, often velocity **v** or energy and pitch angle are used instead of **p**. Versions of the distribution function with reduced dimensionality, such as with only one or two spatial and velocity dimensions, are also used [7, 8].

The distribution function relates to more commonly used terms such as density:

$$n_{\alpha}(\mathbf{r},t) = \int f_{\alpha}(\mathbf{r},\mathbf{p},t)d\mathbf{p}$$
(2.2)

or current:

$$\mathbf{J}(\mathbf{r},t) = \sum_{\alpha} q_{\alpha} \int (\mathbf{p}/m_{\alpha}) f_{\alpha}(\mathbf{r},\mathbf{p},t) d\mathbf{p}$$
(2.3)

where q_{α} and m_{α} are species charge and mass, respectively.

The distribution function obeys the Vlasov Equation:

$$\frac{Df(\mathbf{r}, \mathbf{p}, t)}{Dt} = C[f] \tag{2.4}$$

where D/Dt is the total derivative, and C[f] is the collision operator, which causes diffusion in velocity space. By the chain rule this can be expanded to:

$$\frac{\partial f}{\partial t} + \frac{d\mathbf{r}}{dt} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{d\mathbf{p}}{dt} \cdot \frac{\partial f}{\partial \mathbf{p}} = C[f]$$
(2.5)

where $d\mathbf{r}/dt$ can be written as velocity $\mathbf{v} = \mathbf{p}/m_{\alpha}$ and $d\mathbf{p}/dt$ can be written as force on a particle $q_{\alpha}(\mathbf{E} + \mathbf{v}/c \times \mathbf{B})$. The fields are determined by the Maxwell equations:

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$
 (2.6) $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$ (2.7)

$$\nabla \cdot \mathbf{B} = 0$$
 (2.8) $\nabla \times \mathbf{B} - 1/c^2 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$ (2.9)

where charge and current density can be calculated from the distribution function.

The first two terms on the left hand side of Eq. (2.5) can be recognized as:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f \tag{2.10}$$

which is the advective derivative common to fluid dynamics. In Fourier space, this looks like $\dot{f}_k \sim \sum_{k'} v_{k'} f_{k-k'}$. The advective derivative will be important later because it is common for both the nonlinearity in gyrokinetics and in two dimensional fluid dynamics.

Applying the full 6+1D Vlasov-Maxwell equations directly to plasma physics modeling is only rarely done. This is because directly solving the Vlasov-Maxwell equations is incredibly computationally expensive, as it has 6 dimensions and a wide range of timescales. However, simplifications can be made to get a more tractable system.

The Vlasov equation was introduced because it is conceptually simple; it can be written as only two terms. It also demonstrates the correspondence between single particle motion and the evolution of distribution function. The motion of individual particles are the characteristic curves of the Vlasov equation. All the Vlasov equation says is that particles move along at their velocities, their velocities change according to the forces acting on them, and those forces are generated by electric and magnetic fields which can be calculated from Maxwell's equations. Collisions scatter particles in velocity space, and are not a focus of this thesis. For the most part it is assumed here that the structure of the collisional operator does not change the physics of interest, however collision operators are nuanced and an area of active research [9]. Single particle motion is used in the next subsection to develop intuition for gyrokinetics, a derivation of which will not be included here.

Gyrokinetics

Gyrokinetics is a common formalism for microturbulence theory and simulations, and is used throughout this thesis. As mentioned in Chapter 1, tokamaks have strong magnetic fields, so ignoring perturbations, particles orbit field lines in the perpendicular direction and move freely in the parallel direction. In gyrokinetics, the gyro-orbit angle is averaged over, reducing the perpendicular velocity vector to a scalar and removing the fastest timescales in the plasma. Because of the average, gyrokinetics cannot describe timescales as fast as the cyclotron motion, or normalized background gradient length-scales as small as the gyroradius. The background magnetic field also has to be strong compared to the perturbed electric and magnetic fields. For a review of derivations of gyrokinetics, including the orderings and regimes of validity, see Ref. [10]. There are multiple versions of the gyrokinetic equations depending on the derivation, and they have subtly differing physics, especially in the conserved quantities. For the specifics of the framework used here, see Ref. [11].

There are three main effects of the gyro-average which should be kept in mind for intuition's sake. The first effect consists of drifts, mentioned in the motivation above. An electric field perpendicular to background magnetic field will cause the gyro-orbit to be wider on the lower potential side. Averaging over the orbit, this leads to a constant velocity perpendicular to both the electric and magnetic field ($\mathbf{v}_{E\times B} \propto \mathbf{E} \times \mathbf{B}/B^2$). Figure 2.1 shows the path of an ion in a magnetic field with a perpendicular electric field, as well as the gyro-center path and a magnetic field line. The E × B drift is one of several, including terms due to ∇B and magnetic curvature [12].

Secondly, the magnetic moment μ , sometimes including modifications for higher order convergence [13], is an adiabatic invariant. Because energy due to perpendicular motion is μB and magnetic fields do no work, particles will lose parallel velocity when moving to a region of higher magnetic field. A class of magnetic confinement device known as the magnetic mirror uses this effect to confine plasmas. This single particle motion effect is responsible for the the trapping term $\dot{f} \sim \mu \partial_z B_0 \partial f / \partial v_{\parallel}$ of the gyrokinetic equation.

The last effect occurs because the average effect of a field over the gyro-orbit does not equal the effect at center of the orbit. In Fourier (position) space, the gyro-average introduces a Bessel function in front of the contribution of fields. For example, given a field $\cos(kx)$ and a particle with path $x = r \cos(\theta), y = r \sin(\theta)$, the average is:

$$\int_{0}^{2\pi} \cos(ikr\cos(\theta))d\theta \Big/ \int_{0}^{2\pi} d\theta = J_0(kr), \qquad (2.11)$$

where J_0 is the zeroth order Bessel function. The argument of the Bessel function depends on the ratio of the gyroradius to the wavelength of the field. Because the gyroradius depends on μ , this acts to create fine structure in velocity space, as particles with different μ will respond differently to the fields. The left panel of Figure 2.2 shows circles of various radii overlaid on a sinusoidal field, while the right panels shows the zeroth order Bessel function with marked points corresponding to the average over the circles in the left.



Figure 2.1: Single particle motion, showing the constant drift velocity due to a perpendicular electric and magnetic field. The magnetic field line is shown in blue, while the helical particle path, as well as the center path are both shown in black.



Figure 2.2: Ions feel the average field over the course of their orbit. Plotted are circles corresponding to gyro-radii on a sinusoid, and a Bessel function marked at the corresponding radii in the same color. The magnetic field in the left panel points out of the page.

Gyrokinetic simulations/theory often make two assumptions relevant here. First is the flux-tube approximation. With the flux-tube approximation, only the plasma volume between two flux surfaces, typically chosen around 100 gyroradii apart, is considered. Here, the radial boundary condition is given by connecting the inside and outside flux surface together. For this to be reasonable, the box has to be larger than the correlation length of the turbulence. Studies have been done on how far turbulence from one region can be expected to propagate, giving around 10 gyroradii [14], which is significantly smaller than typical radial box sizes used in gyrokinetic simulations. Geometric terms which scale with the minor over major radius are retained while terms like the difference in minor radius between the inner and outer flux surfaces are not. Gradients are taken to be linear and background quantities are taken to be constant within the volume. Also, only a limited region in the direction perpendicular to both the parallel and radial directions is considered, but given a large enough box with a toroidally symmetric plasma, this does not change the relevant microturbulence physics.

$$f_{\alpha} = F_0 + \tilde{f}, \quad F_0 = cn_0 e^{v^2/2m}$$
 (2.12)

Second, gyrokinetic simulations and theory often assume a Maxwellian-velocity-space background distribution with prescribed gradients which does not relax with heat transport. As a framework, this allows transport and turbulent intensities to be calculated as a function of input parameters at a given radial location. This also introduces a distinction between linear (the perturbed distribution function and its interactions with the background) and nonlinear (perturbed distribution function interacting with itself) physics, which is the subject of the next subsection. Currently, gyrokinetic simulations on the timescales which profiles evolve are infeasibly expensive, but there may be physical insights to be gained from such runs. In the meantime, it is possible to calculate a turbulent transport and then update the gradients in a two step process, which saves resources but is difficult in stiff systems.

The combination of these approximations make gyrokinetic simulations computationally tractable. Simple simulations for linear instability can be performed on laptops, while nonlinear simulations can take $\sim 100,000$ CPUh or more. Numerical and theoretical investigation have revealed a number of microinstabilities, depending on driving gradients and other parameters.

2.2 Linear and Nonlinear Gyrokinetics

The gyrokinetic code used in this thesis is GENE (see Refs. [15, 36]). The equations that GENE solves are introduced in this section before defining a number of metrics which reveal the linear and nonlinear physics. As a prerequisite for that mathematical framework, we define a number of variables. Coordinates, geometric quantities, species-dependent parameters, and the remaining quantities are given in Tables 2.1 to 2.4, in that order. See Ref. [11] for complete definitions and normalizations.

Quantity	Definition
x	Radial coordinate
y	Binormal coordinate
z	Parallel coordinate (from $-\pi$ to π)
v_{\parallel}	Parallel velocity
μ	Magnetic moment
θ	Ballooning angle
k	Perpendicular wavevector (k_x, k_y)
v	Velocity vector (v_{\parallel}, μ)

Table 2.1: Coordinate quantities

Quantity	Definition
R_0	Major radius
q_0	Safety factor $\frac{r \cdot B_t}{R \cdot B_p}$ at r_0
\hat{s}	Magnetic Shear = $(r_0/q_0)(dq/dx)$
$\mathcal{J}(z)$	Jacobian
B_0	Background magnetic field

Table 2.2: Common geometric quantities

- quantity	
j	Species, e.g. electrons, protons,
q_j	Charge
m_j	Mass
T_{j0}	Background temperature
n_{j0}	Background density
v_{Tj}	Thermal velocity
F_{j0}	Background Maxwellian
ω_{Tj}	Normalized temperature gradient = $-(R_0/T_{j0})(dT_{j0}/dx)$
ω_{nj}	Normalized density gradient $= -(R_0/n_{j0})(dn_{j0}/dx)$
Ω_j	Cyclotron frequency $= q_j B_0 / m_j$
$ ho_j$	Gyroradius = $v_{\perp}/ \Omega_j $
f_j	Distribution function $f_j(x, y, z, v_{\parallel}, \mu, t)$
g_j	Nonadiabatic distribution function $g_j = f_j - \frac{2q_j}{m_j v_{Tj}} v_{\parallel} \bar{A}_{\parallel}$
$\Gamma_{x,y}$	$= ik_{x,y}g + (q_j/T_{0j})F_0ik_{x,y}\chi$
Γ_z	$=\partial_z g + (q_j/T_{0j})F_0\partial_z \chi + (v_{Tj}q_j/T_{0j})v_{\parallel}\mu F_0A_{\parallel}\partial_z B_0$
b_j	$= v_{Tj}^2 k_\perp^2 / (2\Omega_j^2)$
λ_j	$= (2B_0 \mu/m_j)^{1/2} (k_\perp/\Omega_j)$

Quantity Definition

Table 2.3: Species dependent parameters and fields

Quantity	Definition
λ_D	Debye length
ω_p	Pressure gradient = $\sum_{j} n_j T_j (\omega_{nj} + \omega_{Tj})$
$K_{x,y}$	Geometric factors (See Ref. [11])
Γ_0	$= e^{-x} \hat{I}_0(x)$

Table 2.4: Remaining quantities

Gyroaveraged quantities (which include the Bessel function factor $J_0(\lambda_j)$) are written with an overbar. Here, the gyrokinetic equation is written in terms of the nonadiabatic distribution function g_j and the modified potential $\chi_j = \bar{\Phi} - v_{Tj}v_{\parallel}\bar{A}_{\parallel}$, where Φ is the electric potential and A_{\parallel} is the parallel magnetic potential. Note that this potential is different for different species, and includes velocity space structure. The use of g and χ instead of f, Φ and A_{\parallel} somewhat simplifies the equations, however, care must be taken to not conflate f and g when doing analysis. GENE simulations can include terms proportional to B_{\parallel} as well, but these terms are not written here and are only included in one of the parameter cases. This is because these terms are small when normalized plasma pressure $\beta = 8\pi n_{\rm e0}T_{\rm e0}/B_0^2$ is small, and the highest β cases investigated here have a β around a few percent.

Firstly, the fields are given by:

$$\Phi = \frac{\sum_{j} n_{0j} \pi q_j B_0 \int J_0(\lambda_j) g_j dv_{\parallel} d\mu}{k_{\perp}^2 \lambda_D^2 + \sum_{j} (q_j^2 / T_{0j}) n_{0j} (1 - \Gamma_0(b_j))}$$
(2.13)

$$A_{\parallel} = \frac{\sum_{j} \frac{\beta}{2} q_{j} n_{j0} v_{Tj} \pi B_{0} \int v_{\parallel} J_{0}(\lambda_{j}) g_{j} dv_{\parallel} d\mu}{k_{\perp}^{2} + \sum_{j} \frac{\beta q_{j}^{2}}{m_{j}} n_{0j} \pi B_{0} \int v_{\parallel}^{2} J_{0}^{2}(\lambda_{j}) F_{0j} dv_{\parallel} d\mu},$$
(2.14)

which obey the superposition principle. There is a factor of β in the equation for A_{\parallel} , so A_{\parallel} physics is only relevant when $\beta \neq 0$.

The adiabatic electron approximation is often used to reduce computation time by using a simplified model of electron physics. Because GENE uses the electron β internally, this approximation requires $\beta = 0$. Under the adiabatic electron approximation, the potential is:

$$\Phi = \frac{\sum_{j} n_{0j} \pi q_j B_0 \int J_0(\lambda_j) g_j dv_{\parallel} d\mu + (q_e^2 n_{0e}/T_e) \langle \Phi \rangle}{(q_e^2 n_{0e}/T_e) + \sum_{j} (q_j^2/T_{0j}) n_{0j} [1 - \Gamma_0(T_{0j} m_j k_{\perp}^2/(q_i^2 B_0^2))]},$$
(2.15)

where the flux surface averaged potential is:

$$\langle \Phi \rangle = \frac{\sum_{j} \pi q_{j} n_{0j} B_{0} \langle \int J_{0}(\lambda_{j}) g_{j} dv_{\parallel} d\mu \rangle}{\sum_{j} q_{j}^{2} n_{0j} / T_{0j} [1 - \langle \Gamma_{0}(T_{0j} m_{j} k_{\perp}^{2} / (q_{i}^{2} B_{0}^{2})) \rangle]}$$
(2.16)

This approximation is used for all of the simulations in Chapters 3 and 4.

The gyrokinetic equation solved by GENE can be split into terms linear and quadratic in the perturbed distribution function and written as:

$$\frac{\partial g}{\partial t} = \mathcal{L}[g] + \mathcal{N}[g], \qquad (2.17)$$

where \mathcal{L} contains all of the linear terms and \mathcal{N} is the nonlinearity due to $\mathbf{v}_{E\times B}$. In detail, the

linear terms are:

$$\mathcal{L}[g] = -\left(\omega_{nj} + (v_{\parallel}^2 + \mu B_0 - 3/2)\omega_{Tj}\right)F_{0j}ik_y\chi + \frac{\beta T_{0j}}{q_j B_0^2}v_{\parallel}^2\omega_p\Gamma_{jy} - \frac{v_{Tj}}{JB_0}v_{\parallel}\Gamma_{jz} - \frac{T_{0j}(2v_{\parallel}^2 + \mu B_0)}{q_j B_0}(K_y\Gamma_{jy} + K_x\Gamma_{jx}) + \frac{v_{Tj}}{2JB_0}\mu\partial_z B_0\frac{\partial f_j}{\partial v_{\parallel}} + \langle C_j(f)\rangle.$$
(2.18)

In order, these are the gradient drive term, the pressure term, the parallel dynamics, the curvature terms, the trapping term and the collisional term¹. The gradient drive term can be recognized as the same $\mathbf{v}_{E\times B}$ term that occurs in the nonlinearity, only due to the perturbed distribution function interacting with the background rather than the perturbed distribution function interacting with itself.

Linear terms do not couple different **k**, except through the parallel boundary condition which depends on the magnetic shear. This enforces $f_{k_x,k_y}(z = \pi) = cf_{k_x+2\pi\hat{s}k_y,k_y}(z = -\pi)$. The quantity c is a phase factor. Coupling through the boundary condition reduces the number of uncoupled modes and puts restrictions on the ratio of the box size in the radial and binormal directions. We specify the distribution function at a wavevector and its parallel connections by writing $g_k(\theta, \mathbf{v})$ instead of $g_k(z, \mathbf{v})$. These modes are labeled by the k_x -center wavenumber, that is, the lowest $|k_x|$ wavenumber in the connected extent.

The gyrokinetic nonlinearity is given by:

$$\mathcal{N}[g] = \sum_{k'} (k'_x k_y - k_x k'_y) \chi_{jk'} g_{jk''}, \qquad (2.19)$$

where $\mathbf{k}'' = \mathbf{k} - \mathbf{k}'$. Unlike the linear terms, the nonlinearity couples all wavevectors such that $|\mathbf{k} \times \mathbf{k}'| \neq 0$. This is an advective nonlinearity in common with fluid dynamics. As a simplification, it suffices to think of the nonlinearity as the action of $\mathbf{v}_{E\times B}$ at \mathbf{k}' , which advects the distribution function at \mathbf{k}'' to \mathbf{k} . Figure 2.3 shows in real space the action of the nonlinearity in fourier space. The $v_{E\times B}$ shear at \mathbf{k}' (the center panel) couples the left and right panels.

¹This term includes numerical dissipation.



Figure 2.3: A schematic representation of the nonlinearity, given by the effect of $v_{E\times B}$ shear from one wavevector on another.

2.3 Turbulence Measures

Eigenmodes and Projections

The time evolution of $g_k(\theta, \mathbf{v})$ can be written as:

$$g_k(\theta, \mathbf{v}, t) = \sum_l \beta_l(t) g_{k,l}^{\text{ev}}(\theta, \mathbf{v}), \qquad (2.20)$$

where mode structures $g_{k,l}^{\text{ev}}$ are the right eigenvectors of \mathcal{L} , while $\beta_l(t)$ are the time-dependent complex coefficients, and l is a mode label, spanning from 1 to the number of gridpoints. In the absence of the nonlinearity, the coefficients evolve according to $\beta_l(t) = \beta_l(0)e^{i\omega_{k,l}t}$ where $\omega_{k,l} = \Re e[\omega_{k,l}] - i\gamma_{k,l}$ is the l^{th} eigenvalue of \mathcal{L} . The eigenmodes grow or damp with time according to the sign of the growth rate $\gamma_{k,l}$.

For typical parameters, eigenmode decompositions return up to several unstable ($\gamma > 0$) and numerous stable ($\gamma < 0$) eigenmodes per wavevector. The stable modes in the unstable wavevector range can be nonlinearly excited and can affect the turbulence through their contributions to transport and energy injection [16, 17, 18]. Computing all of the eigenmodes from \mathcal{L} is possible, although very expensive, so typically only the most unstable eigenmode is found. This is often done by time-evolving the linear system until only the eigenmode with the largest $\gamma_{k,l}$ dominates. The amplitudes β_l of the eigenmodes in an arbitrary distribution function g_k can be found with the projection:

$$\beta_l = \frac{\langle g_{k,l}^{\text{ev}}(\theta, \mathbf{v}) \cdot g_k(\theta, \mathbf{v}) \rangle}{|g_{k,l}^{\text{ev}}(\theta, \mathbf{v})|},$$
(2.21)

where:

$$\langle A \cdot B \rangle = \sum_{j} \int A^{\dagger} B \mathcal{J} d\theta d\mathbf{v}$$
 (2.22)

is the inner product and |A| is the associated norm. Note that as \mathcal{L} is generally non-Hermitian, the eigenmodes are nonorthogonal and the inner product *should* use the left eigenvectors for mode decompositions, to avoid the problems associated with nonorthogonality. The work done here uses the conjugate transpose of the right eigenvectors, which still gives a measure of how strongly a mode is excited in the turbulence, but is complicated by the non-orthogonality².

It will often be useful to talk about the average 'fraction' of the distribution function described by an eigenmode. To obtain this fraction, β_l is time-averaged and normalized by the amplitude of g_k :

$$P(g_{k,l}^{\text{ev}}, g_k) = \int_{t_0}^{t_0 + \Delta T} \left(\frac{\left| \langle g_{k,l}^{\text{ev}}(\theta, \mathbf{v}) \cdot g_k(\theta, \mathbf{v}) \rangle \right|}{|g_{k,l}^{\text{ev}}(\theta, \mathbf{v})| |g_k(\theta, \mathbf{v})|} \right) dt \middle/ \Delta T$$
(2.23)

where ΔT should be taken to be a long enough time for the average to converge. The average fraction of the distribution function described by a single eigenmode $P(g_{k,l}^{ev}, g_k)$ spans between 0 and 1 depending on how similar the distribution function in the turbulence is to the eigenmode. As mentioned previously, in the absence of the nonlinear term, this quantity would end up being completely dominated by the most unstable mode. As such, measuring the unstable and stable mode fractions provides a tool to examine in detail the role of the nonlinearity in the turbulence. A caveat first: *Eigenmodes are, in general, nonorthogonal*. Because of this, the sum of multiple eigenmode fractions can be larger than one, as this sums contributions of the nonorthogonal parts of modes multiple times, and thus one cannot rely on a projection of 1 for a subset of modes preventing other modes from having non-zero projection.

²See Ref. [19] for a discussion on the orthogonality of left and right eigenvectors.

Energy: Injection, Dissipation and Nonlinear Transfer

The energy-conserving gyrokinetic nonlinearity transfers energy (sometimes called entropy) within interacting triplets $\mathbf{k}, \mathbf{k}', \mathbf{k}'' = \mathbf{k} - \mathbf{k}'$, with energy defined as [20, 18, 19]:

$$E_{k} = \Re e \bigg\{ \sum_{j} \int \frac{n_{j0} T_{j0}}{F_{j0}} \bigg[g_{jk} + \frac{q_{j} F_{j0}}{T_{j0}} \chi_{jk} \bigg]^{*} g_{jk} dz d\mathbf{v} \bigg\}.$$
(2.24)

The energy injection/dissipation rate can be calculated using the rate of change of the energy operator and $\partial g_k/\partial t$. Components of $\partial E_k/\partial t$ from each term in $\partial g_k/\partial t$ can be evaluated separately, which is how the nonlinearity can be shown to conserve energy. When used on the components of \mathcal{L} , this shows that only the drive term and dissipation term inject/dissipate energy. The rate of change of energy at **k** due to these terms is given by:

$$\frac{\partial \mathbf{E}_{k}}{\partial t}\Big|_{\mathbf{N.C.}} = \sum_{j} \left\{ -2\Re \mathbf{e} \Big[\int \pi n_{j0} T_{j0} v_{*} i k_{y} \Gamma_{jk}^{*} \chi_{k} d\theta dv_{\parallel} d\mu \Big] + 2\Re \mathbf{e} \Big[\int \pi n_{j0} T_{j0} v_{*} \Gamma_{jk}^{*} C_{j}(f_{jk}) d\theta dv_{\parallel} d\mu \Big] \right\},$$

$$(2.25)$$

where $v_* = \omega_{nj} + \omega_{Tj}(v_{\parallel}^2 + \mu B_0 - 3/2)$, $\Gamma_{jk} = g_{jk} + (q_j F_{j0}/T_{j0})\chi_{jk}$, and $C_j(f_{jk})$ is the collision operator which also includes numerical dissipation. The first term on the RHS is due to the gradient drive and the second is due to collisions and numerical dissipation. As the first term depends on $\Re e[ig_{jk}^*\chi_{jk}]$, it can be either positive or negative depending on the phase relation between g_{jk} and χ_{jk} .

In the eigenmode basis, the rate of change in energy has the form:

$$\sum_{l} 2\gamma_l |\beta_l|^2 - \sum_{l} \sum_{m} \Re e[c_{lm} \beta_l^* \beta_m], \qquad (2.26)$$

where the terms $\sim \beta_l^* \beta_m$ are the cross terms due to nonorthogonality³ and c_{lm} is a complex number dependent on the mode structures. These are all complex quantities and the

³Nonorthogonality depends on the relevant operator, which is the energy operator in this case. Even if left eigenvectors are used for the eigenmode decomposition, there will still be energy effects proportional to $\beta_l^*\beta_m$. This is because the energy and the time evolution operators do not commute. If they did commute, and were both diagonalizable, they would be simultaneously diagonalizable.

contribution to energy production depends on their relative phases. In some cases, the cross terms can increase energy production relative to that from just the unstable mode, or even cause net energy injection in a case without an instability. If this effect is of limited duration it is called transient amplification, while in cases where the nonlinearity maintains a phase relation between eigenmodes which causes energy injection in the absence of linear instability, it is called nonlinear instability or subcritical turbulence [21]. In the velocity-space basis instead of the eigenmode basis, this effect is due to an increased $\Re e[ig_{jk}^*\chi_{jk}]$ relative to that from the eigenmode with highest γ alone. Stable eigenmodes can be dissipative due to the drive term or dissipation.

These effects motivate the comparison of the linear growth rates with the quantity:

$$\gamma_{\text{eff}} = \frac{\partial E_k / \partial t|_{\text{N.C.}}}{2E_k},\tag{2.27}$$

which is equal to the growth rate of the eigenmode when evaluated with the eigenmode structure as the distribution function. It gives a measure of the net effect of the stable eigenmodes excited in the turbulent state. This quantity also represents the instantaneous growth rate if the nonlinearity were to be turned off.

To measure the importance of the drive and dissipation terms in the turbulence, we will also split the nonconservative terms between the two:

$$\frac{\partial E_k}{\partial t}\Big|_{\text{N.C.}} = \frac{\partial E_k}{\partial t}\Big|_{\text{drive}} + \frac{\partial E_k}{\partial t}\Big|_{\text{diss}},\tag{2.28}$$

where $\partial E_k / \partial t |_{\text{drive/diss}}$ is due to the first/second term on the RHS of 2.25, to calculate an effective growth rate because of each:

$$\gamma_{\rm drive/diss} = \frac{\partial E_k / \partial t |_{\rm drive/diss}}{2E_k}.$$
(2.29)

These terms sum to γ_{eff} .

The rate of change of energy at \mathbf{k} due to the nonlinearity is given by the sum of couplings to different wavevectors [22]:

$$T_{k,k'} = 2\Re e \bigg\{ \sum_{j} \int \frac{n_{j0} T_{j0}}{F_{j0}} \big[g_{jk} + \frac{q_j F_{j0}}{T_{j0}} \chi_{jk} \big]^* (k'_x k_y - k_x k'_y) \Big[\chi_{jk'} g_{jk''} \Big] dz d\mathbf{v} \bigg\}.$$
(2.30)

This conserves energy within wavenumber triplets $\mathbf{k}, \mathbf{k}', \mathbf{k}''$, that is, the sum of energy transfer

over all permutations of those wavevectors is zero. As such, the nonlinearity conserves energy overall. For an unstable mode to saturate, the energy injected by it must be balanced over time average by nonlinear energy transfer. Because of this, it is useful to measure energy transfer to and from individual mode structures, given by:

$$T_{k,k'}^{\text{ev}} = 2\Re e \bigg\{ \sum_{j} \int \frac{n_{j0} T_{j0}}{F_{j0}} \big[\beta_l g_{jk}^{\text{ev}} + \frac{q_j F_{j0}}{T_{j0}} \beta_l \chi_{jk}^{\text{ev}} \big]^* (k'_x k_y - k_x k'_y) \Big[\chi_{jk'} g_{jk''} \Big] dz d\mathbf{v} \bigg\}.$$
(2.31)

Here, g_{jk} and χ_{jk} have been replaced by the projection of g_{jk} onto the eigenmode structure and the fields from it, respectively. Similar to the effects described earlier on projections and energy injection, eigenmode nonorthogonality can cause the sum of transfers $T_{k,k'}^{ev}$ to different eigenmodes at a wavevector to overpredict total energy transfer to that wavevector.

Throughout this thesis, energy transfer (when split for eigenmodes) is only split into transfer to the unstable ITG mode $T_{k,k'}^{u}$, which uses the unstable mode for g_{jk}^{ev} , and a remainder given by:

$$T_{k,k'}^{\rm s} = T_{k,k'} - T_{k,k'}^{\rm u}.$$
(2.32)

The circumvents the issue of nonorthogonality. For cases with only one unstable mode, $T_{k,k'}^{s}$ is energy transfer to the combined stable modes at that wavevector. Energy transfer *due* to specific eigenmodes instead of to specific eigenmodes could be calculated by substituting their mode structure at \mathbf{k}' or \mathbf{k}'' , however this increases the difficulty of properly accounting for nonorthogonality and is not done here.

Nonlinear energy transfer can also be split in ways other than mode structure. There are two energy components, a $g^2/F_0 \approx f^2/F_0$ (entropy-like) term, and a $\chi^*g \approx C\Phi^2$ (field) term, where C is a constant. These approximations are exact at $\beta = 0$. The contribution of the nonlinearity to the change in each of these are:

$$T_{k,k'}^{g} = 2\Re e \bigg\{ \sum_{j} \int \frac{n_{j0} T_{j0}}{F_{j0}} g_{jk}^{*} (k_{x}' k_{y} - k_{x} k_{y}') \chi_{jk'} g_{jk''} dz d\mathbf{v} \bigg\},$$
(2.33)

for the entropy term and:

$$T_{k,k'}^{\chi} = 2\Re e \bigg\{ \sum_{j} \int n_{j0} q_j \chi_{jk}^* (k'_x k_y - k_x k'_y) \chi_{jk'} g_{jk''} dz d\mathbf{v} \bigg\}.$$
(2.34)

for the field term. They are each separately conserved within triplets.

The mode at $\mathbf{k}', \chi_{k'}$ is also a composite quantity. $T_{k,k'}$ can also be split into transfer $T^{\Phi}_{k,k'}$

due to coupling with Φ at \mathbf{k}' :

$$2\Re e \left\{ \sum_{j} \int \frac{n_{j0} T_{j0}}{F_{j0}} \left[g_{jk} + \frac{q_j F_{j0}}{T_{j0}} \chi_{jk} \right]^* (k'_x k_y - k_x k'_y) \left[\bar{\Phi}_{jk'} g_{jk''} \right] dz d\mathbf{v} \right\}$$
(2.35)

and transfer $T_{k,k'}^{A_{\parallel}}$ due to A_{\parallel} at \mathbf{k}' :

$$2\Re e \Big\{ \sum_{j} \int \frac{n_{j0} T_{j0}}{F_{j0}} \Big[g_{jk} + \frac{q_j F_{j0}}{T_{j0}} \chi_{jk} \Big]^* (k'_x k_y - k_x k'_y) \Big[(-\bar{A}_{\parallel jk'} v_{Tj} v_{\parallel}) g_{jk''} \Big] dz d\mathbf{v} \Big\}.$$
(2.36)

The combination of all of these quantities provides a much better description of instability saturation and the turbulent state than the total transfer alone.

Triplet Correlation Times



Figure 2.4: Energy transfer as a function of time for two different nonlinear couplings.

The sign of energy transfer rate is tied to the complex phase of the interacting modes and varies between positive and negative values on short time scales. Energy transfer consistently follows a path from sources to sinks only when averaged over many turbulent correlation times. Figure 2.4 shows energy transfer as a function of time for two different nonlinear couplings. Both energy transfers mostly average out over time, so the instantaneous energy transfer is typically much larger in magnitude than the average value. Average transfer

is governed by the time averages $\langle g_{jk}^* \chi_{jk'} g_{jk''} \rangle$ and $\langle \chi_{jk}^* \chi_{jk'} g_{jk''} \rangle$, which are sensitive to the correlation time $\tau_{k,k'}$ of the triplet interaction. This time is given by

$$\tau_{k,k'} = -\frac{i}{[\hat{\omega}_{k-k'} + \hat{\omega}_{k'} - \hat{\omega}_k^*]},\tag{2.37}$$

where $\hat{\omega}_k$ is the complex nonlinear frequency at **k**, which includes linear and nonlinear components. Average energy transfer rates are proportional to $\tau_{k,k'}$. Equation (2.37) can be derived from closure theory [23], which shows that saturated turbulence levels and fluxes are inversely proportional to $\Re e[\tau_{k,k'}]$.

The quantity $\tau_{k,k'}$ is universal to fluid models with quadratic nonlinearities. As an example, consider $\langle \Phi_{-k} \Phi_{k'} \Phi_{k''} \rangle$, where Φ is a quantity in the fluid model, and the linear and nonlinear evolution can be represented as:

$$\dot{\Phi}_k + i\omega_k \Phi_k = \sum_{k'} C_{k,k'} \Phi_{k'} \Phi_{k''}.$$
 (2.38)

Here, $C_{k,k'}$ is a nonlinear coupling coefficient. The time evolution of the triplet correlation is constructed from the equations for $\dot{\Phi}_{-k}$, $\dot{\Phi}_{k'}$, and $\dot{\Phi}_{k''}$, yielding

$$\frac{\partial \langle \Phi_{-k} \Phi_{k'} \Phi_{k''} \rangle}{\partial t} + i [\omega_{k-k'} + \omega_{k'} - \omega_k^*] \langle \Phi_{-k} \Phi_{k'} \Phi_{k''} \rangle \sim \langle \Phi^4 \rangle.$$
(2.39)

This expression is part of a moment hierarchy that is closed at third order in statistical closure theory [24, 25]. The closure assumes quasi-Gaussian statistics to write $\langle \Phi^4 \rangle$ as $3 \langle \Phi^2 \rangle \langle \Phi^2 \rangle$. Part of the closed nonlinearity renormalizes the frequencies with a nonlinear component only preset at finite-amplitude. When the amplitude-dependent component is included, we write $\hat{\omega}_k$ for the frequency. In the steady state, Eq. (2.39) becomes

$$T_{k,k'}^{(\Phi)} = \langle \Phi_{-k} \Phi_{k'} \Phi_{k''} \rangle \sim [\hat{\omega}_{k-k'} + \hat{\omega}_{k'} - \hat{\omega}_k^*]^{-1} \langle \Phi^2 \rangle \langle \Phi^2 \rangle.$$
(2.40)

where $T_{k,k'}^{(\Phi)}$ represents nonlinear transfer of Φ^2 . Energy balance implies that injection ($\sim \gamma \langle \Phi^2 \rangle$) balances transfer $(T_{k,k'}^{(\Phi)} \sim \langle \Phi_{-k} \Phi_{k'} \Phi_{k''} \rangle)$, from which one obtains $\gamma \langle \Phi^2 \rangle \sim T_{k,k'}^{(\Phi)} \sim [\hat{\omega}_{k-k'} + \hat{\omega}_{k'} - \hat{\omega}_k^*]^{-1} 3C_{k,k'}^2 \langle \Phi^2 \rangle \langle \Phi^2 \rangle$, or

$$\langle \Phi^2 \rangle \sim \frac{\gamma}{3\tau_{k,k'}C_{k,k'}^2}.$$
(2.41)

Energy transfer scales with $\tau_{k,k'}$ and fluctuation levels inversely with $\tau_{k,k'}$. This is true in general for saturation by quadratic nonlinearities [23, 55].

The nonlinear component of $\hat{\omega}_k$ is derived in closure theory [23] and is also measurable in simulation. Such measurements are based on the Fourier transform of the autocorrelation function of Φ at a given wavevector [26], or from the response to a small perturbation [27]. A common assumption is that the Fourier-transformed autocorrelation function can be fitted by a Lorenzian whose peak is the real frequency $\Re e[\hat{\omega}_k]$ and whose width is $\Im [\hat{\omega}_k]$. In principle, other fluid moments or even eigenmode amplitudes could be used instead of Φ for calculating the autocorrelation function. A $\tau_{k,k'}$ exists for every set of coupled eigenmodes, which makes properly incorporating the quantity even more difficult.

Fluxes and Quasilinear Weights

Ultimately, one of the major applications of gyrokinetic modeling is for predicting fluxes of particles and energy. This thesis focuses on electrostatic (due to the fluctuating Φ component) ion heat flux, given by $Q_i^{\text{es}} = \Re e[\int i k_y \Phi_k \int v^2 f_{ik}^* dv_{\parallel} d\mu dz]$. The $i k_y \Phi_k$ can be identified as the radial component of $\mathbf{v}_{\text{E}\times\text{B}}$. Like the energy, the flux is a quadratic quantity where the sign and magnitude depend on the complex phase of fluctuating fields.

Stable modes affect fluxes by modifying the cross phase between fluctuating quantities and by changing the saturated amplitudes of the turbulence, through their effects on energy injection/dissipation. In the eigenmode basis, the flux is given by $\sum_{l} \alpha_{l} |\beta_{l}^{2}| - \sum_{l} \sum_{m} \Re[d_{lm}\beta_{l}^{*}\beta_{m}]$, where α_{l} gives flux per mode amplitude and d_{lm} are cross terms. This is like energy injection but with different coefficients. Transport can be approximated without including the cross terms, which is referred to as a 'quasilinear' estimate, not to be confused with quasilinear mixing-length transport models referenced later. This approximation has been shown to not agree well with actual transport in some cases [28], indicating that the cross terms can be important for the true flux levels.

The net contribution of stable modes to transport can be measured in a very similar way to how their effect on energy injection is measured. The quantity

$$w_k = Q_i^{\rm es} / \langle \Phi^2 \rangle, \qquad (2.42)$$

where $\langle \Phi^2 \rangle$ is the field line integrated potential, is called the quasilinear weight [32]. In it, the amplitudes are normalized out of the flux, which produces a flux-per-mode-amplitude, when evaluated for an eigenmode (w_k^{lin}) , or a flux-per-turbulent-amplitude when evaluated in the turbulence (w_k^{nl}) . When the quasilinear weight is lower in the turbulence than linearly, it is a sign that stable modes are reducing transport, either by diluting the effect of the unstable
mode and contributing to $\langle \Phi^2 \rangle$ or directly through an associated pinch effect (inward flux component) which reduces $Q_i^{\rm es}$.

3.1 Instabilities and Saturation

As mentioned in Chapter 2, fusion plasmas suffer from a number of microinstabilities, which are classified into types according to their properties. This thesis focuses on the Ion-Temperature-Gradient-Driven instability (ITG) (see Refs. [29, 30]). The ITG mode is an ion-frequency, ballooning-parity¹ instability, which is common in fusion plasmas.

Without the nonlinearity, instabilities would grow to arbitrary amplitudes. In actuality, the nonlinearity causes the system to evolve into a quasi-stationary state where the time-averages of turbulent quantities converge. In the energy framework, the instabilities gain energy from the background gradients through the drive term in Eq. (2.25), which is then nonlinearly transferred to other wavevectors through Eq. (2.30), where energy can be transferred further or is removed by the drive or dissipation terms of the same Eq. (2.25). When evaluated over a long enough time-average, all of these effects-energy injection, transfer, and dissipation-must balance.

Saturation physics is both interesting in its own right and is useful. With no understanding of saturation, the linear physics of microinstabilities gives limited insight into what can be expected in the turbulent state. In contrast, even a rough understanding of saturation can be very useful. Quasilinear mixing-length models (to be discussed in Chapter 6) simply take turbulent amplitudes to scale as γ/k_{\perp}^2 and are relatively successful [31] despite being orders of magnitude computationally cheaper. The frequencies and transport crossphases can be calculated from linear information, which has been tested in gyrokinetics and found to not change much in the nonlinear state [32]. In this chapter, results of energetic analyses are presented for ITG.

3.2 ITG Saturation

Previous Work

Previous work has shown that ITG saturates through nonlinear coupling involving $k_y = 0$ modes, termed zonal-flows [22, 33, 34]. Zonal flows catalyze energy transfer to higher k_x ,

¹Ballooning parity and tearing parity mean symmetric and anti-symmetric about the outboard midplane, respectively.

both to the unstable eigenmode at the receiving wavevector and to stable eigenmodes which can dissipate energy. This is in contrast both to an earlier description of ITG saturation where the zonal flows directly served as a primary dissipation mechanism [35] and to typical hydrodynamic systems, where energy can be dissipated only after being transferred (in wavevector space) through a large, almost undamped inertial range. Technically, zonal flows are flows (due to $v_{E\times B}$ here) which are constant along a flux surface, and as such have constant Φ both in the y and the z directions. However, the distinction between effects due to the flux-surface-averaged flow and the distribution function at $k_y = 0$ is often not made, either computationally or experimentally.

There are several motivations from theory as to why zonal flows would be important in the turbulence. One approximation that is often made in gyrokinetic simulations is the adiabatic-electron approximation. Under this approximation, electrons are assumed to follow a Gibbs distribution in the potential generated by the ions, partially canceling out the ion contributions. When this approximation is made, the electric potential at a wavevector is given by the ion density multiplied by a constant factor. However, on flux surfaces, the electron contribution cannot cancel out the ion contribution at all, which greatly enhances the field strengths and thus the flows at $k_y = k_z = 0$ [36]. While the adiabatic response is only an approximation, this effect still strengthens zonal potentials when more realistic electron physics is included.

Zonal flows are also low frequency and almost undamped. The weak damping facilitates high amplitudes in the turbulence, and low frequency allows them to catalyze energy efficiently because of the relatively small frequency mismatch in $\tau_{k,k'} = -i[\omega_{k''} + \omega_{\rm ZF} - \omega_k^*]^{-1}$. Their role as an energy transfer catalyst can be inferred from $\tau_{k,k'}$ by permuting the interacting wavevectors in $\tau_{k,k'}$; when ω_k and $\omega_{k''}$ are for an unstable mode and a conjugate mode respectively, they cancel out in $\tau_{k,k'}$. If instead **k** is the zonal wavenumber in the triplet, the $\tau_{k,k'}$ will be much smaller, implying it should receive much less energy.

The last motivation comes from the coupling coefficients in the nonlinearity. There is a factor of $\mathbf{k} \times \mathbf{k}'$ in the nonlinearity, which prevents modes with parallel \mathbf{k}_{\perp} from coupling. Having the two wavevectors be at right angles, as with an unstable mode at $k_x = 0$ and a zonal flow at $k_y = 0$, maximizes the quantity.

Because energy catalyzation by zonal flows scales with zonal flow strength, the factors which set zonal flow amplitudes matter in saturation. The process which excites the zonal flows can be modeled as a secondary instability on top of a primary instability [37]. This framework lacks finite-amplitude effects due to the turbulence, such as a turbulent viscosity. Energy transfer into zonal flows is balanced by linear damping and nonlinear energy transfer out of zonal flows. As the zonal flow regulates the turbulence, zonal flow strength is also limited because the zonal flows cut off their own source of energy, like in predator prey dynamics [38]. In the following subsection, we measure energy injection and dissipation spectra, as well as nonlinear energy transfer in the turbulence. This includes measurements of the transfers which drive the zonal flows.

Given that zonal flows are important in ITG saturation, it is necessary to understand their linear behavior. Unfortunately, linear physics at $k_y = 0$ is not simple. If the distribution function for the zonal flow is initialized as a Maxwellian, \mathcal{L} rapidly changes it. The m = 0and m = 1 modes are coupled because, as the background field is stronger on the inboard side, the flow $v_{\rm E\times B} \propto B_0^{-1}$ is slower there, causing density to build up, which in turn causes currents that modify the electric field [39]. This oscillation is known as the Geodesic Acoustic Mode (GAM) and is damped by Landau damping. The poloidal flow is also reduced by polarization, which cancels out a fraction of it. This leaves only the collisionally damped Rosenbluth-Hinton (RH) residual poloidal flow given by $u_{\rm p} = (1 + 1.6q_0^2/\epsilon^{1/2})^{-1}u(t=0)$ [40, 41]. With typical values of safety factor q_0 and inverse aspect ratio ϵ , the fraction is on the order of 10%. This means that there is a relatively fast oscillation and flow reduction by polarization which weakens the flows by around 90%, and a longer time-scale collisional damping. However, this theory assumes a zonal flow driven by an impulse which is constant in the parallel direction and Maxwellian in velocity space, when the actual distribution function driven at $k_y = 0$ depends on the turbulence. Linear kinetic physics can even amplify the zonal flows; for example, if the driving impulse corresponds to the distribution function which occurs at the zero-crossing during the GAM oscillation, the driven structure would have no potential, yet would evolve to some finite one. It is also worth noting that this type of behavior does not directly correspond to the behavior of any individual eigenmodes and requires the nonorthogonality of eigenmodes with respect to energy. This can be inferred as each eigenmode has a fixed ratio of Φ^2 and f^2/F_0 energy components which (linearly) can only grow or decay exponentially, while the GAM oscillation (also entirely linear) changes energy back and forth between the two forms.

Energetic Description

The Cyclone Base Case (CBC) is a simplified benchmark case based on a shot from DIII-D [42]. All of the following graphs are for ITG turbulence with the modified CBC at $\omega_{Ti} = 7$ as

described in Appendix A. These measurements record a larger number of energy quantities and nonlinear interactions than previous investigations [22, 33, 34], but their merit is mostly to serve as a reference for comparison with material in later chapters. Simulation resolutions are larger than what is shown in the graphed area. Evaluation of the energy production terms is used here to guide later energy transfer measurements.

Figure 3.2 shows the growth rate spectrum in a) and time-averaged energy spectrum in b). The value for each pixel corresponds with that for the wavevector at the center of the pixel. The unstable wavevector region peaks at (0, 0.35) and is much broader in k_x at higher k_y . Most energy resides in the zonal wavevectors (0.086, 0) and (0.172, 0). On the plot, values at $k_y = 0$ are reduced by a factor of 10, so as to not wash out the nonzero k_y modes. For the $k_y \neq 0$ modes, energy is peaked at (0, 0.15) and is mostly confined within the region bounded by $k_y = 0.4$ and $k_x = 0.9$. The energy spectrum is biased towards lower k_y than the growth rates, and is not much broader than the spectrum of unstable wavevectors, which is a sign of dissipation at these scales.



Figure 3.1: a) Growth rate γ_k in wavevector space and b) energy spectrum E_k for ITG turbulence. Energies at $k_y = 0$ (marked with hatching) are plotted at $0.1 \times$ their actual values, so as to be able to visualize the $k_y \neq 0$ energies.

Energy is injected into the system by the drive term (see discussion of Eq. 2.25), which is plotted as a function of **k** in the left panel of Figure 3.2. The energy injection rate peaks around the same location as the peak in turbulent ($k_y \neq 0$) energy, albeit with narrower extent. There is no contribution from the drive term at $k_y = 0$ because it contains a factor of k_y . This plot makes no distinction between the unstable and stable modes, i.e. it sums over the positive contributions of the unstable mode and the contributions of the stable modes, which could be of either sign.

The right panel of Figure 3.2 shows the spectrum of energy dissipation, which also has contributions from both the unstable eigenmode and stable modes. The $k_x = 0.172$ zonal flow dissipates the most, but the total dissipation of the $k_y \neq 0$ modes is much greater. Energy dissipation is much more diffuse than the energy injection.



Figure 3.2: Energy a) injection and b) dissipation spectrum by the drive and dissipation terms, respectively, as a function of (k_x, k_y) .

The sum of the nonconservative terms are balanced by the nonlinearity (Eq. 2.30), which is plotted in Figure 3.3. This sums over all \mathbf{k}' that couple to the mode at \mathbf{k} . The plot shows a region of net nonlinear energy transfer out of the range of energy injection, and transfer of energy into zonal modes. Because transfer is not resolved by \mathbf{k}' or eigenmode, the actual path of energy cannot be inferred. Stable mode effects can be seen, however, because part of the region above $k_y = 0.4$ has an unstable eigenmode but net negative energy production, which is only possible with stable eigenmode excitation.



Figure 3.3: Spectrum of the contributions to $\partial \mathbf{E}_k / \partial t$ due to the nonlinear term $T_{k,k'}$, summed over $\mathbf{k'}$.

The effects of stable modes on energy come from a combination of their effects on the drive and dissipative terms. The nonconservative terms in Eq. (2.27) can be broken down into these parts, which will sum to the growth rate in the linear system or the effective growth rate in the nonlinear one. Figure 3.4 shows these components. In red and black are the drive and dissipative rates for the ITG eigenmode respectively, showing that regardless of k_y , the reduction in growth rates due to dissipation is small. The quantities in the nonlinear state, which include the effect of stable modes, are in magenta and blue for the drive and dissipative terms respectively. The effect of stable modes on each term is roughly equally responsible for the reduction in γ_{eff} compared to γ . It is likely that the ratio depends on the respective strengths of the driving gradients and the collisionality, which could be investigated in future work.



Figure 3.4: Components of γ and γ_{eff} as a function of k_y . red: $\gamma_{\text{drive}}^{\text{lin}}$, black: $\gamma_{\text{diss}}^{\text{lin}}$, magenta: $\gamma_{\text{drive}}^{\text{nl}}$, blue: $\gamma_{\text{diss}}^{\text{nl}}$. See Eq. 2.29 for details.

Nonlinear energy transfer depends on both \mathbf{k} and \mathbf{k}' , which makes it four dimensional and presents issues with visualization. A solution is to specify one mode in the triplet and examine $T_{k,k'}$ over the plane of the unspecified wavevector. This is demonstrated schematically in Figure 3.5. Gridpoints correspond to (k_x, k_y) pairs. In this case, \mathbf{k}' is chosen to be (0, 4). In the following plots, \mathbf{k}' will usually be a wavevector with an unstable eigenmode and net energy production. Energy transfer from that mode can be plotted as a function of (k_x, k_y) , where the color on the graph corresponds to energy transfer to that wavevector. When represented this way, transfer can be split in all of the ways described in Section 2.3.

Figure 3.6 shows a plot of this type for ITG turbulence, from a mode near the peak in energy injection ($\mathbf{k}' = (0.0, 0.1)$, marked in grey). The left plot shows energy transfer at a representative instant, while the right one shows time averaged energy transfer, which requires ~ $1000R/c_s$ to converge. Only the first quadrant in (k_x, k_y) is shown, because with \hat{s} - α geometry there is symmetry between positive and negative k_x . Most energy transfer is due to zonal couplings (given by wavevectors at the same k_y as k'_y) in either case. However, for the instantaneous plot, energy transfer magnitudes are much larger than those of the average. The sign of the strongest transfer also reverses between the two plots. That they do not match up at a particular instant is reasonable, given that energy transfer scales with the product of the phases of the three interacting modes, which rotate in the complex plane at approximately their linear frequencies. While this is not visible on the plot, the ratio of nonzonal energy transfers to zonal transfers decreases significantly in the time average. The preference for zonal energy transfer on the time average relates to the $\tau_{k,k'}$ factor discussed in Section 3.2. Zonal flows act as catalysts as they receive very little of the energy transferred to higher k_x . What we refer to as catalyzation is also occasionally called scattering.



Figure 3.5: Energy transfer $T_{k,k'}$ to mode **k** due to **k'** coupling with **k''** can be plotted as a function of **k** (red circle). For the example vectors: $\mathbf{k} = (4, 5), -\mathbf{k'} = (0, 4)$, and $\mathbf{k''} = (4, 1)$. Because every **k** is another triplet's **k''**, plots of this form show all of the couplings to **k'** and the sum gives $-\partial \mathbf{E}_{k'}/\partial t|_{\mathrm{NL}}$.



Figure 3.6: a) Instantaneous $(T_{k,k'})$ and b) time-averaged $(\langle T_{k,k'} \rangle)$ energy transfer due to coupling with $\mathbf{k}' = (0, 0.1)$.

Transfers to the higher- k_y wavevectors (0, 0.2) and (0, 0.4) are shown in Figure 3.7 left and right panels respectively. Again, almost all energy transfer is due to the $k_x = 0.081$ zonal flow, and energy transfer is appropriately lower to balance the lower rate of energy injection at these wavevectors.



Figure 3.7: $\langle T_{k,k'} \rangle$ due to coupling with a) (0,0.2) and b) (0,0.4).

Almost all energy transfer is zonal-catalyzed, so the vast majority of energy transfer can be tracked by specifying \mathbf{k}' to be the zonal mode responsible for the most energy transfer. As zonal flows have $k_y = 0$, this is a cascade to higher k_x at the same k_y . Figure 3.8 shows schematically how the cascade can be measured from $T_{k,k'}$, with arrows representing energy transfers and boxes representing modes. Transfer to the unstable mode at higher wavenumber is shown with the red arrows, while transfer to the stable modes are shown with blue ones. The transfers which drive the zonal flows are shown in green.



Figure 3.8: Schematic of nonlinear energy transfer catalyzed by zonal flows. In this case, $\mathbf{k}' = (-k_{\rm ZF}, 0)$, so energy is transferred from (k_x, k_y) to $(k_x + k_{\rm ZF}, k_y)$, forming chains to higher- k_x . This is the k_x cascade, which can be split between energy transfer to unstable (red arrows) and stable (blue arrows). Energy transfers which drive the zonal flows can also be visualized as a function of (k_x, k_y) (green arrows). The transfer represented by an arrow is plotted at the graph point given by the left wavevector in the triplet. For example, a transfer from the top left triplet would be plotted at $(0, k_{ymin})$.



Figure 3.9: The k_x -cascade, split into energy transfer to the higher- k_x a) unstable mode $T_{k,k'}^{u}$ and b) stable modes $T_{k,k'}^{s}$. This is for $k_{\text{ZF}}\rho_{\text{s}} = 0.04$.

The k_x -cascade, split between unstable and stable as in Eq. (2.31), is shown in Figure 3.9. The k_x -cascade is a chain of energy transfers to higher k_x , starting at $(0, k_y)$, each link separated by $\Delta k_x = k_{\rm ZF}$. Transfer to stable modes is comparable to that to unstable modes, and generally peaked further down the cascade (i.e. at higher k_x). Energy transferred to the stable modes can then be dissipated, with stable modes acting as an energy sink in the unstable wavevector region. There is a high k_x region of zero amplitude on the left plot. For these wavevectors, there is no instability and thus no transfer to the unstable mode.

Figure 3.10 shows the same cascade, except with energy transfer split into the entropy-like component (f^2/F_0) in a) and the field component (Φ^2) , in b) as in Eq. (2.33). Transfer of the entropy-like energy is of similar magnitude and peaks at higher k_x than the field energy. Associated with any given mode structure, there is a specific ratio of these energies and their injection/dissipation rates. Consequently, these quantities aid in obtaining a better understanding of stable mode physics, by measuring the discrepency between the actual ratio of energy components and the ratio given by the unstable mode alone. Also, as they are seperately nonlinearly conserved, an unstable eigenmode with a higher fraction of field energy could drive zonal flows more efficiently, as there is more Φ^2 to go to the zonal flows. This could be tested with a secondary instability analysis over a parameter scan to see if the zonal flow growth rate correlates with $\Phi^2/(f^2/F_0)$ of the unstable mode.



Figure 3.10: The k_x -cascade, split into a) transfer of f^2/F_0 energy $T^g_{k,k'}$ and b) transfer of Φ^2 energy $T^{\chi}_{k,k'}$ to the higher- k_x wavevector.

Figure 3.11 shows transfer of the two energy terms to the zonal wavevector (0, 0.08). This corresponds to the green arrows in Figure 3.8. Transfer of the entropy-like term to the zonal mode is around 5 times as large as transfer of Φ^2 . There is a region of wavevectors around (0.3, 0.2) which transfers entropy into the zonal mode, while the region around (0.0, 0.3) transfers energy out. The net effect is a weaker total entropy transfer into the zonal wavevector, which is balanced by the hyperdissipation in this case. Transfer of Φ^2 into the zonal mode is almost all positive and peaked around the same wavevectors. As energy conserving linear terms can convert these quantities into each other (as in GAM decay), it is possible that the f^2/F_0 term is a contributor to zonal flow strength. This could be tested by evaluating $\Phi[\partial g/\partial t]$ for the linear terms at zonal wavenumbers, to see if the flow dissipation by energy conserving terms is as high as would be expected due to GAM decay.



Figure 3.11: The energy transfers to the zonal wavevector of a) g^2/F_0 energy $T_{k,k'}^g$ and b) transfer of Φ^2 energy $T_{k,k'}^{\chi}$. The Φ^2 term is responsible for driving the zonal flow.

Figure 3.12 sums $\partial E/\partial t|_{\text{N.C.}}$ over k_x and plots it as a function of k_y . Energy transfer balances $\partial E/\partial t|_{\text{N.C.}}$, implying that the integral over k_y in Fig. 3.12 sums to zero, and that energy moves from regions where $\partial E/\partial t|_{\text{N.C.}} > 0$ to regions where $\partial E/\partial t|_{\text{N.C.}} < 0$. For reference, the linear growth rate evaluated at $k_x = 0$ is included in the figure. These quantities have different units and absolute magnitudes should not be compared. Energy spectrum peak is downshifted compared to the growth rate spectrum, which has been noted previously [43]. From this, one might intuitively assume that energy follows an inverse cascade to smaller toroidal wavenumber. However, we find that the region around $k_y = 0.15$ (where the fluctuation spectrum peaks) does not receive energy via transfer from higher k_y , i.e., from where the linear growth rate peaks. Rather, it exports energy to higher k_y . This indicates that the downshift is not caused by energy transfer to those wavenumbers. Non-zonal transfers are always much smaller than the zonal-wavenumber-mediated forward k_x cascade.



Figure 3.12: (Color online) Energy production/dissipation (black crosses), summed over k_x as a function of k_y . Growth rates at $k_x = 0$ (red circles) are shown for comparison. The modes around $k_y = 0.2$ inject net energy into the turbulence, while above $k_y \rho_s = 0.3$ net dissipation is observed.

In summary, the previous graphs reveal the ranges of energy injection and dissipation in ITG turbulence, as well as the energy transfers which saturate the instability. Novel here is the splitting of energy transfer into entropy and field components, which gives clues into the process which regulates the zonal flow strength.

4.1 Motivation

In ITG turbulence, the critical temperature gradient for appreciable heat transport is higher than the critical gradient for instability. For a standard benchmark case, known as the Cyclone Base Case (CBC), this increases the threshold for transport and turbulence by around 50%. Figure 4.1 shows flux and the growth rate as a function of ion temperature gradient $\omega_{Ti} = -(R_0/T_{i0})(dT_{i0}/dx)$ for the CBC, showing the horizontal offset between the two curves. The linear critical gradient is around $\omega_{Ti} = 4.75$, while the nonlinear critical gradient (NLCG) is around $\omega_{Ti} = 6.75$, indicated with the dashed-dotted and dashed grey lines, respectively. The term *Dimits shift* refers to the upshift [42, 44]. The range of normalized temperature gradients with no or very low transport but ITG instability will be referred to here as the *Dimits regime*. Nonlinear simulations starting with low amplitude initial conditions experience a period of exponential growth before the nonlinearity is strong enough to affect the dynamics. In the Dimits parameter regime, zonal modes (modes with $k_y = 0$ develop, and subsequently the turbulence (modes with $k_y \neq 0$) decays to low or zero amplitudes. This can be seen in Figure 4.2, which shows transport (only caused by $k_y > 0$ modes) and zonal flow strength over time at ion temperature gradients below (at $\omega_{Ti} = 5.5$), around (at $\omega_{Ti} = 6.5$), and above (at $\omega_{Ti} = 7.0$) the NLCG. For the $\omega_{Ti} = 6.5$ case, a temporary increase in flux is recorded around t = 1300, which decays shortly thereafter (not shown in the plot).



Figure 4.1: Growth rate γ of the most unstable mode (green) and flux Q_i^{es} (blue) vs. ω_{Ti} for the CBC parameter set. Flux is nonzero in the Dimits regime. Vertical lines denote the linear (dashed-dotted) and nonlinear (dashed) critical gradients.



Figure 4.2: Time trace of flux Q_i^{es} (solid lines) and zonal amplitudes Φ_{ZF} (dashed lines) in the Dimits regime ($\omega_{Ti} = 5.5$, blue), around the NLCG ($\omega_{Ti} = 6.5$, orange), and above it ($\omega_{Ti} = 7$, green).

As an example of a system with a linear instability and (almost) no turbulence, the Dimits shift is interesting. It also poses a problem for reduced transport modeling, such as quasilinear models (the subject of Chapter 6), because they predict transport to scale with growth rates, which does not account for the horizontal offset in critical gradient between the two curves. An accurate method of predicting the Dimits shift could also be used to optimize plasma parameters for better confinement.

Other gyrokinetic systems have upshifts between the critical gradients for transport and linear instability, too. There is a reversed shear NSTX parameter set that is strongly unstable to Electron-Temperature-Gradient-driven (ETG) instability with very low transport [45]. Simulations of density-gradient-driven Trapped Electron Mode turbulence (TEM) in MST without magnetic perturbations (which represent the tearing modes in the experiment) have around a four-fold upshift [46]. Both of these are also turbulence types with strong zonal flows, though zonal flows are not ubiquitous in ETG [36] or TEM turbulence [47]. For ETG turbulence, zonal flows are noted in simulations with reversed shear, as in the NSTX case [22], and for TEM turbulence, where zonal flows are strong in the density gradient driven case (as in MST). The strength of the upshift in the MST case is partly due to the very large RH residual, the physics of which is described in Subsection 3.2.

What *strong* zonal flows means in sentences like the one above is usually not quantitatively defined. Figure 4.3 shows contour plots at representative times for ITG turbulence below and above the NLCG, as well as ETG turbulence, which has much weaker zonal flows in general. This is meant to be illustrative of the differences in potential structures at different levels of zonal flow strength compared to the turbulence. Zonal flows (potential structures constant in the Y direction) are visually evident in both ITG cases.



Figure 4.3: Field line averaged $\Phi(x, y)$ at a representative time for an ETG case (left), an ITG case in the turbulent regime (middle), and an ITG case in the Dimits regime (right). Zonal flows, given by $\langle \partial \Phi / \partial x \rangle$, where $\langle \cdot \rangle$ is the flux surface average are very visually evident in the ITG cases.

As an aside, there are actually systems with critical gradient downshifts as well. This is referred to as subcritical turbulence or nonlinear instability and can exist because the nonlinearity sets a phase relation between fields which injects energy into the turbulence [21], even when no eigenmode would individually inject energy. This requires that the eigenmodes be nonorthogonal (which is usually the case). A possible cause for the Dimits shift could be from the opposite effect, where the nonlinearity holds the phase relation to be one with only marginal energy production, if at all. This possibility is examined later in this chapter and found not to be the case.

To determine the cause of or develop a method to predict the Dimits shift, a simplified model with the same behavior is desirable. Fluid systems have been designed to have critical gradient upshifts, for a gyrofluid ITG model [48], a 1-field ITG fluid model [49], and a TEM fluid model [50], but a detailed comparison has not been made with gyrokinetics. Reduced models offer more promise to reveal the physics behind the upshift, but detailed comparisons must be made to rule out false explanations.

This chapter begins with a review other gyrokinetic investigations of the Dimits shift. Section 4.3 describes the similarities and differences between ITG saturation in the Dimits regime, compared to above the NLCG, using data gathered from a series of gyrokinetic simulations. Results of investigations into several plausible causes of the shift are reported, none of which ultimately explains it. Because zonal modes are critical to the Dimits shift, we also show the results of temperature gradient scans which add a dissipation only on the $k_y = 0$ modes. This isolates the effect of dissipation on the zonal modes, compared to collisions, which necessarily affect the instability as well.

4.2 Review of Other Dimits Shift Investigations

One of the first papers to discuss the Dimits shift (Ref. [42]) was a benchmark comparison of ITG turbulence between several gyrokinetic and gyrofluid codes. It noted a NLCG above the linear critical gradient, with simulations showing strong zonal flows before the transition out of the Dimits regime. This was observed in both gyrokinetic and gyrofluid simulations. These runs all used adiabatic electrons and no collisions. The following is an excerpt:

"The RH components of the zonal flows are linearly undamped except by collisions. The fact that a nonzero χ_i is observed in these collisionless gyrokinetic simulations for $R/L_T > R/L_{\text{Teff}}$ is an indication that nonlinear damping of the RH zonal flows by turbulent viscosity is able to balance the nonlinear drive. One might expect that the turbulent viscosity would increase as R/L_T increases, so that the RH zonal flows would be unimportant relative to the other components of zonal flows when the turbulence is sufficiently strong that the turbulant damping rate of the RH components becomes comparable to the damping rate of the GAM..." [42]

Linear $k_y = 0$ physics, including the GAM and the RH residual are discussed in Section 3.2. Shortly summarized: starting with a Maxwellian impulse, around 90% of the zonal flow is damped on short timescales through energy conserving kinetic effects, leaving an only collisionally damped RH residual flow. Given the weak zonal flow drives in the Dimits regime, one would expect the residual to play the dominant role in zonal flow physics.

In the first two sentences of the excerpt, the authors infer from the nonzero transport that there must be some limit on the zonal flow (residual) strengths, as otherwise they would continue growing until they quench the instability. For the zonal flow to only grow to some limited amplitude, there must be a process which removes energy from it, of which nonlinear energy transfer out (turbulent viscosity/nonlinear damping) is the only possibility in a collisionless simulation. One would also reasonably expect the energy transfer out of zonal flow residuals to grow with turbulent amplitudes, so the GAM component may be more important at those higher amplitudes. However, it is not clear that nonlinear GAM damping would not also increase with turbulent amplitudes.

This reasoning motivates our attention into how the nonlinear energy transfers involving zonal flows change above and below the NLCG, as well as our investigation into the fraction of the turbulent $k_y = 0$ distribution function is described by the residual flow. As a note, some dissipation is almost always included in gyrokinetic simulations, and also we measure the ratio of energy dissipated to nonlinear energy transfer out of the zonal modes. From this, we would expect the nonlinear transfer out of flows to increase until some critical threshold is reached, setting the NLCG.

Reference [42] also noted nonzero transport in the Dimits regime. This affects the dynamics because the turbulence can sustain the zonal flows; if the turbulence died off completely the zonal flows would decay until insufficient to quench the instability, at which point the turbulence would grow and drive the zonal flows again, as in predator prey cycles.

The Dimits shift has been investigated in more realistic plasmas, with collisions and non-adiabatic electrons [44]. Like in the survey in Ref. [42], the Dimits regime was found to have finite transport. The Dimits shift (measured as $\Delta \omega_{Ti}$) for these simulations was independent of collisions. Collisions have also been shown to increase transport in other ITG cases [51] because they damp the zonal flows. Reference [52] argued nonphysical numerical dissipation can be expected to be important at the collisionalities and resolutions commonly used for simulations in the Dimits regime. While simulations at high ω_{Ti} are insensitive to numerical dissipation, it was found that the size of the Dimits shift was strongly affected by dissipation and that heat flux was a discontinuous function of temperature gradient. Again, nonzero heat flux was found below in the Dimits regime, but because it scaled with the artificial dissipation, the actual value was ruled nonphysical. Also noted was that it can take a very long time for a simulation to go into the Dimits regime, appearing to dither between a turbulent state and a flow dominated state. The end of the orange curve in Figure 4.2 shows a bit of this. It is suggested that the Dimits regime ends due to some mechanism which limits the strength of long wavelength zonal flows; the turbulent viscocity described in Ref. [42] could be this mechanism.

More recently, Ref. [53] showed that the discontinuity persists at collisionalities relevant for modern devices. With these the collisionalities, the Dimits regime was shorter in ω_{Ti} . The authors speculate that the weak turbulence in Dimits regime drives zonal flows, but the paper lacks measurements of Reynolds stress or energy transfer, and that the secondary instability driving long wavelength zonal flows saturates through a mechanism involving the turbulent intensity with comparable timescales to the reduction to the RH residual.

Zonal flows and zonal temperature gradients were found to be stable with realistic strengths from simulations around the NLCG [54]. This implies that any energy transfer out of zonal flows has to be analyzed as a finite-amplitude effect.

In summary, previous investigations of the Dimits regime have shown that it has nonzero transport. The actual horizontal offset in critical gradients has strong dependence on details of geometry and dissipation mechanisms. Energy transfer out of the zonal flows has been repeatedly suggested as a limit on zonal flow strengths and a cause for the transition to stronger turbulence, however this cannot be a secondary instability on the flow itself.

4.3 Turbulence in the Dimits Regime

In this section we investigate the turbulence below the NLCG in an attempt to find a cause for the low turbulent amplitudes in the Dimits regime. The following results are for the modified CBC below the critical gradient at $\omega_{Ti} = 5.5$, as described in Appendix A. Scans over ω_{Ti} are presented at the end of this section.



Figure 4.4: Comparison between the ballooning structure at $\omega_{Ti} = 7$ (orange) and $\omega_{Ti} = 4.5$ (blue) for $k_y = 0.15$.

The growth rate spectrum is shown in the left panel of Figure 4.5. Compared with Fig. 3.1, the growth rate has a narrower extent in **k** and a maximum value about half that at $\omega_{Ti} = 7$. Unstable mode structure, shown in Figure 4.4, does not change much with ω_{Ti} and does not appear to have a discontinuity at the NLCG. It is typical to approximate transport as proportional to γ/k_{\perp}^2 , which would predict a reduction of turbulent amplitudes by a factor of two. Turbulent energy in the Dimits regime, shown in the right panel of Figure 4.5, is around a factor of 20 lower than the energy above the NLCG, while the zonal energies are roughly comparable. Note that the reduction of plotted zonal amplitudes on this plot is a factor of 100, not 10 used in Figure 3.2. Other than the lower growth rate and much weaker turbulence, the system in the Dimits regime is roughly similar to that below.



Figure 4.5: Growth rate and energy bispectrum in the Dimits regime. Energies at $k_y = 0$ are plotted at $0.01 \times$ their actual values.

It is plausible that the Dimits regime is due to reduced energy production because of stable eigenmode effects. The sum of the nonconservative terms, which are balanced by the nonlinearity, is shown in Figure 4.6. The regions of net energy injection and dissipation are similar above and below the NLCG. While the energy injection rate is much less than that above the NLCG, this cannot be taken as the cause of the transition, as it can be a product of the difference in amplitudes rather than the cause. Later in this section, we find that reduced energy injection because of stable modes cannot explain the Dimits shift.



Figure 4.6: Nonconservative energy terms $\partial E_k / \partial t \Big|_{NC}$ in the Dimits regime.

Energy transfer from a wavevector with instability to other wavevectors (as in Fig. 3.5) is

shown in Figure 4.7. The left panel is energy transfer from (0, 0.2) and the right panel is transfer from (0, 0.4). In each case, almost all energy transfer is due to the lowest- k_x zonal flow. This is very much like saturation above the NLCG, except scaled to the lower energy injection rate and with a longer wavelength zonal flow.



Figure 4.7: Energy transfer from the unstable wavevector at the black rectangle for a) (0,0.2), b) (0.0,0.4), to (k_x, k_y) at $\omega_{Ti} = 5.5$.

Energy transfer, split into transfer to the unstable and stable eigenmodes, and catalyzed by the (0.086, 0) zonal flow, is shown in Figure 4.8. The corresponding plot above the NLCG is Figure 3.9. A higher fraction of energy transfer is to the stable mode.



Figure 4.8: Energy transfer to the higher k_x mode catalyzed by the $k_x = 0.086$ zonal mode in the Dimits regime, split into stable and unstable.

When split into entropy and field components, shown in Figure 4.9, energy transfer in the Dimits regime is also similar to that above. However, the ratio of entropy transfer to field energy transfer is much higher in the Dimits regime.



Figure 4.9: Energy transfer to the higher- k_x mode, catalyzed by the $k_x = 0.086$ zonal mode in the Dimits regime, split into entropy and field components.



Figure 4.10: Transfer of the f^2/F_0 and Φ^2 energy terms ($T^g_{k,k'}$ and $T^{\chi}_{k,k'}$, respectively) which drive the zonal mode at (0.172, 0) for the $\omega_{Ti} = 5.5$ case.

The energy transfers which excite the zonal flows (Figure 4.10) might reveal the cause of the much weaker turbulence in the Dimits regime. As in the previous transfer plots, when ignoring the scale, transfer to zonal entropy (f^2/F_0) and flow (Φ^2) looks very similar to the case above the NLCG (Figure 4.10). The ratios of the energy injection transfer quantities changed significantly between the Dimits regime and the turbulent parameter case. Table 4.1 shows the maximum energy injection rate, transfer of entropy to higher k_x , transfer of field energy to higher k_x , transfer of entropy to the $k_y = 0$ mode, and transfer of field energy to the zonal flow above and below the NLCG. Entropy transfer both to higher k_x and to the zonal mode is higher than transfer of field energy, and this is stronger above the NLCG.

Several of the references mentioned in the previous section postulated nonlinear energy transfer out of the zonal flows as the physics which limits their amplitude. In none of these cases was transfer of field energy (Φ^2) out of zonal flows appreciable. However, the linear terms at $k_y = 0$ can conservatively transform energy between the entropy and field terms and there is entropy transfer out of the zonal flows in all cases. Entropy transfer out increases with ω_{Ti} . So it is still possible that nonlinear energy transfer out of zonal flows limits their amplitude, but there must also be a linear conversion between the field and entropy terms of energy. This is a kinetic effect and would be difficult to capture accurately in a reduced model. It would be useful to record how energy transforms, which could be done by evaluating $\Phi[\partial g/\partial t]$ for the linear terms at $k_y = 0$. This is left for future work.

ω_{Ti}	$\frac{\partial \mathbf{E}}{\partial t} _{\mathrm{N.C.}}$	$T^g_{k,k'}$ (high- k_x)	$T_{k,k'}^{\chi}$ (high- k_x)	$T^g_{k,k'}$ (ZF)	$T_{k,k'}^{\chi}$ (ZF)
7.0	1.5	1.5	1	0.05	0.01
5.5	0.05	0.4	0.08	0.01	0.00015

Table 4.1: Various energy quantities below ($\omega_{Ti} = 5.5$) and above ($\omega_{Ti} = 7.0$) the NLCG. High- k_x refers to transfer to higher- k_x . As these quantites are all functions of **k**, the reported values are approximate. The default assumption is that they would all scale with $\partial E/\partial t|_{\text{N.C.}}$.

Figure 4.11 shows the ratio between entropy-like energy transfer out of the lowest two zonal wavenumbers to energy transfer into, as a function of ω_{Ti} . The remainder of the energy input must be balanced by linear dissipation in steady state. The ratio increases with ω_{Ti} , suggesting increased that nonlinear eddy damping is more important compared to linear damping as the nonlinear critical gradient is approached.



Figure 4.11: The ratio of total g^2/F_0 energy transferred out of the (0.086,0) (black) and (0.172,0) (red) zonal modes, to the energy transfer into.

Because there is very little energy transfer to the zonal flows in the Dimits regime, it could be expected that flows in it are mostly described by the RH residual. We did a series of zonal flow residual calculations to test this hypothesis, and to see how this changed above the NLCG. These zonal flow residual calculations are done by linearly evolving the distribution function at $k_y = 0$ with time. In Figure 4.12, the response to the standard GAM initial condition is provided for comparison in magenta, showing the $\approx 0.1 \times$ zonal flow residual with this geometry. The traces from nonlinear simulations are the average of 35 individual runs, with initial conditions taken from evenly spaced distribution functions over a time of $\approx 1000R/C_{\rm s}$. For all cases, the flow is almost entirely described by the residual. The case from above the NLCG had a much higher damping rate due to linear terms than the cases from below. This is possible because dissipation rates depend on the amount of fine structure in velocity space, which can change with the turbulence. Turbulent amplitudes are expected to scale with zonal flow damping rates [55], so the increased zonal flow damping above the NLCG likely enhances the turbulence once in the turbulent state.



Figure 4.12: Zonal flow residual calculations using the zonal distribution function from nonlinear simulations (black: $\omega_{Ti} = 4.5$, blue: $\omega_{Ti} = 6.0$, red: $\omega_{Ti} = 7.0$), compared to a standard ZF residual run (magenta).

The zonal mode damping rates also (in terms of energy) increase when the nonlinear critical gradient is passed. Figure 4.13 shows the effective damping rate, evaluated for the $k_y = 0$ modes above and below the NLCG. Except for a single point, the damping rate above the NLCG is always around a factor of two higher. This increase is lower than the change in flow damping rate seen in the previous zonal flow residual calculations.



Figure 4.13: Effective growth (damping) rate γ_{eff} evaluated for the zonal modes for $\omega_{Ti} = 5.0$ (blue) and $\omega_{Ti} = 9.0$ (orange).

While zonal flows are stronger relative to the turbulence in the Dimits regime, the zonal flow shearing rate increases by around a factor of 10 as the NLCG is crossed. The left panel of Figure 4.14 shows shearing rate as a function of ω_{Ti} with the NLCG marked with a dashed line. Given the slope past the NLCG, the intercept without a Dimits regime would be roughly at the linear critical gradient. It can be helpful to compare the shearing rate to the growth rate of the instability to measure the strengths of the two effects. When shearing is much greater than the instability's growth rate shear suppression can be expected to contribute to saturation. The right panel of Figure 4.14 shows the ratio $\omega_{E\times B}/\gamma$, which decreases to a minimum of 2 at $\omega_{Ti} = 6$. Past the NLCG, the value jumps to over 50. From the energy transfer measurements, energy transfer to higher k_x by zonal flows is always the saturation mechanism.



Figure 4.14: a) Zonal flow shearing rate $\omega_{E\times B} = \sum_{k_x} k_x^2 |\Phi_{ZF}|$ as a function of temperature gradient ω_{Ti} . b) Ratio of zonal flow shearing rate $\omega_{E\times B}$ to growth rate of the most unstable mode γ as a function of ω_{Ti} . The linear and nonlinear critical gradients are marked with the dashed-dotted and dotted lines, respectively.

The zonal-flow shearing rate is a sum over the entire spectrum of zonal flows. From Figure 4.15, below the NLCG there is almost no contribution of $\omega_{E\times B}$ from higher- k_x wavevectors. While not obvious from the plot, the higher ω_{Ti} cases are from simulations with a larger L_x and thus smaller $k_{x,\min}$. The spectrum changes past the NLCG, and the discontinuity in shearing rate can be entirely attributed to the excitation of higher- k_x wavevectors. The strength of the (0.086, 0) zonal potential is not monotonic and goes down as the temperature gradient is increased from $\omega_{Ti} = 8$ to $\omega_{Ti} = 9$. This could indicate some limit of low-wavevector zonal flow strengths as the cause, as suggested in the previous work section.



Figure 4.15: Zonal potential spectrum $\Phi_{ZF}(k_x)$ for temperature gradients ω_{Ti} between 5.5 and 9.0

As ω_{Ti} increases, the peak γ moves to higher k_y . At the same time, the peak in energy (ignoring the zonal energy) moves to lower k_y . This occurs monotonically as ω_{Ti} is increased from 4.5 to 11. Figure 4.16 plots both E and γ , for a parameter case below and above the NLCG, normalized so that the highest value for each is 1.



Figure 4.16: Growth rate spectrum $\gamma(k_y)$ (solid lines) and energy spectrum $E(k_y)$ (dashed lines) for $\omega_{Ti} = 5.5$ (blue) and $\omega_{Ti} = 10$ (green).

The effect of stable eigenmodes on energy injection can be found by comparing γ_{eff} (Eq. 2.27) to γ . In the Dimits regime (at $\omega_{Ti} = 5$), γ_{eff} very closely tracks γ , indicating that the energy injection rate is close to what it would be without any stable mode excitation.

Above the NLCG (at $\omega_{Ti} = 11$), γ_{eff} is lower than γ , especially at high k_y . Above $k_y = 0.5$, there is actually a negative γ_{eff} , despite the presense of the instability, indicating that stable eigenmodes cause this wavevector to dissipate energy on net. This rules out increased stable mode dissipation as a possible cause for the Dimits regime.



Figure 4.17: Growth rates γ (solid lines) and effective growth rates γ_{eff} (dashed lines) for temperature gradients $\omega_{Ti} = 5$ (blue) and $\omega_{Ti} = 11$ (green).

Outside of the Dimits regime, the ratio of energy dissipated by the turbulence and energy dissipated by the zonal modes holds to within several percent. This trend is also present in the electromagnetic runs discussed in Chapter 5. Below the NLCG, this does not hold and the zonal modes dissipate proportionally more energy. Figure 4.18 shows the sum of all energy dissipated by the $k_y \neq 0$ wavevectors as a function of energy dissipated by the $k_y = 0$ wavevectors, with a trendline for comparison. With this it is possible to relate energy dissipated by the $k_y \neq 0$ wavevectors, energy dissipated by those wavevectors, and energy dissipated by the zonal modes to within a few percent all throughout the investigated range about the NLCG.



Figure 4.18: Energy dissipation (through the term including collisions and hyperdissipation) by $k_y = 0$ modes vs. $k_y \neq 0$ modes, over the ω_{Ti} scan. The blue line is for reference.

Because none of the previous results explain the cause of the Dimits shift, we have repeated the scans with an artificial zonal flow dissipation. This is because zonal flow dissipation can shorten the Dimits regime, which could give indications of why the Dimits shift exists. We applied a second-order dissipation, just on the $k_y = 0$ distribution function, which dissipates not just the field energy but the entropy-like term as well.



Figure 4.19: Flux (left panel) and zonal flow shearing rate (right panel) for the standard Dimits shift scan (blue), as well as with two levels of zonal dissipation, low (red) and high (magenta).

Figure 4.19 shows flux and zonal flow shearing rate over a scan of ω_{Ti} with two values for zonal dissipation. Above the NLCG, these levels of zonal dissipation only increase flux by around 20%. With both levels, there is no discontinuity in flux as a function of ω_{Ti} . The

Dimits shift persists in both, but its extent is reduced to around one third of that in the simulations without artificial zonal dissipation. Zonal shearing rates are higher with the dissipation (below $\omega_{Ti} = 7$) because the Dimits shift is shorter, while above the nominal NLCG, the shearing rate is much lower when the dissipation is added. It is noteworthy that the fluxes are very similar here despite the factor of 5 difference in shearing rate.



Figure 4.20: Energy dissipation by $k_y = 0$ modes vs. $k_y \neq 0$ modes of two levels of dissipation (none: black, low: red, high: magenta).

The ratio of zonal to nonzonal energy dissipation is shown in Figure 4.20, and the zonal flows always dissipates a higher fraction of the energy with the artificial dissipation.

Dimits Regime Summary

The process of saturation above the NLCG is qualitatively very similar to that below. Energy injected by the instability is transferred to higher- k_x stable and unstable modes, with a small fraction going to the zonal flow. As in the references discussed in Section 4.2, there is weak turbulence sustaining the zonal flows throughout the Dimits regime. This also confirms reports of finer-scale zonal flow excitation above the NLCG [52].

Several new details of the Dimits regime have been described here. For one, there is energy transfer out of zonal modes. However, it was only in the entropy-like energy component and not in the field component. This transfer increases with ω_{Ti} . As energy can be converted between the forms, this could be the sign of a turbulent viscosity which limits the zonal flow amplitudes and ultimately causes the end of the Dimits regime, but the results presented here are not conclusive.

The effects of stable modes on energy production were actually much weaker in the Dimits regime than above the NLCG. This rules out an explanation of the Dimits regime as a reverse situation to subcritical turbulence, where the nonlinearity allows the system to be in a state which injects energy despite having no unstable eigenmodes.

We investigated the turbulent distribution function and flow at $k_y = 0$ and found that in all cases, the vast majority of the flow is residual. The case above the NLCG has a faster damping rate, which can only be due to either the excitation of finer structures in velocity space which are more collisionally damped, or transformation of flow into entropy-like energy.

Above the NLCG, energy dissipated by the zonal modes and the turbulence holds a very nearly fixed ratio. This is worthy of note, because it is rare that quantities maintain such close relationships.

Purely artificial $k_y = 0$ dissipation removed the discontinuity in flux and shearing rate at the nominal NLCG. Above the NLCG it had little ($\approx 20\%$) effect on the flux, but decreased zonal flow shearing rate by around a factor of 5. This implies that zonal shearing rate is not a good measure of the importance of zonal flows in saturation. Energy transfer by zonal flows is strongest for the lowest- k_x modes, while the factor of k_x^2 in shearing rate gives a stronger weight to the higher- k_x modes. The dissipation, being second order in k_x , affects the zonal modes important for saturation much less than those that contribute to the shearing rate, which causes a five-fold reduction in shearing rate, but only a slight increase in transport.

5.1 Introduction

All of the parameter cases used in Chapters 3 and 4 were idealized to have zero normalized plasma pressure:

$$\beta \equiv 8\pi n_{\rm e0} T_{\rm e0} / B_0^2 = 0 \tag{5.1}$$

In reality, all plasmas have nonzero β , and high β (around one or a few percent) is desirable for fusion energy because it increases the fusion yield. The subject of this chapter is the effect of β on ITG turbulence.

The fluctuating magnetic potential A_{\parallel} is zero at $\beta = 0$, so turbulence in that regime is electrostatic. When β is nonzero, the turbulence can affect the magnetic fields, so that regime is electromagnetic; the terms finite- β and electromagnetic are often used in interchangibly in gyrokinetics.

In gyrokinetics, plasma β affects a linear term directly and a number of terms indirectly through the fields. The pressure term in the gyrokinetic equation ($\sim \beta v_{\parallel}^2 \omega_p \partial_{x,y} g$) is the only linear term directly dependent on β . Linearly, increasing β reduces ITG growth rates¹ and, at high enough levels, destabilizes microtearing and kinetic ballooning modes [57, 58, 59]. The A_{\parallel} component of χ and g affects both the linear terms and the nonlinearity, and in the nonlinear case transport due to ITG is reduced significantly more than the reduction in γ . However, this could be a side effect of changes to linear physics rather than because of the change to the nonlinearity itself. In the specific parameter scan used for the following energy transfer measurements, transport was reduced by 95% while the growth rates were only reduced by 50%. As one generally expects transport to scale as γ/k^2 , we investigate a number of possible causes for this phenomenon.

This chapter begins with an investigation into how energy quantities change as β is increased. The picture of saturation is very similar regardless of β ; the instability is saturated by zonal flow mediated energy transfer to higher- k_x stable and unstable modes. To determine whether increased stable eigenmode effects at high β cause the transport reduction, we compare the quasilinear transport weight and the effective growth rate in the nonlinear state to that from the linear instability. While stable mode effects are prominent and increase

¹This is actually due to field line bending and is thus a consequence of A_{\parallel} instead of the pressure term; see Ref. [56] for linear electromagnetic ITG physics.
with β , they only increase $\approx 5\%$ across the β range, which is much too weak to explain the nonlinear transport reduction. We then report on the properties of the triplet correlation time² τ measured in the turbulence, noting that this time is effectively an energy transfer efficiency. A higher τ implies that lower secondary mode amplitudes are sufficient to transfer energy at the same rate, so that the instability will saturate at a lower amplitude. The Triplet correlation time increases with β by an appropriate amount to explain most of the nonlinear transport reduction. This is tested quantitatively in Chapter 6, which finds that a linear proxy for τ predicts 50 – 100% of the nonlinearly enhanced electromagnetic transport reduction across a wide variety of parameter cases.

5.2 Energy Measurements and the Saturated State

The following energy transfer measurements are from a modified CBC parameter scan over ω_{Ti} and β , as described in Appendix A. The runs use kinetic electrons, have a reduced density gradient and no electron temperature gradient, as compared to the CBC. The modified gradients are such that the only unstable mode throughout the parameter range is ITG and the gradients were specifically chosen for this property because the presence of multiple different instabilities could change the saturation process. At the end of this chapter there is a short discussion of saturation with multiple instabilities, but the subject is primarily left for future work.

Features of the linear growth rate spectrum may contribute to nonlinear electromagnetic stabilization. For example, growth rates at nonzero k_x -center³ may be more strongly stabilized by β than the ones at $k_x = 0$. Figure 5.1 shows the two dimensional growth rate spectrum for a low and high β case, showing nothing that could explain the transport reduction. The angled feature showing unstable eigenmodes at high k_x stems from the fact that those wavevectors are not linearly independent, so that the linearly connected mode repeats every $(6k_y/k_y^{\min}) k_x$ points. Energy spectra, shown in Figure 5.2, are also similar between the two cases, except the $k_y \neq 0$ energy levels are reduced significantly more than the $k_y = 0$ energy levels for $\beta = 0.75\%$. The turbulent energy levels are reduced similarly to the transport reduction, which shows that nonlinearly enhanced stabilization has to be due to changes in energy and saturation rather than just due to changes to transport.

²See the discussion around Eq. (2.37).

³See discussion of linearly coupled modes around Eq. (2.12).



Figure 5.1: Growth rate spectrum for a) $\beta = 0.01\%$ and b) $\beta = 0.75\%$ at $\omega_{Ti} = 8$.



Figure 5.2: Energy spectrum for a) $\beta = 0.01\%$ and b) $\beta = 0.75\%$ at $\omega_{Ti} = 8$. Zonal amplitudes on the right plot (hatched region) are reduced by $10 \times$ for visualization purposes.

The spectra of the nonconservative terms, shown in Figure 5.3, are also qualitatively similar to each other and to the nonadiabatic case. Dissipation at the $k_y = 0$ wavenumbers is weak compared to the turbulence in all of the adiabatic-electron cases (see Figures 3.2 and 4.6) and is even weaker here.



Figure 5.3: Spectrum of nonconservative energy terms for a) $\beta = 0.01\%$ and b) $\beta = 0.75\%$ at $\omega_{Ti} = 8$.

Figure 5.4 shows time-averaged energy transfers for two different wavevectors at each β value. Compared to the results in Chapter 3, zonal flow mediated energy transfer is not as dominant, especially at $\beta = 0.01\%$. However, this is only the case for the (0, 0.3) wavevector, which is well above (0, 0.15)-the wavevector responsible for the most flux. The zonal flows which transfer significant amounts of energy are typically below $k_x = 0.5$, which is a much broader spectrum than in the adiabatic electron cases where almost all of the energy transfer was by zonal flows under $k_x = 0.2$. The nonzonal energy transfer to (0, 0.3) is almost all into the mode for wavenumbers at lower k_y , and out of it for wavenumbers at higher k_y , which is a weak cascade to higher k_y . A k_y -cascade was also observed in Chapter 3, which discussed the downshift in k_y between the peak growth rate and the peak in turbulent amplitude. Nonzonal energy transfer is not expected to affect the turbulence strongly, as it is much weaker than zonal flow catalyzed energy transfer in the region responsible for most energy production and flux.



Figure 5.4: Energy transfer from the unstable wavevector at the grey rectangle ((0, 0.1) (top row) or (0, 0.3) (bottom row)) to (k_x, k_y) at $\beta = 0.01\%$ (left column) and $\beta = 0.75\%$ (right column) for the $\omega_{Ti} = 8$ case.

Figure 5.5 shows energy transfer from the wavenumber responsible for the most energy injection as a function of the zonal flow wavenumber, split into unstable and stable components. The (0,0.15) wavevector actually receives energy from the stable modes at (0.086, 0.15), which may be because those stable modes are driven by energy transfer through other nonlinear couplings or because the stable modes at those wavenumbers contribute to energy injection through nonorthogonality. Energy transfer to stable modes is higher for higher- k_x zonal flows. There is no transfer to the unstable component past $k_x \approx 0.25$ because that is the end of the region of instability.



Figure 5.5: Energy transfer to $(k_x, k_y) = (k_{\text{ZF}}, 0.15)$, from $\mathbf{k}' = (0, 0.15)$ and $\mathbf{k}'' = (k_{\text{ZF}}, 0)$, broken down into transfer to the unstable modes $T^{\text{u}}_{k,k'}$ (black circles) and $T^{\text{s}}_{k,k'}$ (red crosses). This is with $\beta = 0.5\%$.

The nonzero A_{\parallel} component is the only direct effect of β on the nonlinearity, and as such is a candidate cause for nonlinearly enhanced electromagnetic stabilization. Figure 5.6 shows energy transfer from (0,0.1), split into the Φ and A_{\parallel} components at low and high β , as in Eq. (2.35). In all cases, transfer by A_{\parallel} is much smaller than transfer by Φ , which is not unexpected as A_{\parallel} has a factor of β and the high- β case is still only at $\beta = 0.75\%$. The fraction of energy transfer by the A_{\parallel} component also grows with β . Its absolute value is actually higher in the low- β case, because the total energy transfer goes down more with β than the fraction goes up. Energy transfer by A_{\parallel} is likely to have very little effect on the saturation process given its low magnitude.



Figure 5.6: Energy transfer catalyzed by the Φ (top rpw) and A_{\parallel} (bottom row) components of \mathbf{k}' to $\mathbf{k} = (0, 0.1)$ at $\beta = 0.01\%$ (left column) and $\beta = 0.75\%$ (right column).

Because a broad spectrum of zonal flows is important for energy transfer at nonzero β , individual k_x -cascade plots show a smaller portion of transferred energy than in the adiabatic electron runs from the previous two Chapters. However, the features of energy transfer due to different zonal flows are very similar within the range of important zonal flows. Figure 5.7 shows energy transfer due to the two lowest- k_x zonal flow wavevectors. In either case, energy transfer is prominent around the range of energy injection shown in Figure 5.3, extending only a bit further in k_x . This is a sign of stable mode dissipation at these wavenumbers, as without dissipation there would be an inertial range of relatively constant energy transfer before a dissipation scale is reached.



Figure 5.7: Energy transfer catalyzed by the zonal mode (top: $\mathbf{k}' = (0.086, 0)$, bottom $\mathbf{k}' = (0.172, 0)$) to higher k_x at $\beta = 0.01\%$ (left column) and $\beta = 0.75\%$ (right column).

The spectrum of zonal flows changes with β , which brings attention to the nonlinear energy transfer driving zonal flows. Figure 5.8 shows transfer of entropy and field energy to the lowest k_x mode. In this case, field energy contains both Φ^2 and A_{\parallel}^2 components, but in all of these cases $E^{\Phi} \gg E^{A_{\parallel}}$. The transfers driving the flows occur in a smaller region of wavenumber space than in the adiabatic-electron cases (see Figure 4.10 for comparison), and unlike with adiabatic electrons, some interactions transfer field energy out of the zonal wavenumbers. At both β values, the ratio of entropy to field-like energy transfer is the same.



Figure 5.8: Energy transfer to the zonal mode $\mathbf{k} = (0.086, 0)$, split into transfer to the pressure component $\sim g^2/F_0$ (top row) and transfer to the field component $\sim g\chi$ (bottom row) at $\beta = 0.01\%$ (left column) and $\beta = 0.75\%$ (right column).

As in the adiabatic electron runs, the ratio of energy dissipated by the turbulence $(k_y \neq 0)$ to energy dissipated by the zonal wavenumbers $(k_y = 0)$ maintains a near-constant ratio, regardless of β and ω_{Ti} . Figure 5.9 plots these two quantities against each other for every case within the scan. The slope is different compared to the adiabatic-electron runs, but this is also possibly a result of the different values for numerical dissipation for these runs. This is not necessarily physical, but the quality of the fit is noteworthy.



Figure 5.9: Energy dissipated by the dissipation term for the $k_y \neq 0$ modes compared to energy dissipated by the $k_y = 0$ modes, across the entire β and ω_{Ti} range.

Overall, the saturation process is qualitatively the same in the low and high β cases. The changes described above cannot explain the reduction in transport with β .

Energy Transfer Scaling with Φ_{ZF}

Energy transfer due to coupling with a zonal flow has a factor of $k_{\rm ZF}\Phi_{\rm ZF}$ in it. We test the extent to which the other factors in energy transfer, such as τ , matter by comparing the spectrum of energy transfers to the zonal flow spectrum $k_{\rm ZF}\Phi_{\rm ZF}$. If those other factors do not depend on $k_{\rm ZF}$, the two spectra should match. To measure energy transfer due to a flow, we sum the transfers from the $(0, k_y)$ wavenumbers over $k_y \in [0.05, 0.6]$. Figure 5.10 plots the comparison between the values at low and high β . While features from the zonal flow spectrum are visible in the energy transfer spectrum, the match is not good. The energy transfer falls off faster with $k_{\rm ZF}$ than the $k_{\rm ZF}\Phi_{\rm ZF}$ spectrum, which may be related to τ , as the damping rate for those flows is higher, which would make τ less resonant.



Figure 5.10: Energy transfer as a function of the k_x of the catalyzing zonal flow (dashed lines) compared to $k_x \Phi_{\rm ZF}$ (solid lines) at $\beta = 0.01\%$ (black) and $\beta = 0.75\%$ (magenta). All spectra are normalized to be 1 at their peaks, as energy transfer also scales with energy injection rate, which changes between the two cases.

5.3 Stable Mode Excitation

Increased stable eigenmode excitation, leading to reduced energy injection and flux, is a plausible cause of nonlinear electromagnetic stabilization. The stable mode fraction alone does not measure the effects of stable modes on the turbulence, but a nonzero stable mode fraction is a requirement for stable modes to play a role in saturation. Figure 5.11 shows the time-averaged fraction of the distribution function described by the unstable eigenmode at (0, 0.2) and (0, 0.4), as in Eq. (2.23). The difference between this quantity and one is the fraction associated with stable eigenmodes. The stable mode fraction increases by around 10% over the β range, which is a much smaller percentage than the decrease in transport.



Figure 5.11: The average fraction of the turbulent distribution function described by the unstable eigenmode (see Eq. (2.23)) across the entire ω_{Ti} , β range. ($\beta = 0.01\%$: black, $\beta = 0.25\%$: red, $\beta = 0.5\%$: blue, $\beta = 0.75\%$: magenta).

Measuring the stable mode fraction directly requires access to the turbulent distribution function, which is usually not outputted from simulations because eigenmode decompositions are rarely used and because of the large amount of data involved. As a proxy, the potential structures in the turbulence are sometimes compared to those from the eigenmode [63]. This approach uses the same formula as in Eq. (2.23), except the g structure is replaced by the Φ ballooning structure. Figure 5.12 shows the unstable mode fraction as a function of k_y for the modified CBC, as well as a comparison between the inner product calculated with $g_j(\theta, v_{\parallel}, \mu)$ and with $\Phi(\theta)$. Because there exists a space of g structures that will all give the same Φ structure, the Φ metric can produce a larger fraction than actually described by g^{ev} . The Φ metric is not an upper bound on the g metric either, because the ions and electrons have opposite contributions in it. Like the g metric, the Φ metric is also bounded between 0 and 1. Figure 5.12, as well as the later plots of γ_{eff} and w_k , also contain results from parameter cases based on reconstructions of a AUG shot 29197 [75] and JET shot 75225 [73]. These are fully described in Appendix A.

The g and Φ metrics correspond roughly for the mCBC. The unstable mode fraction is around 70% at the wavevectors which contribute the most to flux, and decreases at higher k_y . For the AUG case, the unstable mode fraction is always close to 90%. For the JET case, the fraction starts at around 20% and increases to 60% at higher k_y . Between each of these cases, the stable mode fraction changes dramatically, implying that some variables other than β (such as collisionality or geometry) affect stable mode excitation. This is natural, as stable modes are excited by nonlinear interactions with the instability, and the structure of the eigenmode affects details of those interactions. It also suggests that quasilinear transport models may yield poor results when extrapolating fluxes across different devices. In each of these plots, the stable mode fraction increases around only 5% with β , which is small compared to the decrease in flux ($\approx 50\%$ in AUG and $\approx 95\%$ in JET). However, this does not measure which stable eigenmodes are excited, and it is entirely plausible (until γ_{eff} and w_k^{nl} are measured in the following subsections) that the eigenmodes which are excited at higher β are more dissipative or contribute more negatively to transport.



Figure 5.12: The unstable mode fraction (solid lines) and the value based on the Φ proxy (dashed lines) for low (black) and high (magenta) β as a function of k_y for the three cases: a) mCBC, b) AUG, c) JET.

Understanding stable mode excitation mechanisms can be useful to explain why the stable mode fraction changes, and to help predict changes in transport with parameters. Here, we sketch a linear proxy, which approximates the nonlinear coupling coefficients for energy transfer in the eigenmode basis, and compare to the energy transfer rates and stable mode proportions. Like τ , the coupling coefficient can be referred to as an efficiency; however, the two should not be confused, as this is an efficiency based on eigenmode structure, while τ is an efficiency due to frequency matching. Transfer of entropy-like energy ($\sim g^2/F_0$) to an eigenmode g_k^{ev} at $\mathbf{k} = (-k_{\text{ZF}}, k_y)$ due to the nonlinear interaction of the zonal flow $\Phi_{k'}$ at $\mathbf{k}' = (k_{\text{ZF}}, 0)$ and the distribution function described by the ITG mode $g_{k''}^{\text{ITG}}$ at $\mathbf{k}'' = (0, k_y)$ is:

$$T_{k,k'}^{\text{ev}} \sim \Re e \left[\int g_k^{\text{ev}*} \bar{\Phi}_{k'} g_{k''}^{\text{ITG}} F_0^{-1} dv_{\parallel} d\mu dz \right],$$
(5.2)

where g_k^{ev} could be an unstable or a stable mode structure. For the purposes here, we approximate $\bar{\Phi}_{k'}$ with $\Phi_{k'}$ and take it to be constant in z. The amplitude information can be factored out, producing:

$$T_{k,k'}^{\text{ev}} \sim \Re e \left[\left(\int \frac{g_k^{\text{ev*}}}{\|g_k^{\text{ev}}\|} \frac{g_{k''}^{\text{ITG}}}{\|g_{k''}^{\text{ITG}}\|} F_0^{-1} dv_{\parallel} d\mu dz \right) \frac{\Phi_{k'}}{|\Phi_{k'}|} \right] \|g_k^{\text{ev}}\| |\Phi_{k'}| \|g_{k''}^{\text{ITG}}\|,$$
(5.3)

where $\|\cdot\|$ is the energy norm. The real part can then be written in polar representation, with $M(g_k^{\text{ev}}, g_{k''}^{\text{ITG}})$ as its magnitude and $\theta_{k,k'}$ as the complex phase of the product:

$$T_{k,k'}^{\text{ev}} \sim M(g_k^{\text{ev*}}, g_{k''}^{\text{ITG}}) \cos(\theta_{k,k'}) \|g_k^{\text{ev}}\| \|g_{k''}^{\text{ITG}}\| |\Phi_{k'}|,$$
(5.4)

where

$$M(g_k^{\text{ev}}, g_{k''}^{\text{ITG}}) = \left| \int \frac{g_k^{\text{ev*}}}{\|g_k^{\text{ev}}\|} \frac{g_{k''}^{\text{ITG}}}{\|g_{k''}^{\text{ITG}}\|} F_0^{-1} dv_{\parallel} d\mu dz \right|.$$
(5.5)

Energy transfer to a specific eigenmode scales with $M(g_k^{\text{ev}}, g_{k''}^{\text{ITG}})$, which can be calculated from the linear modes alone. The coupling coefficient M can be calculated more correctly by including the field energy and the Bessel function from the gyroaverage. Because we lack specific stable modes of interest out of the ≈ 10000 present, we calculate the coefficient for transfer to the unstable mode at **k** and assume that the fraction of energy transfer to unstable modes compared to stable modes scales with it. Alternatively, a lower coefficient for coupling to unstable modes would predict more transfer to stable modes.

Figure 5.13 shows M as a function of zonal flow wavevector and β . This quantity starts near one at $k_x = 0.05$ and decreases to 60% at $k_x = 0.25$. It increases 5-10% with β . The prediction from this coupling coefficient agrees with the previous observation that higher- k_x zonal flows transfer a higher fraction of energy to stable modes and disagrees with the observed stable mode fraction as a function of β , which weakly increases in the nonlinear system.



Figure 5.13: Proxy for the nonlinear coupling coefficient between the unstable mode at $k_x = 0$ and $k_x = k_{\rm ZF}$ for the four β values. $\beta = 0.01\%$: black, $\beta = 0.25\%$: red, $\beta = 0.5\%$: blue, $\beta = 0.75\%$: magenta.

5.4 Stable Mode Effects on Energy

For a given stable-mode fraction, there are a wide range of possible energy injection rates, depending on which stable modes the fraction is comprised of. The net effects are also not just given by the sum of contributions by the individual eigenmodes, because the cross correlations between eigenmodes affect energy injection. As such, the easiest way to measure stable mode effects on energy is to directly measure energy injection in the turbulence by using γ_{eff} (see Eq. (2.27). Figure 5.14 shows γ_{eff} compared to the linear growth rate γ as a function of k_y at $k_x = 0$ for the mCBC, AUG, and JET cases. For the mCBC case, γ_{eff} tracks γ closely at low k_y , while there is not energy dissipation towards the end of the unstable range. This corresponds to the behavior of the stable mode fraction, which was low at low k_{y} and high at high k_y (see Fig. 5.12). For the AUG case, $\gamma_{\rm eff}$ closely tracks γ , which corresponds to the consistently low stable mode fraction in that case. In the JET case, which has a high stable mode fraction, γ_{eff} is always much lower than the linear γ . Overall, stable mode effects on energy are prominent and correspond to the stable mode fraction, which depends on parameter case. These quantities all depend on k_x as well as k_y , and Figure 5.15 shows the same comparison for the mCBC, but as a function of k_x at the k_y responsible for the most flux and energy injection. The range in k_x with energy injection is actually broader than the region of instability, showing that stable modes at these wavenumbers must be contributing to energy injection through nonorthogonality.



Figure 5.14: γ (solid lines) and γ_{eff} (dashed lines) as functions of k_y for three of the investigated cases. Symbols represent β values, with black pluses for low β and magenta circles for high β . Specifically, the values of $\beta/\%$ are a) 0.01,0.75 for the mCBC, b) 0.81,1.54 for AUG, and c) 0.001,1.75 for JET.



Figure 5.15: γ and γ_{eff} as functions of k_x for $k_y = 0.15$ in the mCBC.

5.5 Stable Mode Effects on Transport

Apart from stable mode effects on energy, stable modes can also affect transport directly through the cross-phase in Q_i^{es} . To measure the net effects of stable modes on transport, we compare the quasilinear weight (Eq. 2.42) in the turbulent case to that of the unstable eigenmode. These comparisons are shown in Figure 5.16 for the mCBC, AUG and JET cases. Linearly, β increases the quasilinear weight in the mCBC and AUG case, but not in the JET case. In both the mCBC and the JET case, stable mode effects reduce w_k relative to the linear value, however the reduction does not strengthen with β enough to explain the nonlinear transport reduction. In the AUG case, the quasilinear weight in the turbulence is around twice that from the linear eigenmode. This is unexpected and unresolved, but it may be related to the external $v_{\text{E\timesB}}$ in those simulations.



Figure 5.16: Quasilinear weights w_k^{lin} (solid lines) and w_k^{nl} (dashed lines) as functions of k_y for the investigated cases. Symbols represent which β value was used, with black plusses for low β and magenta circles for high β . Specifically, the values of $\beta/\%$ are a) 0.01, 0.75 for the mCBC b) 0.81, 1.54 for AUG, and 0.001, 1.75 for JET.

5.6 Triplet Correlation Time

None of the previously discussed stable mode effects can explain a 95% reduction in transport given a 50% reduction in growth rate. As discussed around Eq. (2.37), the triplet correlation time $\tau_{k,k'}$ acts as an energy transfer efficiency, and the saturated turbulent amplitudes can be expected to scale inversely with it. It is given by the difference in nonlinear complex frequencies of the interacting modes, which can be measured by taking the Fourier transform of the autocorrelation function of quantities in the turbulence [23]. We measure the nonlinear frequencies using the z-averaged electric potential in the turbulent state, but other moments or even eigenmode amplitudes could be used as well. Using Φ conflates the frequency effects of having multiple excited eigenmodes at a wavenumber (because velocity space information is lost) with the direct effect of the nonlinearity on the mode phase. Because GENE uses an adaptive timestep, we interpolate the real and imaginary parts of the potential time-series to have constant time spacing. Figure 5.17 shows the comparison between the interpolated potential and the actual value, which is acceptable because the timestep for outputting the field data is much shorter than the timescales of interest.



Figure 5.17: Comparison of the interpolated real part of Φ (black) at a wavenumber with the actual value (green).

The autocorrelation function of Φ and its Fourier transform are shown for (0,0.15) at $\beta = 0.01\%$ and $\omega_{Ti} = 8$ in Figure 5.18. These are each calculated using data from $\approx 1000 R/C_s$

time units, started after the simulation has reached a saturated state. The noise in the Fourier transform decreases with the time used for integration. The linear and nonlinear frequencies match well at this wavenumber, justifying using a linear proxy for frequencies in $\tau_{k,k'}$ at the wavenumbers around the peak in energy injection rate. The match gets progressively worse at higher **k**.



Figure 5.18: Left: the autocorrelation function of Φ at (0,0.15). Right: The Fourier transform of the autocorrelation function for Φ in the turbulent state. The linear frequency is marked with the orange dashed line. The least-squares Lorenzian fit is marked in red. The nonlinear frequencies are extracted from the Lorenzian parameters.

Figure 5.19 shows the nonlinear $\tau_{k,k'}$ values as a function of the coupled k_x from the (0, 0.15) wavevector. $\tau_{k,k'}$ is highest for interactions involving the lowest k_x zonal flows, and increases roughly a factor of 2 over the β range. As transport can be expected to scale inversely with τ , this would explain a large fraction of the difference between the growth rate and transport scalings. Other wavevectors (not shown) have similar dependence on β and k_x . The k_x dependence also might explain why energy transfer as a function of k_x decreases much faster than $k_x \Phi_{\rm ZF}$, as was seen in Figure 5.9. The real part of $\tau_{k,k'}$ is much larger than the imaginary part, such that it is almost equal to the absolute value. This implies $\tau_{k,k'}$ is primarily from the difference in the growth rates of nonlinearly interacting modes and not the frequencies.



Figure 5.19: Nonlinear triplet correlation time $\tau_{k,k'}$ for $\mathbf{k} = (0, 0.15)$ and $\mathbf{k'} = (-k_x, 0)$ for four β values in the modified CBC scan. Black: 0.01%, red: 0.25%, blue: 0.5%, magenta: 0.75%.

5.7 Multimode Effects

Stable modes are driven by energy received from the instability, so changes to the instability can affect which stable modes are excited. The AUG 29197 case has a microtearing (MT) instability with similar growth rates as the ITG (see Figure A.1). However, this mode has a much broader extent in the ballooning direction, so it may be expected to be weaker in the turbulence. The $\langle k_{\perp}^2 \rangle$ (Eq. (6.2))for these modes are roughly an order of magnitude larger than the ITG mode. Microtearing modes are orthogonal to ITG modes, as ITG has ballooning parity (symmetry about z = 0) while microtearing has tearing parity (antisymmetric about z = 0). As such, for both metrics, the fraction of the two modes can not add up to more than one. Figure 5.20 shows the proportion at $k_y = 0.4$ calculated using the Φ projection, showing that the MT mode always has a low fraction, and the two fractions are anticorrelated. This was only a cursory investigation, meant to provide justification for ignoring the MT contribution in Chapter 6. Understanding saturation in general when multiple types of instabilities coexist is an open problem.



Figure 5.20: The fraction of the potential structure described by the ITG (blue) and MT (red) as a function of time.

5.8 Chapter Summary

Electromagnetic effects strongly reduce turbulence and transport in typical ITG regimes, but do not qualitatively change the saturation process. The instability saturates by energy transfer, catalyzed by zonal flows, to higher- k_x stable and unstable modes. The largest difference in the turbulent physics at high β is that some nonlinear couplings transfer field energy out of the zonal flows, which did not occur in any of the adiabatic-electron simulations. This may be related to the nonzonal transition [61], which sets a critical β above which the instability does no longer saturates, or saturates at nonphysically high amplitudes, because of radial electron flow due to field line flutter eroding the zonal flows.

Stable mode amplitudes and effects increase slightly with β . This cannot be explained by the change to the coupling coefficient between the unstable mode at $k_x = 0$ and at nonzero k_x , but the coupling coefficient agreed with the stronger effect that the fraction of energy to stable modes was higher with higher- k_x zonal flows. There are three possible reasons for the discrepancy. Firstly, this analysis only evaluates the coefficient for transfer to the unstable mode, and evaluating the coupling coefficient for stable eigenmodes may show stronger energy transfer at higher β . Secondly, this used only an approximation of the coupling coefficient. Thirdly, this lacks frequency matching information, and it is possible that there is a stronger β dependent effect on τ . Stable mode effects also vary strongly between the parameter cases, implying other parameters must have a strong effect. This could be investigated in future work.

Ultimately, the strongest difference at high β was a much increased triplet correlation time, which can explain most of the transport reduction exceeding the linear stabilization of ITG.

6.1 Quasilinear Transport Models

Introduction

Reduced models are used for transport prediction because nonlinear simulations are too expensive for large, multidimensional parameter scans. The most commonly used reduced models are quasilinear mixing-length models, which predict transport from linear eigenmode properties and are calibrated to a single nonlinear simulation. In ITG turbulence, increasing β strongly reduces transport, an effect which quasilinear models greatly underpredict [58, 57, 62]. The discrepancy indicates that β has some strong effect on the saturation physics missing in quasilinear models. This is what originally motivated the investigations in Chapter 5. While stable mode excitation was found to be slightly higher at high $\beta ~(\approx 5\%)$, the strongest effect was an increase in the triplet correlation time $\tau_{k,k'}$ by almost a factor of two. From Eq. (2.41), turbulent amplitudes can be expected to scale inversely with $\tau_{k,k'}$, indicating that much of the transport reduction at high β can be explained by lower turbulent amplitudes due to more efficient energy transfer. This chapter addresses two ambiguities in quasilinear modeling with electromagnetic simulations, specifically: the evaluation of the effective wavenumber $\langle k_{\perp}^2 \rangle$, and the quasilinear weight normalization. These ambiguities are described before testing a standard quasilinear model modified to include $\tau_{k,k'}$ across six parameter cases, which are all described fully in Appendix A. The modification accounts for 50% - 100% of the enhanced stabilization in all parameters cases but one.

Quasilinear Model Definition

The model used here [63] writes flux in terms of a Fick's law ($Q_i^{\text{es}} = \chi_i^{\text{es}} \omega_{T_i}$), with diffusivity given by a mixing length rule γ/k^2 . Specifically, it is:

$$Q_{i}^{es} = \left(\mathcal{C} \sum_{k_{y}} \frac{w_{k}^{lin} \gamma_{k}}{|\tau_{k,k'}^{lin}| \langle k_{\perp}^{2} \rangle} \right) \omega_{Ti}.$$

$$(6.1)$$

The model has a calibration constant C in front of a sum of contributions of eigenmodes at wavevectors $\mathbf{k} = (0, k_y)$. The phase information for transport is included in the quasilinear weight $w_k^{\text{lin}} = Q_i^{\text{es}}(k)|_{\text{lin}}/\Phi^2(k)|_{\text{lin}}$, a function of the heat flux generated by the unstable eigenmode at k_y and the square of the ballooning averaged potential of the same mode. The model also depends on the growth rate γ_k , the triplet correlation time $\tau_{k,k'}^{\text{lin}}$, and the effective wavenumber $\langle k_{\perp}^2 \rangle$. When evaluating the improvement due to τ , we compare to the same model without τ and with a recomputed calibration constant C. The effective wavenumber is given by [64, 37]:

$$\langle k_{\perp}^2 \rangle = \left\langle \frac{k_y^2 [1 + [g^{xy} + \hat{s}\theta_0(k_x)g^{xx}]^2]}{g^{xx}} \right\rangle,$$
 (6.2)

where g^{xy} and g^{xx} are metric coefficients, $\theta_0(k_x)$ is the extended ballooning angle at the low-field side for a given k_x , and the average $\langle A \rangle$ is defined as

$$\langle A \rangle = \frac{\int_{-3\pi}^{3\pi} A |\Phi|^2 d\theta}{\int_{-3\pi}^{3\pi} |\Phi|^2 d\theta}.$$
(6.3)

The average includes only a single connected radial wavevector on each side, which corresponds to an integral in ballooning angle from -3π to 3π . This choice is explained in the next subsection.

Quasilinear models either focus on only the most unstable eigenmode or account for multiple unstable eigenmodes at a wavevector by summing over their flux contributions [37]. The choice of what values to use in $\tau_{k,k'}^{\rm lin}$ complicates this, especially given that some instabilities do not saturate through coupling to zonal flows. For these instabilities, the choice of \mathbf{k}' cannot be narrowed down in the way it is here. The AUG case has a MT instability which contributes less than 10% to $Q_i^{\rm es}$ in the quasilinear model without $\tau_{k,k'}^{\rm lin}$. The amplitude of the MT mode was measured in the turbulence and was only excited to, on average, several percent. In contrast, the ITG mode represented 90% of the turbulence. The electrostatic, no-fast-ions case of the JET 75225 discharge has an ion-frequency, tearing-parity mode which contributes only several percent to quasilinear predictions. Because of the relatively minor contributions of subdominant eigenmodes seen in the present cases, and the ambiguous nature of applying the $\tau_{k,k'}^{\rm lin}$ factor to them, the quasilinear model used here neglects these modes. Because nonlinearly enhanced electromagnetic stabilization is much stronger than a 10% effect, this approach is satisfactory in evaluating the performance of the quasilinear model at nonzero β .

Evaluation of $\langle k_{\perp}^2 \rangle$

The use of the effective wavenumber $\langle k_{\perp}^2 \rangle$ over the binormal wavevector k_y^2 in Eq. (6.1) is made to account for the extent of eigenmodes in the parallel direction, which connects different k_x values. It is the Φ^2 weighted average of $k_x^2 + k_y^2$. For ITG simulations with adiabatic electrons, Φ^2 dies off much faster than θ so $\langle k_{\perp}^2 \rangle$ converges very quickly in the number of connected wavevectors-often only one is sufficient. With active electrons, however, electrons (and thus Φ) form a radially narrow structure at the resonant surface, which corresponds to an extended structure in the ballooning angle [65, 66]. For example, a plot of the unstable mode structure in the x, z plane is given in Figure 6.1.



Figure 6.1: Φ structure of an ITG mode in the x, z plane from a simulation with active electrons. There is a radially narrow structure at z = 0, which stems from the electrons at the rational surface.

The corresponding ballooning structure is shown in Figure 6.2. It is visually evident that the square of Φ (which becomes the denominator in $\langle k_{\perp}^2 \rangle$) converges rapidly. The figure also shows the same function with the factor of θ present in Eq. (6.3). The integral of its square would be utterly dominated by the tails, and may not even converge as resolution is increased. This is not an issue in the turbulent case, as the turbulent viscosity narrows the modes by acting as a k^2 damping term, and as nonlinear simulations are converged at much lower k_x resolutions than are included in this figure.



Figure 6.2: Ballooning Φ structure of an ITG mode (blue), compared with $\theta\Phi$ (orange), which is used in Eq. (6.3).

To remedy the long tails, we tried several modifications. Manually trimming the structure was not valid because it produced much different results between aggressive and conservative trimming. 'Censoring' the rational surface by replacing the structure with the interpolation between the edges could be justified as similar to nonlinear resonance broadening, however the tail structure persisted after the replacement. Justified by the nonlinear damping, we tried various levels of hyperdiffusion, but sufficient dissipation to remove the tails also affected other properties of the unstable eigenmode.

There are gyrofluid quasilinear simulations which solve for mode structure assuming a Gaussian shape (See Qualikiz references: [67, 68]). Fitting the mode structure with a Gaussian produced unambiguous results but did not capture any of the mode broadening with β .

Weighting the field line average by Φ^4 produced very similar results to truncating the mode structure to only include a single connection, and in the end we chose the truncation as it is more similar to what is typically done.

Quasilinear Transport Weight Normalization

Under the adiabatic electron approximation, $n_i^2(\mathbf{k})$ and $\Phi^2(\mathbf{k})$ are related by a constant factor (which is a different constant only at $k_y = k_z = 0$). This is no longer true with nonzero β and kinetic electrons. In some references [37, 63], w_k^{lin} is normalized with the ion density n_i instead of Φ as in Eq. (6.1) or Ref. [32]. The use of Φ^2 here was chosen to have more applicable results to the majority of quasilinear transport predictions. Other energy-like quantities for spectral weight and flux normalization are possible; however, accurate transport predictions require that the linear proxies for these quantities reflect the nonlinear results. At very high β , terms accounting for A_{\parallel} could be required, and the energy would be a natural way to include both components of Φ and A_{\parallel} . Any modification to the normalization should be tested against the wide region of parameter space where quasilinear modeling is known to produce accurate results.

Figure 6.3 shows the quasilinear transport weight with either normalization at low and high β . Any constant multiple to this factor would be rolled into the calibration constant and would not effect the transport prediction. However, the spectra and the dependence on β do matter in the transport estimate. With n_i^2 the weight slightly decreases with β , while with the Φ^2 definition the weight increases $\approx 20\%$. Because of this, the Φ^2 normalization will predict 20% less transport reduction, which is why the transport agreement at nonzero β here is worse than in Ref. [63].



Figure 6.3: Comparison of Φ^2 normalized (dots) and n_i^2 normalized (crosses) quasilinear transport weights at low and high β ($\beta = 0.01\%$: black, $\beta = 0.75\%$: magenta).

Triplet Correlation Time

Equation (6.1) is a standard quasilinear model modified to include the element of saturation physics which changes most strongly with β , the triplet correlation time $\tau_{k,k'}$. The choice of \mathbf{k}' can be narrowed down to the zonal mode wavenumber usually responsible for the most energy transfer. For the mCBC, the first three zonal wavevectors transport the most energy, of which $k_x = 0.1$ is chosen as a typical value, and is used in all cases, noting that the several longest zonal-flow wavelengths produced similar β scalings for $\tau_{k,k'}$. It is possible to tailor the characteristic \mathbf{k}' as a function of both k_y and parameter set, which could be expected to produce more accurate transport estimates. However, this requires at least one nonlinear simulation per region of parameter space, lessening the advantages of quasilinear modeling.

To include the physics stemming from $\tau_{k,k'}$ in quasilinear transport models, this quantity should be constructed from information obtainable through linear simulations alone. This implies using linear proxies for the nonlinear frequencies $\hat{\omega}$, which is justified when comparing frequency broadening in nonlinear simulations with linear growth rates [69]. One proxy uses the complex linear frequency of the unstable mode for $\hat{\omega}_k$, sets the zonal-flow frequency $\hat{\omega}_{k'} = 0$, and substitutes the complex frequency of the unstable mode at k - k' for $\hat{\omega}_{k-k'}$, so that $\tau_{k,k'}^{\text{lin}} = -i[\omega_{k-k'} - \omega_k^*]^{-1}$.

A variety of improvements to this linear proxy are conceivable but are not tested here,

such as a nonlinear correction to linear frequencies as done with eddy damping rates [70, 31]. A second possibility would seek to include a more realistic zonal-flow damping rate and frequency. A third improvement sums over different eigenmodes for the mode at k - k'; however, this may not be practical due to the large number of stable modes and the cost of computing them, as well as the possibility of a continuum of stable modes close to resonance where $\tau \to \infty$. Nonlinear resonance broadening could be expected to prevent the latter possibility.

6.2 Quasilinear Results



Figure 6.4: Transport and quasilinear estimates for mCBC parameters as a function of β . Heat flux: Q_{i}^{es} (blue). Quasilinear without τ : $C_{1}\sum_{k} \frac{w_{k}\gamma_{k}}{\langle k_{\perp}^{2} \rangle}$ (orange). Quasilinear with τ : $C_{2}\sum_{k} \frac{w_{k}\gamma_{k}}{|\tau_{k,k'}|\langle k_{\perp,m}^{2} \rangle}$ (red)

Nonlinear and quasilinear results for transport in the mCBC parameter case are shown in Fig. 6.4. Increasing β from 0.01% to 0.75% reduces the actual, nonlinear flux by 95%, while the quasilinear model without $\tau_{k,k'}$ only predicts a 50% reduction. Owing to the change with β of the nonlinear transfer efficiency, the model with $\tau_{k,k'}$ captures a 75% reduction, which is less than the reduction presented in Ref. [63], because Q_i^{es}/Φ^2 increases with β while Q_i^{es}/n_i^2 does not. Some under-prediction of stabilization is to be expected, because this model does not include the increased stable mode excitation, discussed in Chapter 5, which reduces flux and saturated amplitudes further. Note that typical statistical error bars for the nonlinear fluxes are typically around 10% for this and the subsequent cases.



Figure 6.5: Transport and quasilinear estimates for a) AUG , b) QA, and c) CMOD cases. Heat flux: Q_i^{es} (blue). Quasilinear without τ : $C_1 \sum_k \frac{w_k \gamma_k}{\langle k_{\perp}^2 \rangle}$ (orange). Quasilinear with τ : $C_2 \sum_k \frac{w_k \gamma_k}{|\tau_{k,k'}| \langle k_{\perp}^2 \rangle}$ (red).

Figure 6.5 shows turbulent flux and quasilinear predictions for the AUG, QA, and CMOD cases. In each, β nonlinearly reduces transport by around half. The quasilinear model without $\tau_{k,k'}^{\text{lin}}$ overpredicts transport in all cases, capturing only between 10% (AUG) and 50% (CMOD) of the transport reduction due to electromagnetic stabilization. The quasilinear transport reduction is improved in each case with $\tau_{k,k'}^{\text{lin}}$. The difference between the quasilinear model and true flux is reduced between half-fold in the AUG case and completely in the CMOD case.



Figure 6.6: JET case (a) 75225 and (b) 73224. ES is for $\beta \approx 0$, while EM is for $\beta \neq 0$. FP indicates the presence of fast ions. Heat flux: Q_i^{es} (blue). Quasilinear without τ : $C_1 \sum_k \frac{w_k \gamma_k}{\langle k_\perp^2 \rangle}$ (orange). Quasilinear with τ : $C_2 \sum_k \frac{w_k \gamma_k}{|\tau_{k,k'}| \langle k_\perp^2 \rangle}$ (red)

Figure 6.6 shows transport and quasilinear predictions with and without fast ions and electromagnetic effects for the JET 75225 and 73224 cases. For the 75225 case, quasilinear flux without the $\tau_{k,k'}$ factor underpredicts the transport reduction for fast ions, electromagnetic effects, and the combination. The $\tau_{k,k'}$ factor improves the accuracy for all of these situations.

With the 73224 case, fast ions reduce transport, but both quasilinear models overpredict this reduction at $\beta \approx 0$. Unlike the other cases, finite β increases nonlinear transport when only thermal species are included, which is not well-captured in either quasilinear model. A parameter case which is slightly modified from the 73224 discharge, including a different safety factor, has largely different saturation physics because of a toroidal Alfvén eigenmode [6]. This could explain the failure of quasilinear models for that case. However, the combination of both fast ions and electromagnetic effects results in a nearly complete reduction in transport, which is approximately predicted by the quasilinear model without $\tau_{k,k'}$ and very closely predicted by the model with it.

An advantage of a reduced model like Eq. (6.1) is that reduced models allow the transport reduction to be attributed to specific physical mechanisms. Table 6.1 shows the percent change in the quasilinear flux estimate due to the effect of β on the various factors for the cases where the quasilinear flux accurately predicts the transport reduction with β . The quasilinear weight increases with β and generally opposes the transport reduction. The strongest contributors to stabilization are γ decreasing and $\tau_{k,k'}$ increasing. Mode width broadening, captured with $\langle k_{\perp}^2 \rangle$, only played a small part in the transport reduction in all cases. Thus, the claim is justified that nonlinear electromagnetic stabilization, on top of (quasi-)linear stabilization, occurs in large part due to enhanced energy transfer to stable eigenmodes as captured by the triplet correlation time τ .

parameter case	γ	w_k	$\langle k_{\perp}^2 \rangle$	$ au_{k,k'}$
mCBC	-50%	+20%	-20%	-45%
AUG	-35%	+50%	-5%	-15%
CMOD	-35%	+15%	-5%	-35%
QA	-20%	+10%	0%	-25%
JET 75225 NOFP	-50%	0%	-10%	-55%

Table 6.1: For each parameter case: the percent change of the quasilinear transport prediction of flux between the low- and high- β cases due to various factors. These are: growth rate γ , quasilinear weight w_k , mode width $\langle k_{\perp}^2 \rangle$, and triplet correlation time $\tau_{k,k'}$. These values are calculated by substituting just the high- β factor of interest into the formula for the otherwise low- β case. Because the spectrum can change, the total reduction does not exactly equal the product of reductions due to each factor.

The increase in $\tau_{k,k'}^{\text{lin}}$ is due to a reduced dependence of γ on k_x . This was investigated in a fluid model and found to be due to changes in finite gyroradius effects with β [86].

6.3 Chapter Summary

The transport reduction for ITG turbulence as β is increased can be mostly explained by better frequency matching between nonlinearly interacting modes, as measured by the triplet correlation time τ . This is a fundamentally nonlinear phenomenon, but it arises due to changes in linear physics. Quasilinear models can be modified in a straightforward, physics-based way to include τ , which greatly increased their accuracy at nonzero β for a wide range of parameter cases.

Quasilinear transport models can also be used to attribute changes in the nonlinear state to the sum over contributions of individual factors. For all of the parameter cases where the modified quasilinear model produced accurate results the largest contributions to stabilization were the reduction in linear growth rate γ and increased triplet correlation time τ .

7.1 Conclusions

This thesis is an investigation into the nonlinear, turbulent state which arises from the ITG instability. In all the examined parameter cases, zonal flows form which saturate the instability by transferring energy to higher- k_x stable and unstable modes. The role of stable eigenmodes was investigated by measuring their amplitudes in the turbulence, as well as their direct effect on energy injection rate and heat flux. Those effects vary strongly between parameter cases with different geometries and collisional dissipation levels, implying that these parameters have a strong effect on the stable modes.

The nonlinear critical temperature-gradient upshift was examined in an attempt to find a physics-based mechanism which could predict the magnitude of the shift. Direct stable mode effects on energy injection are not responsible, as the normalized energy injection rate closely tracks the unstable mode's growth rate. It has been suggested that the nonlinear threshold could be due to nonlinear eddy damping setting a limit on the zonal flow strength. Energy transfer out of the $k_y = 0$ wavenumbers increases with temperature gradient; however, the transfer of Φ^2 energy does not. This could limit the zonal flow strength, but it would require that the linear terms at $k_y = 0$ transform energy between the two forms. The fraction of the zonal distribution function described by the RH residual was always close to 100%, and the linear damping rate of the residual flow was different by around a factor of five between above and below the nonlinear critical gradient. This does not seem to be the cause for the transition, but this effect would be expected to strengthen the turbulence above the nonlinear critical gradient.

Nonlinear electromagnetic stabilization was investigated and found to be largely due to increased triplet correlation lifetime, which acts as a nonlinear energy transfer efficiency. When incorporated in a standard quasilinear model, it reduced the error by 50-100% across five parameter cases.

7.2 Future Work

There are a number of research topics which could be investigated to follow up the work in this thesis, especially relating to stable mode and zonal flow physics.

Stable Eigenmodes

In gyrokinetics, stable mode analysis in the eigenmode basis is rather difficult because of the number of stable eigenmodes, the cost of complete eigenmode decompositions, the sensitivity of individual eigenmodes to both physical input parameters and numerical effects¹, the complexity added by nonorthogonality, and the cost of doing projections on the turbulent state. This is why a strong preference was given here to techniques which bypass these difficulties. For example, the effective (nonlinear) growth rate can be compared to the linear growth rate to measure energy injection modified by the stable eigenmodes present in the turbulence, yet it does not require knowledge of any stable eigenmode structures or properties. These same techniques can be repeated in a straightforward fashion with any type of instability-driven microturbulence, and would provide insights into the saturation process for those other instabilities. Individual stable eigenmodes can be identified in gyrokinetics where they play a clear role, as when a stable microtearing mode contributes to electromagnetic transport in ITG turbulence [19], but this is might be an exception rather than the norm.

For any system, there exists a maximum possible normalized energy injection rate, caused by an associated perturbation, which is referred to the instantaneous optimal perturbation [72]. Because this sets an upper bound on energy injection rate, it gives a limit to the increased energy injection rate due to stable eigenmode effects/nonorthogonality [21]. The instantaneous optimal is dependent on the linear operator alone. It would be illuminating to evaluate and compare this rate to the most unstable eigenmode's growth rate, as well as to the effective growth rate in the turbulence across several experimentally-relevant cases. The increased energy injection rate due to stable modes/nonorthogonality could be expected to be stronger in cases where the optimal energy injection rate is much higher than that of the linear eigenmode, while if there is very little difference between the rates then stable eigenmodes could only decrease the energy injection rate.

Stable modes remove energy from the turbulence through both the drive term and the dissipation term. In Chapter 3, it was shown that for the CBC, these effects have roughly equal strengths. A straightforward followup would be to measure the breakdown between the two terms across a range of collisionalities to investigate if the nature of energy dissipation by stable modes changes with the parameter case. This could also be compared to results from reduced fluid models and analytical theory.

¹See Ref. [71], Figure 5.12. Two eigenmode decompositions with differing hyperdiffusivities yield much different stable eigenmodes as well as amplitudes in the turbulence. However, the heat flux and spectra are not modified much by the hyperdiffusivity

Zonal Flows

Zonal flows are not prominent in all types of microturbulence, but have importance beyond just ITG saturation. There are opportunities for theory with more realistic nonlinear drive and damping as well as comparisons between theory and computation. Figure 4.12 reveals that the zonal flow damping rate due to the linear terms can indirectly² change as ω_{Ti} changes. A straightforward followup would be to measure the effect of the individual linear terms on $\Phi(k_y = 0)$, which can be done by evaluating $\Phi[\partial g/\partial t|_{term}]$ in the gyrokinetic code.

There is a simple model of zonal flow excitation which could be tested computationally. Electrostatic potential is nonlinearly transferred to the $k_y = 0$ wavenumbers, where it decays through GAM damping to the more weakly damped RH residual. The fraction of each in the turbulence can be inferred from their relative damping rates from theory. It would be revealing to compare this simple model with the actual value from a nonlinear simulation, as the theory both lacks a realistic nonlinear drive and nonlinear eddy damping.

Because the field energy term in Eq. (2.24) is nonlinearly conserved, it could be expected that unstable modes with a higher fraction of field energy to entropy-like energy could drive zonal flows more efficiently. This could be studied by finding a parameter which affects the ratio, and conducting a secondary instability analysis to determine zonal flow growth rates. If zonal flow growth rates correlate with the field energy fraction, it supports this hypothesis.

The range in ω_{Ti} of the Dimits shift depends on geometry [52]. It could be investigated if this is because the RH residual flow proportion also depends on geometry.

The distinction between the true zonal flow (which has no parallel structure) and the $k_y = 0$ flow was not made in this thesis. There is still a question of how much the $k_z \neq 0$ component of the flow at $k_y = 0$ is responsible for nonlinear energy transfer. This distinction could be studied by decomposing the $k_y = 0$ flow in the nonlinear energy transfer diagnostic in GENE into a flux-surface-averaged part and a remainder, and measuring the energy transfer breakdown that way. This also ties into energy transfer by the GAM, as the GAM oscillation has structure along the field line.

²This must be an indirect effect, as the linear terms with ω_{Ti} are zero at $k_y = 0$. The difference must lie in the nonlinearity which sets the $k_y = 0$ distribution function in the turbulence.

Coupling coefficients

The nonlinear coupling coefficient investigation in Chapter 5 predicted the strong effect that higher- k_x zonal flows transfer a larger fraction of energy to stable eigenmodes, while incorrectly predicting a much weaker effect that energy to stable modes would decrease with β . The failure with β could be because the coupling coefficient was only approximate³, because it left out phase-matching effects, or because it was only measuring the coupling coefficient to the unstable mode at higher- k_x . Each of these could be addressed in a more accurate manner, and if accurate, this analysis would reveal each effect's importance. The coupling coefficients could then be used to investigate why stable mode fraction varied so much between the different parameter cases in Chapter 5. Such analyses could then be used to improve the predictive capabilities of reduced models across wide varieties of parameter space.

³The coupling coefficient in Chapter 5 was for transfer of entropy-like energy, left out the Bessel function in the gyroaverage, and assumed a $k_z = 0$ structure for the $k_y = 0$ flow.

We used a variety of parameter sets unstable to ITG. These include some parameter sets which are modified from the Cyclone Base Case (CBC), which is from an idealized D-IIID equilibrium [42], as well as sets from a quasi-axisymmetric stellarator equilibrium (QA), and equilibria that model discharges from the experiments ASDEX Upgrade (AUG), JET, and Alcator CMOD (CMOD).

Selected simulation parameters for the cases are provided in tables. Lists enclosed with curly brackets denote scans. Further information is provided on a case by case basis in the following paragraphs. Most of these parameter cases are in published papers; references are in the individual descriptions. Definitions of resolution quantities are in Table A.1. The resolutions and box sizes are provided in Table A.2. The geometry type, safety factor, magnetic shear, $\alpha_{\rm MHD} = -q^2 R(d\beta/dr)$ and inverse aspect ratio $\epsilon = r/R$ are provided in Table A.3. The β values, normalized gradients of ion temperature, electron temperature, and electron density, are given in Table A.4. Some of these cases have multiple ion species, in which case the dominant species is the one reported in the table and detailed information is given in the individual description, as well as in the references.

Parameter	Definition
L_x	Radial box size
L_y	Binormal box size
n_x	Radial resolution
n_{ky}	Binormal wavenumber resolution
n_z	Parallel resolution
n_v	Parallel velocity resolution
n_{μ}	Magnetic moment resolution

Table A.1: Definitions of resolution and box size quantities.
case	L_x	L_y	n_x	n_{ky}	n_z	n_v	n_{μ}
CBC-DIMITS	75.3	125.7	128	16	24	48	8
INC-GRAD	125.6	125.7	192	16	24	48	8
mCBC	150.7	125.7	128	16	24	48	8
AUG	97.3	133.8	256	48	32	32	16
JET 75225	280.7	125.7	256	32	32	48	12
JET 73224	175.3	125.7	192	48	24	32	8
QA	182	62.8	128	24	128	48	8
CMOD	120	125.6	256	24	32	32	16

Table A.2: Box size and resolutions for the various parameter cases.

case	geometry	q_0	\hat{s}	α_{MHD}	ϵ
CBC-DIMITS	$\hat{s} - \alpha$	1.4	0.796	0	0.18
INC-GRAD	$\hat{s} - \alpha$	1.4	0.796	0	0.18
mCBC	$\hat{s} - \alpha$	1.4	0.796	0	0.18
AUG	tracer	1.48	1.31	0	0.191
JET 75225	miller	1.14	0.159	0.62	0.116
JET 73224	miller	1.74	0.523	0.39	0.121
QA	gist	1.8	-0.549	N.A.	0.16
CMOD	miller	1.166	1.133	0.128	0.19

Table A.3: Geometry type and geometric coefficients for the parameter cases. Additional miller parameters are left out for brevity. Miller parameters for the JET cases are in Refs. [73, 74], while for the CMOD case are in the text.

case	eta/%	$\omega_{T\mathrm{i}}$	ω_{Te}	$\omega_{n\mathrm{e}}$
CBC-DIMITS	0	$\{4.5, 5.0, 5.5, 6.0, 6.5\}$	N.A.	2.2
INC-GRAD	0	$\{7, 8, 9, 10, 11\}$	N.A.	2.2
mCBC	$\{0.01, 0.25, 0.5, 0.75\}$	$\{6,7,8\}$	0	1
AUG	$\{0.81, 1.5\}$	5.76	5.76	0.61
JET 75225	$\{0.001, 1.75\}$	8.1	4.23	2.94
JET 73224	$\{0.001, 0.33\}$	11.1	6.96	1.31
QA	$\{0.05,1\}$	3	1	1
CMOD	$\{0.01, 1.3\}$	8.42	6.04	0.82

Table A.4: Electron β values and normalized gradients for the various parameter cases.

Chapter 3 and Chapter 4, which study ITG saturation and the Dimits shift respectively, used results from the CBC-DIMITS and the INC-GRAD scans. These parameter cases are the CBC with a modified temperature gradient. They only model the ion species and use the adiabatic electron approximation. In the Dimits regime, radial-box-scale zonal flows dominated energy transfer. Convergence in the Dimits regime was tested with a $\omega_{Ti} = 5$ run with twice the box size; the same zonal flow wavenumber still dominated. However, for future work, larger box sizes should be preferred to avoid artificial numerical effects.

The mCBC scan was used in Chapter 5 to investigate ITG at nonzero β . It has active electrons and is taken from Ref. [57]. This scan changes the electron temperature and density gradients compared to the standard CBC. If this is not done, other electromagnetic instabilities are destabilized which mixes the effects of β on ITG and on the other modes.

Chapter 5 also included parameter scans modeled after AUG shot 29197 and JET shot 75225. The AUG β scan uses geometry and profile data from a reconstruction, with the same parameters as in Ref. [75]. Experimentally, this case corresponds to a β scan where an attempt was made to vary β with other normalized parameters unchanged. AUG has major and minor radii of 1.65 and 0.65 meters, respectively. Shot 29197 had an on-axis toroidal field of 2.17 T, a plasma current of 0.94 MA and had an on-axis temperature around 4keV for both ions and electrons. Unlike the mCBC, this simulation case has realistic geometry, impurity species, external $v_{E\times B}$ and collisions. In addition to ITG modes, it is also unstable to microtearing modes (MTM). As stable modes are ultimately driven by unstable modes, competing unstable eigenmodes could be expected to involve different saturation physics than the cases with a single unstable eigenmode per wavevector. Growth rates and frequencies for both eigenmodes are shown in Figure A.1.



Figure A.1: Growth rates and frequencies for both eigenmodes of the AUG case as a function of k_y

Fast ions stabilize ITG [76], and must be included for accurate simulations of some JET discharges. The JET runs used here are from shot 73224, which was part of an investigation of ion profile stiffness with plasma rotation and low magnetic shear [77], and shot 75225, which is from an improved-confinement regime [78]. Each case has been used previously to study fast-ion effects in JET discharges [73, 74]. Shot 73224 is a special case, in that the stabilization is dominated by the fast ion contribution, and has been studied elsewhere [6]. Physically, JET has major radius of 3 m and a minor radius of 1.25 m. The 75225 shot had an on-axis toroidal field of 2T, a current of 1.7MA, and a q_{95} of 3.94. The on-axis electron density was $3.24 \ 10^{19} \text{m}^{-3}$, while the ion and electron temperatures were 10 keV and 5 keV respectively. For the purpose of these studies, both JET cases are modified to remove external $v_{E\times B}$ shearing, and made into four cases by using each combination of high/low β with/without fast ions. When fast ions are removed, the ion density gradients were changed to preserve ambipolarity in the gradients. The cases differ strongly by their shear values. Growth rates and frequencies of the ITG mode for the JET 75225 parameter case are shown in Figure A.2. Flux time traces for each run are shown in Figure A.3.



Figure A.2: Growth rates γ and frequencies of the ITG mode in the JET 75225 case as a function of k_y



Figure A.3: Time traces of flux for both β values of 75225, with and without fast ions.

The QA case affords the opportunity to test quasilinear models in a 3D geometry, which affects mode structure and in turn mode coupling. Quasi-axisymmetry refers to a family of quasi-symmetric stellarator configurations in which a $(m \neq 0, n = 0)$ mode dominates the equilibrium magnetic spectrum, where m and n are poloidal and toroidal mode numbers respectively. In this case m = 1, n = 0 [79]. The specific QA configuration used is the baseline NCSX geometry [80, 81]. The NCSX equilibrium has a total normalized plasma pressure $\beta \approx 4\%$, three field periods, and mean magnetic field $\langle B \rangle \approx 1.6$ T.

We also tested quasilinear models using a parameter case based on an Alcator CMOD run. This discharge was part of a CMOD experiment with reduced toroidal fields to test ITER-like scenarios at relevant $\beta_N = 1.8$ and Greenwald fraction $f_{\rm GW} > 0.5$ with dominant electron heating and low torque (via ICRH) [82]. CMOD has major and minor radii of 0.68 and 0.22 m, respectively. The modeled shot had an on-axis toroidal field of 2.6T, a plasma current of 0.6MA and a q_{95} around 3. On-axis densities were around 3 10^{20} m⁻³, and ion and electron temperatures were both around 1.5keV. The original simulation work [83] was motivated as validation with ITER-like parameters. Using Miller geometry and GENE units, the geometry factors are as follows: major radius $R_0/a = 3.163$, minor radius $r_0/a = 0.6$, elongation $\kappa = 1.35$, triangularity $\delta = 0.156$, squareness $\zeta = -0.011$, and their shears, respectively: $s_{\kappa} = 0.104$, $s_{\delta} = 0.206$, $s_{\zeta} = -0.03$. The major radius shift of the flux surface was dr/dR = -0.151, and $\rho^* = \rho_{\rm ref}/L_{\rm ref} = 0.005$. The Miller geometry quantities are defined in Ref. [84]. These runs had 3 species, deuterium ($\omega_n = 0.26, \omega_T = 2.66, T_0 = 0.99$, $n_0 = 0.91$), boron ($\omega_n = 0.26, \omega_T = 2.64, T_0 = 0.99, n_0 = 0.005$), and electrons ($\omega_n = 0.26, \omega_T = 0.26, \omega_T = 0.91$) $\omega_T = 1.91, T_0 = 1, n_0 = 1$). The collision frequency (see Ref. [85]) was $\nu_c = 0.002$. The growth rate and frequency spectrum for the two β values are shown in Figure A.4.



Figure A.4: Growth rates and frequencies for both eigenmodes of the CMOD case as a function of k_y

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