DIAGNOSIS OF EQUILIBRIUM MAGNETIC PROFILES, CURRENT TRANSPORT, AND INTERNAL STRUCTURES IN A REVERSED-FIELD PINCH USING ELECTRON TEMPERATURE FLUCTUATIONS

by

Eli Parke

A dissertation submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

(Physics)

at the

UNIVERSITY OF WISCONSIN-MADISON

2014

Date of final oral examination: 08/01/14

The dissertation is approved by the following members of the Final Oral Committee: Daniel J. Den Hartog, Research Professor, Physics

Chris Hegna, Professor, Engineering Physics

Vladimir Mirnov, Staff Scientist, Physics

Carl Sovinec, Professor, Engineering Physics

John Sarff, Professor, Physics

Jan Egedal, Professor, Physics

© Copyright by Eli Parke 2014 All Rights Reserved

Acknowledgments

I'll be fine. I'm a scientist. Cecil, a scientist is usually fine.

— Carlos (Welcome to Night Vale Ep. 49)

Next to Daniel's office door, a copy of the article "The importance of stupidity in scientific research" (Martin A. Schwartz, *Journal of Cell Science*, 2008) has been posted. I still remember the first time I read it, while waiting for a meeting. I have re-read that article many times over the past several years, and my appreciation of it has not waned. Working for Daniel and the MST team has been a rewarding experience; providing such a friendly atmosphere while challenging students to push the boundaries of their abilities and still allowing them the flexibility to learn from their mistakes is not an easy balance to achieve, but the MST team makes it look easy.

I'd like to thank the entire TS team, which has included many people during my time here: Daniel Den Hartog, Mike Borchardt, Rob O'Connell, Adam Falkowski, Josh Reusch, Hillary Stephens, You Ming Yang, Lucas Morton, Steph Kubala, Wayne Harris, William Young, and the army of undergrads (Adam Beardsley, Cale Kasten, Curt Johnson, Noah Hurst, Alex Plunckett, and Josh Goglio).

For diagnostic services and interesting physics, the NBI team: Jay Anderson, Mark Nornberg, Josh Reusch (again), Scott Eilerman, Jon Koliner, Jeff Waksman, Bill Capecchi, and Jungha Kim.

For listening to my results and analysis many times and offering suggestions: the entire IDA group.

For many illuminating discussions: Vladimir Mirnov, Chris Hegna, Carl Sovinec, Karsten McCollam, Brett Chapman, Abdul Almagri, and Weixing Ding. Without their help and advice, much of the work presented here would not have been possible.

For engineering support and invaluable technical expertise: Bill Zimmerman, Peter Weix, Don Holly, Jim Adney, Mark Thomas, Mikhail Reyfman, David Deicher, and everybody at the Instrument Shop. For computer support and hacking-fu: Paul Wilhite, Andrey Levochkin, and Alex Squitieri. For APD calibration services: Rick Gerson at CNRC-INMS. For excellent administrative support: Jane Schwantz, Russ Rylander, and Christine Hrenak.

For reading and editing this thesis of his own free will: Jason Loch.

For making Madison such a wonderful place to live, work, and play for the last seven years: all of my wonderful friends (even the friends I had to travel thousands of miles to hang out with—that's you, Trenton). For support and encouragement: my parents, sister, and family. I could not have survived graduate school without all of you.

Finally, I must thank my husband, Tamvana, for putting up with me while I wrote this thesis. Despite the long hours and the stress, he has always been there for me.

Abstract

Due to long fast ion confinement times, neutral beam injection (NBI) on the Madison Symmetric Torus (MST) yields large fast ion populations with substantial density gradients. Novel application of the unique high-rep-rate (>10 kHz) Thomson scattering diagnostic on MST has enabled characterization of a newly observed beam-driven instability, and detailed measurement of equilibrium changes caused by the fast ion population. While previous work has focused on high-frequency energetic particle modes (EPMs), recent observations indicate that fast ions drive a bursting instability near the plasma rotation frequency under appropriate conditions. The mode chirps strongly, with a frequency of approximately 7 kHz in the plasma reference frame at peak amplitude. Bursts are correlated with EPM activity and core neutral particle analyzer signals drop by 30% during a burst, suggesting that this mode participates in avalanches of the higher frequency EPMs and drives enhanced fast ion transport. Electron temperature fluctuations correlated with this low-frequency mode exhibit a core-peaked structure with a sensitive dependence on the safety factor q. Although this mode has not yet been positively identified, its characteristics and internal structure are suggestive of an internal kink (fishbone) or beta-induced Alfvén eigenmode. In addition to driving EPMs, the large fast ion population also modifies the current profile. An increase in on-axis current density driven by NBI is offset by a reduction in the mid-radius, leading to net-zero current drive. This results in a slight flattening of the safety factor profile, observed by precise measurement of the rational surface locations of the dominant tearing modes; these are identified from the phase flip in correlated electron temperature fluctuations recorded by Thomson scattering. For the core n = 6 rational surface, an inward shift of 1.1 ± 0.6 cm is observed, with an estimated reduction in q₀ of 5%. This technique provides a powerful tool for measuring the equilibrium magnetic field in the RFP; the phase of the temperature fluctuations also enables an estimate of pressure-fluctuation contributions to the dynamo electric field. An examination of the effect of pressure on electron momentum balance indicates that anisotropy is crucial to the kinetic dynamo.

Contents

- Abstract iii
- Contents v
- List of Tables viii

List of Figures ix

- **1** Introduction 1
 - 1.1 *MST and the RFP* 3
 - 1.2 Neutral beam injection 5
 - 1.3 *Thomson scattering* 6
 - 1.4 Thesis 8
 - 1.5 *References* 11

2 Thomson Scattering and Calibration 14

- 2.1 *Thomson scattering* 15
- 2.2 Depolarization of Thomson scattered light 20
- 2.3 *TS hardware* 33
- 2.4 *Calculating electron temperature* 39
- 2.5 *Calibration for fluctuation measurements* 42
- 2.6 Correlating TS fluctuation measurements with other diagnostics 49
- 2.7 Magnetic coil arrays 58
- 2.8 References 60

3	Tear	ring Modes and Fast Ions in the RFP 64
	3.1	<i>Origin and characteristics of tearing modes</i> 65
	3.2	Magnetic diagnosis of tearing modes 67
	3.3	<i>Temperature flattening and fluctuations</i> 70
	3.4	<i>Fast ions in the RFP</i> 75
	3.5	Stabilization of MHD activity by fast ions: theoretical predictions 78
	3.6	References 83
4	Safe	ty Factor Modification with NBI 89
	4.1	Correlated T_e fluctuations 90
	4.2	<i>Rational surface constraint with tearing modes</i> 104
	4.3	<i>Current redistribution and q-profile modification</i> 111
	4.4	Comparison to reversed discharges 114
	4.5	Discussion 118
	4.6	References 120
5	Cur	rent Transport and the Dynamo 123
	5.1	The phase of \tilde{B}_r 124
	5.2	Radial profiles for \tilde{B}_r 128
	5.3	<i>Temperature fluctuation driven current transport</i> 129
	5.4	<i>The RFP dynamo</i> 133
	5.5	<i>The kinetic dynamo</i> 136
	5.6	Discussion 145
	5.7	References 147
6	NBI	and Instability 150
	6.1	<i>Energetic particle modes in the RFP</i> 151
	6.2	$ ilde{T}_{e}$ correlated with EPMs and chirping modes 157
	6.3	<i>Ultra-low-frequency instabilities in reversed discharges</i> 162
	6.4	$ ilde{T}_{e}$ correlated with ultra-low-frequency instabilities 181
	6.5	Discussion 187
	6.6	References 190

7 Conclusion 196

- 7.1 Summary of results 197
- 7.2 Suggestions for future work 199
- 7.3 Final remarks 202
- 7.4 References 202

Colophon 203

List of Tables

4.1	$n = 5$ Filters and Parameter Values $\dots \dots \dots$
4.2	$n = 6$ Filters and Parameter Values $\dots \dots \dots$
4.3	$n = 7$ Filters and Parameter Values $\dots \dots \dots$
4.4	$n = 8$ Filters and Parameter Values $\dots \dots \dots$
4.5	Island Model Fit Parameters
4.6	Rational surface locations
6.1	EPM Filter Parameters159
6.2	Chirp Filter Parameters
6.3	n = 5 Burst Filter Parameters
6.4	EPM Frequency Filters 180
6.5	$n = 5$ Burst Filter Parameters for \tilde{T}_e Correlation
6.6	Burst Time Windows 182

List of Figures

1.1	RFP magnetic configuration	4
1.2	Sawtooth behavior in reversed MST discharges	5
1.3	Top-view of NBI on MST	7
2.1	Thomson scattering of laser radiation.	16
2.2	Poloidal cross section of MST with TS laser chord	18
2.3	Thomson scattering spectra	19
2.4	Degree of depolarization for linearly polarized incident light	23
2.5	Depolarization and relative error as functions of ellipticity and orienta-	
	tion angle.	24
2.6	TS polarimeter detector design	25
2.7	Predicted error in polarimetric TS measurement for various incident	
	laser polarization states	28
2.8	Error in temperature measurement for polarimeter with finite wave-	
	length range	31
2.9	Thomson scattering laser operational modes.	35
2.10	Voltage signal showing TS scattered light and time-changing background.	38
2.11	Background and Scattered Fits to Voltage Signal	40
2.12	Two-dimensional T_e/n_e probability grid for fit TS data	43
2.13	APD spectral responsivity curve.	45
2.14	Dark and pulsed signal integrals	46
2.15	Gain-noise product for reference APD	48
2.16	Polychromator transmission functions.	50

2.17	Temperature data with sinusoidal fluctuation model superimposed	52
2.18	Two-dimensional $T_{e,0}/\tilde{T}_e$ probability grid $\ldots \ldots \ldots \ldots \ldots \ldots \ldots$	53
2.19	Evolution of probability distribution	54
2.20	Shafranov shift of flux surfaces	55
2.21	Phase scan of correlated temperature fluctuations	57
3.1	MST safety factor	67
3.2	Tearing mode reconnection and island formation	68
3.3	q-profile evolution over a sawtooth cycle	69
3.4	Equilibrium and fluctuation profiles for $n = 6$ correlated \tilde{T}_e	70
3.5	Characteristic temperature fluctuation profile (left) for magnetic islands	
	with complete flattening of the equilibrium profile (right). The vertical	
	dashed lines indicate the rational surface location.	71
3.6	Ion-guiding-center safety factor profile	76
3.7	TRANSP results for NBI into RFP plasmas	77
3.8	Ensemble average time traces of $I_p, P_{NBI}, n_e,$ neutron flux, and $\tilde{B}_{\theta,5}$	78
3.9	Mode suppression factor vs. toroidal mode number for $F = 0$ and PPCD	
	plasmas	79
4.1	n = 5 correlated T _e fluctuation amplitude and phase	93
4.2	$n = 6$ correlated T_e fluctuation amplitude and phase	96
4.3	Island flux surfaces and TS measurement chords	97
4.4	Helical model fits to $n = 6$ fluctuations w/out NBI	100
4.5	$n = 7$ correlated T_e fluctuation amplitude and phase	102
4.6	$n = 8$ correlated T_e fluctuation amplitude and phase $\ldots \ldots \ldots$	104
4.7	Rational surface locations w/out NBI	106
4.8	Reconstructed B profiles without NBI	112
4.9	Reconstructed parallel current profiles w/out NBI	112
4.10	Redistribution of parallel current with NBI	113
4.11	Safety factor modification with NBI	114
4.12	Suppression of $\tilde{B}_{\theta}(a)$ in reversed discharges	115
4.13	Suppression of \tilde{T}_e in reversed discharges	116

4.14	n = 5 mode Time-of-Death vs density	117
4.15	n = 6 rational surface and island width evolution in reversed discharges	118
5.1	Modeled \tilde{B}_r profiles	128
5.2	Correlated product Π_T for $n = 5$ fluctuations	130
5.3	Correlated product Π_T for $n = 6$ fluctuations	130
5.4	Correlated product Π_T for $n = 7$ fluctuations	131
5.5	Correlated product Π_T for $n = 8$ fluctuations	131
5.6	Opposing dynamo terms summed over multiple modes	132
5.7	Total correlated product Π_T for $n = 5-8$ fluctuations $\ldots \ldots \ldots \ldots$	132
5.8	$E_{\parallel} \text{ versus } J_{\parallel} \dots \dots$	134
6.1	Toroidicity-induced gap in Alfvén continuum	153
6.2	Spectrograms of EPM activity in non-reversed plasmas	155
6.3	FIR density fluctuations for EPMs in the Alfvén continuum	156
6.4	Chirping $n = 4$ activity in non-reversed discharges with NBI	158
6.5	\tilde{T}_e correlated with $n = 5$ EPMs in non-reversed discharges	160
6.6	\tilde{T}_e correlated with $n = 4$ chirps in non-reversed discharges	162
6.7	Spectrogram of $n = 5$ activity in a reversed discharge with NBI	163
6.8	Spectrogram and time series of $n = 5$ activity in a reversed discharge	
	with NBI	165
6.9	Ensemble averaged \tilde{B}_{θ} and \tilde{B}_{φ} of $n = 5$ bursts	167
6.10	Comparison of active/passive IDS with $n = 5$ kink-tearing velocity (sin-	
	gle shot)	169
6.11	Ensemble average of passive IDS and $n = 5$ kink-tearing velocity at	
	time-of-death	170
6.12	Single shot and ensemble average burst velocity in comparison to equi-	
	librium flow	171
6.13	Tearing mode velocity/amplitude time evolution through burst-event	
	for $n = 6-10$	174
6.14	Velocity/amplitude perturbation through burst event	175
6.15	Impurity ion heating during $n = 5$ burst event	177

6.16	Single-shot ANPA signal through a full sawtooth period and $n = 5$
	spectrogram
6.17	Ensemble averaged ANPA signal through an $n = 5$ burst $\dots \dots \dots$
6.18	Ensemble average EPM \dot{B}_{θ} signals during an $n = 5$ burst
6.19	\tilde{T}_e correlated amplitudes and phases with $n = 5$ bursts in F = -0.12 plasmas183
6.20	\tilde{T}_e correlated amplitudes and phases with $n = 5$ bursts in $F = -0.2$ plasmas185
6.21	\tilde{T}_e correlated amplitudes and phases with $n = 5$ bursts in $F = -0.3$ plasmas186
6.22	Speculated $n = 5$ stability diagram in $q_0 - \beta_{hot}$ parameter space 189

Chapter 1

Introduction

In the sun, fusion of hydrogen isotopes into helium supplies the energy required to counteract gravitational forces and maintain stability against its own crushing weight. The energy released also constitutes a significant fraction of the input energy for Earth's biosphere and makes life as we know it possible. Laboratory plasmas seek to duplicate this process with a variety of techniques since gravitational compression is not a viable option on Earth. Electrostatic, magnetic, and inertial confinement devices have been constructed using a variety of geometries. Toroidal magnetic confinement devices (specifically tokamaks) are the current front-runners in this quest, having achieved plasmas with durations up to six minutes and thirty seconds, power output greater than power input, and the highest 'fusion triple product' to date (the product of density, temperature, and energy confinement time).

In a fusion reactor, the fusion of hydrogen isotopes into helium produces neutrons and alpha particles with MeV energies. For example, in a common reactor scheme:

$$D + T \rightarrow {}^{4}He_{3.5MeV} + n_{14.1MeV}$$
 (1.1)

The energetic alpha particles are expected to provide significant heating to sustain the reactor. Furthermore, external heating via cyclotron resonance waves or neutral beam injection also produce significant populations of energetic ions. Therefore, the dynamics of not just high temperature plasmas but also the superthermal energetic particles are of great interest.

While early magnetic confinement research focused on stellarators, mirrors, and pinches, the stabilizing influence of energetic particles was suggested as early as 1953 by Nicholas Christofilos with his Astron proposal [1]. Injecting energetic electrons into a magnetic mirror would lead to the formation of a current layer near the edge of the device that would generate a magnetic field opposing the externally applied field. This field, it was hoped, would close the field lines at the ends of the mirror and improve particle confinement, while the energetic electrons themselves would heat the plasma. Although the Astron was never successful, energetic particles have continued to play a role in fusion research. A later experiment, the ELMO Bumpy Torus, also attempted to use energetic electron rings to stabilize a configuration of magnetic mirrors closed into a torus [2].

Unfortunately, some of the earliest experimental studies of energetic particles in tokamaks revealed only destabilizing effects. The fast-ion population established with neutral beam injection on the Princeton Poloidal Divertor experiment interacted with low frequency magnetohydrodynamic (MHD) modes, n = m = 1 kink modes, driving them unstable [3]. These modes, which have come to be known as fishbones, in turn drove significant fast-ion transport. It was not until several years later that RF produced fast ions would be observed to stabilize MHD activity. During studies of toroidal Alfvén eigenmodes on the Tokamak Fusion Test Reactor, intense localized particle losses actually led to a vacuum breach [4].

In the decades since, energetic particle studies have flourished. A diverse zoology of particle driven instabilities has been catalogued, ranging from Alfvén eigenmodes to fishbones and other energetic particle modes. A variety of mechanisms for stabilizing/destabilizing MHD activity have been identified—primarily these involve trapped particle effects or more brute force current drive, but passing particles and other effects have also been implicated. The Madison Symmetric Torus (MST), a device with an alternative toroidal magnetic configuration known as a reversed-field pinch (RFP), has recently installed a neutral beam injection system for energetic particle studies. As the only major RFP with a high-power neutral beam, this provides a unique platform for studying energetic particle dynamics and interaction with MHD activity in a low-field, high-shear configuration.

1.1 MST and the RFP

The Madison Symmetric Torus has a major radius of 1.5 m and a minor radius of 0.52 m [5]. Plasmas densities are most commonly near $n_e \sim 1 \cdot 10^{13} \text{ cm}^{-3}$, with core electron temperatures in the 100 eV to 2000 eV range. Unlike tokamaks, which have a high toroidal field (typically several Tesla), the RFP is characterized by much lower toroidal fields; MST operates with field strength below 0.6 T for most discharges. Due to the low overall field strength, the toroidal field is comparable in magnitude to the poloidal field. Self-organization of the plasma spontaneously produces a magnetic configuration shown in Figure 1.1, where the toroidal field at the edge of the plasma points in the opposite direction of the toroidal field at the core of the plasma, giving the RFP its name. These configurations have been sustained for time scales much longer than the resistive time that might be expected.

A general description of plasma self-organization has been provided by relaxation theory [6]. According to this description, the total magnetic helicity and toroidal flux are conserved. If the magnetic energy is significantly larger than the kinetic energy of the plasma, minimization of the total energy through variational techniques predicts that the Lorentz force vanishes: $\vec{J} \times \vec{B} = 0$. As a result, $\vec{J} = \mu \vec{B}$ where μ is a constant. Solution of $\vec{J} \times \vec{B} = 0$ in a cylinder results in a description of the magnetic equilibrium known as the Bessel function model:

$$B_{r} = 0$$

$$B_{\theta} = B_{0}J_{1}(\mu r)$$

$$B_{\phi} = B_{0}J_{0}(\mu r)$$
(1.2)

where B_r , B_θ , and B_ϕ are the radial, poloidal, and toroidal magnetic field components, respectively, J_0 and J_1 are Bessel functions, and B_0 is also a constant. For appropriate conditions, this solution leads to a minimum energy state which qualitatively describes field reversal in the RFP. Quantitatively, the most prominent



Figure 1.1: Magnetic configuration for a reversed-field pinch. The poloidal and toroidal fields are comparable in magnitude, and the toroidal component changes sign moving from the core to the edge.

deficiency of this model is the prediction of uniform J_{\parallel}/B across the entire minor radius; experimental profiles universally exhibit zero current at the edge. This formulation of relaxation theory, for various reasons, turns out to be less applicable to tokamaks under most conditions.

The sustainment of an RFP discharge beyond resistive time scales originates in a cyclic competition between resistive diffusion and MHD instabilities [7]. While the Bessel function model produces a configuration in a minimum energy state, diffusion leads to a deviation from this state. Diffusion produces equilibrium gradients that act as a source of free energy for MHD instabilities, which in turn act to restore the relaxed profiles through magnetic reconnection. This cycle is observed on many different signals during an RFP discharge—for example, the average toroidal magnetic field in Figure 1.2; the sawtooth-like behavior observed over a dissipation/relaxation cycle leads to the name 'sawtooth cycle'. Additionally, sustainment of RFP discharges beyond resistive time scales through relaxation is known as the RFP dynamo. These processes have made the RFP an appealing platform for studying reconnection and dynamo physics.



Figure 1.2: Behavior of the average toroidal magnetic field during a reversed discharge (shot 1130920034). Cyclic reconnection events produce the sawtooth-like behavior observed in many plasma parameters.

1.2 Neutral beam injection

Neutral beams are routinely utilized during MST discharges to study fast-ion physics in the reversed-field pinch. A wide range of phenomena, from heating and torque to energetic particle instabilities, are present in neutral-beam-heated MST plasmas, making them rich targets for study. The RFP magnetic configuration, with low total magnetic field and strong shear, presents a stark contrast to the tokamak and a unique opportunity for fast ion studies. Although the typical RFP exhibits greater stochasticity than other configurations, fast ions have been found to decouple from the dominant magnetic perturbations. The resulting fast-ion confinement times are much longer than bulk confinement, permitting studies of fast-ion behavior in a remarkably diverse set of discharges.

The Neutral Beam Injector (NBI) on MST is installed tangentially and can inject either co-current or counter-current beams by inducing plasma current in the appropriate direction, see Figure 1.3. Capable of 1 MW of neutral power at energies up to 25 keV, both the beam current and power are variable. The full energy component of the beam is 86%, with the rest of the beam at lower energy. Beam composition is typically majority hydrogen (95-97%) with trace deuterium (3-5%) for monitoring fast-ion content via beam-target fusion generated neutrons. Deuterium has been used as a majority species, however, for injection into both hydrogen and deuterium plasmas. In typical MST discharges, the fast ions are slightly super-Alfvénic ($v_{fi} \sim 1.25v_A$) and have Larmor radii extending across a significant fraction of the minor radius (4 cm or nearly 10%). The discharges studied in this thesis all utilize co-current injection at full beam energy (25 keV, 50 A). For almost all of the discharges, the beam content is majority hydrogen; only one exception, with full deuterium beam, is noted in Chapter 6.

1.3 Thomson scattering

Electron temperature measurements in many devices, including MST, are routinely made using Thomson scattering (TS) diagnostics. Thomson scattering offers a local, non-perturbative measurement viable across a wide range of plasma temperatures. The MST Thomson scattering system is state-of-the-art, capable of high time resolution (up to 25 kHz with the present system) and covers temperatures ranging from 10 eV to 5 keV [8, 9, 10]. These capabilities allow Thomson scattering to accurately measure the highly dynamic profiles frequently encountered in MST discharges. While a repetition rate of 25 kHz makes the MST Thomson scattering diagnostic one



Figure 1.3: Top view of MST with the Neutral Beam Injector installed near 50° toroidal in the top left. Energetic particles are injected tangentially.

of the fastest in the world (if not the fastest), development of a next-generation laser system capable of 250 kHz repetition rates is in progress [11]. This laser system is expected to dramatically improve the range of physical phenomena diagnosable with Thomson scattering. For the work presented here, however, only the 25 kHz system is used.

Radial profiles of equilibrium electron temperature are the primary output for Thomson scattering diagnostics. These profiles are highly informative, but fluctuations in electron temperature provide a great deal of additional information about plasma processes. Even with the fast repetition rates achieved on MST, analysis of electron temperature fluctuations relying on traditional Fourier spectral decomposition is limited to frequencies below the range of interest for most phenomena. Bayesian analysis techniques permit fluctuation measurements at frequencies well above the Nyquist limit or even the diagnostic sampling rate. A striking example of this approach is found in the search for extra-solar planets. Due to weather and limited telescope time, astronomical measurements are frequently both sparsely sampled and irregularly sampled, confounding Fourier analysis. The effect of a planet on a star's measured radial velocity is well understood, though, permitting inference of likely planetary orbital parameters via forward modeling. A Bayesian statistical framework has been successfully applied to predict the most likely parameters for identification of candidate exoplanets [12].

Thomson scattering measurements represent a similar sparsely sampled dataset, with fluctuation contributions from many real, physical sources as well as noise. Forward modeling of electron temperature fluctuations correlated with coherent magnetic structures has likewise been successfully demonstrated as a powerful tool for measuring internal structures associated with MHD activity [13]. In this case, edge magnetic signals with much higher time resolution provide critical additional information; combination of both the magnetic data and electron temperature data under a Bayesian framework makes possible the inferrence of temperature fluctuations much smaller than the total fluctuation power found in the signal. Since the development of this technique, correlated fluctuation analysis has become a regularly utilized tool for diagnosis of the internal structure of MHD activity in MST.

1.4 Thesis

The novel technique developed for correlation of temperature fluctuations with edge magnetic signals has proven to be a powerful approach for sparsely sampled or irregularly sampled data. The goal of this thesis is to formally expand the existing Bayesian correlation analysis framework to fully describe both the fluctuation power present in Thomson scattering signals as well as the spatial phase of those fluctuations. The primary thesis statement is: **Statement 1.** Bayesian correlation techniques between \tilde{T}_e and \tilde{B} can resolve both the amplitude $|\tilde{T}_e|$ and the relative phase δ of temperature fluctuations correlated with coherent magnetic structures.

The Bayesian statistical framework used for correlated temperature fluctuations is discussed in Chapter 2. Similar to pseudospectral techniques used in probe ensemble averaging, both cosine and sine correlations are used to determine the total fluctuation amplitude and phase.

Several applications are chosen for diagnosis of neutral-beam-heated plasmas. The neutral beam diagnostic and energetic particle studies on MST represent a rapidly expanding field with many phenomena relevant to a fusion RFP reactor as well as fundamental physics. Thomson scattering measurements of correlated temperature fluctuations are applied to a broad range of behavior in a demonstration of diagnostic versatility: equilibrium profiles, transport, and beam-driven instabilities.

Statement 2. *Rational surface measurements via tearing mode correlated temperature fluctuations afford precise determination of the safety factor in the core of MST discharges.*

Measurements of temperature fluctuations correlated with the dominant tearing modes are presented in Chapter 4. Resolution of rational surface locations with 3-6 mm accuracy is demonstrated for both reversed and non-reversed discharges. Applying these measurements to axisymmetric equilibrium reconstruction allows inferrence of the safety factor and parallel current density on axis with error bars of only a few percent (~3.5% for q_0 and ~5% for $J_{\parallel,0}$). Neutral beam heating has been shown to stabilize core MHD activity in a variety of plasma conditions, including reversed and non-reversed discharges. One proposed mechanism for this is modification of the safety factor with beam-driven current. These measurements show significant differences in neutral beam current drive between reversed and non-reversed discharges. Safety factor modification alone may be inadequate to explain observed MHD stabilization.

Statement 3. Based on deviations of δ from the phase of \tilde{B}_{θ} at the wall, the correlated product of temperature fluctuations with radial magnetic perturbations $\langle \tilde{T}_e \tilde{B}_r \rangle$ can be measured.

Due to the predicted and measured $\pi/2$ phase difference between \tilde{B}_{θ} and \tilde{B}_{r} for magnetic perturbations at the edge of MST, temperature fluctuations which deviate from a relative phase of 0 or π have non-zero correlation with radial magnetic perturbations. Focusing on non-reversed discharges without neutral beam heating, the first measurements of the correlated product $\langle \tilde{T}_e \tilde{B}_r \rangle$ in the core of MST are presented in Chapter 5. This correlated product has been connected with current transport by previous work. Although contributions from individual tearing modes appear to drive non-zero transport, different modes produce opposing current transport which largely cancels; no net current transport is observed. An analytical comparison of the impact of isotropic and anisotropic pressure tensors on the electron momentum balance indicates that anisotropy is critical for net current transport and dynamo generation. NBI is also observed to eliminate the correlated product $\langle \tilde{T}_e \tilde{B}_r \rangle$ for all modes.

Statement 4. *Temperature fluctuations correlated with low-frequency beam-driven instabilities reveal core-peaked structures.*

A new beam-driven, bursting instability has been observed near the plasma rotation frequency in reversed discharges. The first characterization of these bursts is performed in Chapter 6. A number of features distinguish these bursts from other MHD activity which is stationary in the plasma reference frame:

- Association with chirping behavior, followed by propagation only a few kHz above the plasma rotation
- Fast ion transport and participation in energetic particle mode avalanches
- Anomalous momentum generation and increased tearing mode amplitude

Correlation of electron temperature fluctuations with these bursts reveals corepeaked structures with fluctuation amplitudes of 10-15 eV. The structures are localized to within r/a = 0.2 in shallow reversal conditions, with the structure moving further inward with deeper reversal. Possible ion heating is also observed. Some speculation about the identity of this instability is presented. Theoretical predictions for fast ion effects in RFPs have largely focused on Alfvén eigenmodes, with few predictions available for energetic particle modes. The measured internal structure may serve a valuable role in validating future theoretical work.

Finally, one last note is worth mentioning. MHD activity with tearing characteristics is commonly measured and predicted in the RFP. However, for MST discharges, the mode with toroidal number n = 5 fails to meet these characteristics. Previous work has indicated internal structure more consistent with a kink instability, and several of the measurements presented here provide additional evidence supporting this identification. Kink activity also plays a critical role in the speculation of Chapter 6 regarding the identity of the bursting instability discussed above. While no definitive identification of this mode is made in this thesis, a closer evaluation of kink stability in the RFP is strongly encouraged.

1.5 References

- [1] Elisheva Coleman. Greek Fire: Nicholas Christofilos and the Astron Project in America's fusion program. Spring Junior Paper, Princeton University, 2004.
- [2] EBT Group. ELMO Bumpy Torus programme. *Nucl. Fusion*, 25(9):1271, 1985.
 doi: doi:10.1088/0029-5515/25/9/046.
- [3] K. McGuire, R. Goldston, M. Bell, M. Bitter, K. Bol, K. Brau, D. Buchenauer, T. Crowley, S. Davis, F. Dylla, H. Eubank, H. Fishman, R. Fonck, B. Grek, R. Grimm, R. Hawryluk, H. Hsuan, R. Hulse, R. Izzo, R. Kaita, S. Kaye, H. Kugel, D. Johnson, J. Manickam, D. Manos, D. Mansfield, E. Mazzucato, R. McCann, D. McCune, D. Monticello, R. Motley, D. Mueller, K. Oasa, M. Okabayashi, K. Owens, W. Park, M. Reusch, N. Sauthoff, G. Schmidt, S. Ses-

nic, J. Strachan, C. Surko, R. Slusher, H. Takahashi, F. Tenney, P. Thomas, H. Towner, J. Valley, and R. White. Study of high-beta magnetohydrodynamic modes and fast-ion losses in PDX. *Phys. Rev. Lett.*, 50(12):891, 1983. doi: 10.1103/PhysRevLett.50.891.

- [4] R. B. White, E. Fredrickson, D. Darrow, M. Zarnstorff, R. Wilson, S. Zweben, K. Hill, Yang Chen, and Guoyong Fu. Toroidal Alfvén eigenmode-induced ripple trapping. *Phys. Plasmas*, 2(8):2871, 1995. doi: http://dx.doi.org/10. 1063/1.871452.
- [5] R. N. Dexter, D. W. Kerst, T. W. Lovell, S. C. Prager, and J. C. Sprott. The Madison Symmetric Torus. *Fusion Sci. Technol.*, 19(1):131, 1991.
- [6] J. B. Taylor. Relaxation and magnetic reconnection in plasmas. *Rev. Mod. Phys.*, 58(3):741, 1986. doi: 10.1103/RevModPhys.58.741.
- S. Ortolani and D.D. Schnack. *Magnetohydrodynamics of Plasma Relaxation*. World Scientific, River Edge, NJ, 1993. ISBN 9789810208608.
- [8] D. J. Den Hartog, J. R. Ambuel, M. T. Borchardt, J. A. Reusch, P. E. Robl, and Y. M. Yang. Pulse-burst operation of standard Nd:YAG lasers. *JPCS*, 227(1): 012023, 2010.
- [9] J. A. Reusch, M. T. Borchardt, D. J. Den Hartog, A. F. Falkowski, D. J. Holly, R. O'Connell, and H. D. Stephens. Multipoint Thomson scattering diagnostic for the Madison Symmetric Torus reversed-field pinch. *Rev. Sci. Instrum.*, 79 (10):10E733, 2008. doi: 10.1063/1.2956742.
- [10] H. D. Stephens, M. T. Borchardt, D. J. Den Hartog, A. F. Falkowski, D. J. Holly, R. O'Connell, and J. A. Reusch. Calibration of a Thomson scattering diagnostic for fluctuation measurements. *Rev. Sci. Instrum.*, 79(10):10E734, 2008. doi: 10.1063/1.2964229.

- [11] W. C. Young, L. A. Morton, E. Parke, and D. J. Den Hartog. High-repetition-rate laser for Thomson scattering on the MST reversed-field pinch. *Bull. Am. Phys. Soc.*, 58(16):CP8.96, 2013.
- [12] P. C. Gregory. A Bayesian analysis of extrasolar planet data for HD 73526. *Astrophys. J.*, 631(2):1198, 2005. doi: http://dx.doi.org/10.1086/432594.
- [13] H. D. Stephens. Electron Temperature Structures Associated With Magnetic Tearing Modes in the Madison Symmetric Torus. PhD thesis, University of Wisconsin-Madison, 2010.

Chapter 2

Thomson Scattering and Calibration

The focus of the diagnostic work in this thesis is on electron temperature fluctuations measured via Thomson scattering. While electron temperature fluctuation measurements are routinely used on other devices, such measurements are typically obtained through electron cyclotron emission. ECE diagnostics offer higher time resolution than standard Thomson scattering diagnostics, which are primarily utilized to obtain equilibrium profiles. Even with the high-repetition rates available with the MST Thomson system, however, resolving fluctuations is technically challenging. Nonetheless, advanced Bayesian analysis techniques have been successfully used to combine diagnostic information from Thomson scattering and edge magnetic coils; this approach has made possible the measurement of temperature fluctuations as small as 1-2% of the mean value.

The physics of the Thomson scattering process is described in Section 2.1. In Section 2.2, recent work evaluating the feasibility of Thomson scattering polarimetry at high temperatures is summarized. Although Thomson polarimetry would not be viable on MST and is largely irrelevant to the rest of the work presented in this thesis, these results represent a significant advance in high-temperature plasma diagnostic theory. Section 2.3 provides an overview of both the laser hardware and the detection system for measuring scattered light. Analysis of the scattered light and Bayesian inferrence of electron temperature is outlined in Section 2.4. Section 2.5 details techniques for calibration of the TS diagnostic for fine resolution of temperature fluctuations. Finally, Section 2.6 describes the Bayesian analysis of temperature fluctuations. The previously developed method for obtaining the amplitude of temperature fluctuations correlated with magnetic signals is presented; improvements to this approach, which allow measurement of both phase and amplitude, are also presented.

2.1 Thomson scattering

Thomson scattering diagnostics achieve non-perturbative, internal measurements of electron temperature through plasma scattering of electromagnetic waves [1, 2]. Classically, the scattering process can be described as the acceleration of charged particles within the plasma by an incident wave and the subsequent re-radiation of light (the scattered wave) by the accelerating particles (see Figure 2.1). Although the process was first described in the classical limit by Sir J. J. Thomson OM, FRS (where the photon energy is much smaller than the particle rest mass, $\hbar \omega \ll mc^2$), it was not a viable plasma diagnostic until the advent of the laser [3, 4]. The Thomson scattering cross section is exceedingly small:

$$\sigma_{\text{Thomson}} = \frac{8\pi}{3} r_e^2 = 6.65 \cdot 10^{-29} \text{m}^2 \tag{2.1}$$

where r_e is the classical electron radius. Given the inefficiency of the scattering process, a large amount of laser power is required in order to measure a sufficient number of scattered photons for a reliable temperature estimate—typical photon densities in a TS laser pulse exceed the plasma electron densities in MST by a few orders of magnitude. In fact, the electromagnetic fields of the laser are significantly larger than the fields inside the plasma. Given 2 J pulses with widths of approximately 12 ns and a beam waist of approximately 5 mm inside the plasma, the power per unit area is given by the Poynting vector: $\langle S \rangle = \frac{1}{2} \varepsilon_0 c E_0^2$. This yields laser electric fields on the order of 10^7 V/m and magnetic fields (with $B_0 = \frac{1}{c} E_0$) on the order of 0.1 T. Compared to the plasma electric (~1 V) and magnetic (~0.1 T) fields during typical quiescent periods, $E_{\text{laser}} \gg E_{\text{plasma}}$, $B_{\text{laser}} \sim B_{\text{plasma}}$.

The electromagnetic fields of the incident laser cause the charged particles to oscillate rapidly at the laser frequency. For low-temperature plasmas the laser electric field dominates the acceleration process, but for the plasmas found in MST and other high-temperature devices the particle velocities become sufficiently relativistic for $\vec{v} \times \vec{B}$ forces to contribute significantly to particle motion. Although both electrons and ions are accelerated, the ions are much heavier and scatter negligible radiation in comparison to the electrons. The characteristics of the scattered light (wavelength, polarization, angular distribution) depend strongly on electron velocity. Since the temperature is a moment of the velocity distribution, with $\frac{1}{2}mv_{th}^2 = k_BT$, the spectrum of scattered light can be used as a measure of electron temperature.



Figure 2.1: Thomson scattering of laser radiation.

A poloidal cross section of MST is shown in Figure 2.2, with the Thomson scattering laser direction of propagation and polarization indicated. The laser is directed vertically through the center of the machine and the electric field is aligned toroidally. For electrons at rest, the angular distribution of scattered light

is the well known $\sin^2 \chi$ dipole radiation pattern, where χ is the angle between the incident electric field and the scattered wave vector \vec{k}_s . Detectors viewing in the plane defined by the incident, \vec{k}_i , and scattered, \vec{k}_s , wave vectors see maximum scattered intensity ($\chi = 90^\circ$). The vertical orientation of the laser prevents direct measurements of the temperature at the magnetic center of the device, which does not correspond to the geometric center. Due to the nature of the experimental setup, only one dimension of the three dimensional velocity distribution can be sampled. The scattering vector,

$$\vec{k} = \vec{k}_s - \vec{k}_i \tag{2.2}$$

defines the dimension along which the distribution is sampled. Note that, for different radial positions, the laser propagation \vec{k}_i does not change, while \vec{k}_s varies slightly from point to point—the portion of the velocity distribution sampled by the Thomson scattering diagnostic changes across the minor radius. Near the core of the plasma measurements predominantly sample the velocity distribution perpendicular to the magnetic field while the edge measurements pick up significant contributions from both the parallel and perpendicular distributions. However, the electron temperature equilibrates rapidly in typical magnetic confinement devices, leading to the assumption that the temperature is isotropic. For most situations, then, the single dimension of the velocity distribution that is sampled through Thomson scattering is sufficient.

For electrons at rest, the scattered photons have the same wavelength and polarization as the incident photons—the only change is the direction of propagation. Electrons in MST, however, are moving. The electron velocity Doppler shifts the scattered light:

$$\omega_{\rm s} = \omega_{\rm i} \frac{1 - \beta_{\rm i}}{1 - \beta_{\rm s}} \tag{2.3}$$

where β is the electron velocity relative to the speed of light and ω is the angular frequency. The scattered spectrum is thermally broadened via the Doppler shift. Additionally, headlighting effects due to relativistic electron velocities preferentially direct blue-shifted light toward the observer and red-shifted light away, skewing the scattered spectrum. Both the thermal broadening and the blue-shift yield



Figure 2.2: Path of the TS laser through MST. Incident and scattered wavevectors are indicated in green, as is the orientation of the incident electric field. Red lines indicate ray tracing of light from the scattering volumes to the fiber image plane.

information about the electron temperature. The scattered spectrum, originally derived by Zhuravlev and Petrov [5] and expressed in a computationally friendly form by Selden [6, 7], is given by:

$$S_{\omega}(\lambda_{s},\theta,\mu,n_{e}) = \frac{n_{e}r_{e}^{2}x^{4} \cdot q(\lambda,\mu,\theta)}{2\lambda_{i}K_{2}(\mu)\sqrt{1+x^{2}-2x\cos\theta}} \exp\left(-\mu\sqrt{\frac{1+x^{2}-2x\cos\theta}{2x(1-\cos\theta)}}\right)$$
(2.4)

where $x = \lambda_i / \lambda_s$ (note that Selden and others frequently express this in terms of $\varepsilon = \frac{1}{x} - 1$), $\mu = \frac{m_e c^2}{T_e}$ and $K_2(\mu)$ is the modified Bessel function of the second kind (with order 2) originating from the normalization of the relativistic Maxwellian distribution function. The scattering angle is defined as the angle between the incident and scattered wavevectors: $\cos \theta = \vec{k}_i \cdot \vec{k}_s / |k_i| |k_s|$. The wavelength of the

scattered radiation is λ_s , while T_e and n_e are the electron temperature and density. The factor $q(\lambda, \mu, \theta)$ represents the depolarization effects. Sample spectra for several temperatures are shown in Figure 2.3.

This spectrum is fit to measurements to infer the electron temperature (see Section 2.4 for a description of the fitting process). The term $q(\lambda, \mu, \theta)$ arises from a combination of electron headlighting effects and polarization shifts. For fully polarized incident light, the scattered light becomes partially polarized; this effect must be accounted for at temperatures above ~5 keV, where electron velocities are sufficiently relativistic. The net impact of the headlighting and polarization effects described by q is a reduction in scattered spectral intensity. For typical temperatures, though, the degree of depolarization can exceed 10% and an accurate description of q must be included in spectral calculations.



Figure 2.3: Power spectra vs wavelength for several temperatures at a scattering angle of $\pi/2$.

2.2 Depolarization of Thomson scattered light

Due to the dependence of the depolarization of Thomson scattered light on electron temperature, polarimetric techniques have been suggested as an alternative to the usual spectral decomposition approach to Thomson scattering for plasmas above 10 keV [8, 9]. The technique proposed in Ref. [8] involves up to four measurements of the polarization properties of the scattered light—the Stokes vector components—and has the potential to be simpler to implement. Furthermore, polarimetric Thomson scattering measurements could serve as an independent calibration or cross-check of spectral Thomson scattering measurements. Expected diagnostic error bars calculated in Ref. [8] compared well with spectrally resolved Thomson predictions for fusion-grade temperatures. However, a number of assumptions and constraints limit the results in Ref. [9]. They are not universally valid for the whole range of experimental parameters such as electron temperature, scattering angle, and incident laser polarization state. These restrictions prevent a full optimization of the diagnostic scheme. Furthermore, the work in Ref. [9] contains an error in the weighting factor for averaging over the electron distribution function.

Following the previously developed Stokes vector approach, the degree of polarization for incoherent Thomson scattering can be calculated analytically. Exact results valid for the full range of incident laser polarization states, scattering angles, and electron temperatures have been obtained. The results and analysis presented in this section are adapted from a paper presented at the 16th Annual Laser-Aided Plasma Diagnostics Conference in 2013 [10] and based on the exact description of the degree of polarization presented at the IAEA Fusion Energy Conference in 2012 and the International Conference on Fusion Reactor Diagnostics in 2013 [11, 12]. A brief outline of the approach and key theoretical results are provided. These results are then applied to the diagnostic proposed in Ref. [8].

Theoretical results

For the typical, wavelength resolved Thomson scattering diagnostic, the wave scattered by a single electron is calculated in the far-field zone, Fourier transformed, converted to a power spectrum, and then integrated over the electron distribution function [1, 2]. This yields the spectrum of Eqn. 2.4. A more general approach following Stokes vector and Mueller matrix formalism allows description of the conversion of fully polarized incident light to partially polarized scattered radiation.

Both the scattered field, $S^{(s)}$, and the incident field, $S^{(i)}$, are expressed in Stokesvector form as $\mathbf{S} = (S_0, S_1, S_2, S_3)$. Here, the S_0 component corresponds to the total intensity of the wave and the remaining components describe the polarization properties. The polarization is characterized by two parameters: the ellipticity, χ , where $\chi = 0$ for linear light and $\chi = \pi/4$ for circular light, and the orientation angle, ψ , between the the major axis of the polarization ellipse and the normal vector to the scattering plane ($\psi = 0$ for light aligned perpendicular to the scattering plane and $\psi = \pi/2$ for light parallel to the scattering plane). Writing the Stokes vector in terms of χ and ψ yields $\mathbf{S} = (S_0, S_0 \cos 2\chi \cos 2\psi, S_0 \cos 2\chi \sin 2\psi, S_0 \sin 2\chi)$ for completely polarized light. From the incident and scattered electric fields, a 4×4 Mueller matrix describing the scattering process can be constructed: $\mathbf{S}^{(s)} = \mathbf{M} \cdot \mathbf{S}^{(i)}$. This matrix is then integrated over the electron distribution function (assumed here to be an isotropic, relativistic Maxwellian). Most of the matrix elements are zero or integrate to zero. The Mueller matrix can be written $\mathbf{M} = \mathbf{C} \cdot \mathbf{m}$, where $C = \frac{1}{2}r_e^2 n_e V$ is a normalization constant. The relevant terms of the (unnormalized) Mueller matrix are:

$$\begin{split} m_{00} &= 1 + u^2 - 2G(\mu)(u^2 + 4u - 3) + (16/\mu^2)(1 - u)^2 \\ m_{01} &= m_{10} = 1 - u^2 \\ m_{11} &= 1 + u^2 + 2G(\mu)(u^2 - 4u + 1) + (12/\mu^2)(1 - u)^2 \\ m_{22} &= 2u - 4G(\mu)(u^2 - u + 1) - (12/\mu^2)(1 - u)^2 \\ m_{33} &= 2u - 4G(\mu)u(2u - 1) - (8/\mu^2)(1 - u)^2 \end{split}$$
 (2.5)

Where $u = \cos(\theta)$ represents the scattering angle dependence and

$$G(\mu) = \frac{K_1(\mu)}{\mu K_2(\mu)}$$
(2.6)

represents the temperature dependence, with $\mu = m_e c^2/T_e$ as before. K₁ and K₂ are modified Bessel functions of the second kind.

The work in Ref [9] contains an error in the power of the term $\kappa = (1 - \beta_s)$ originating in the Liénard-Wiechert expression for the scattered field. While there is contention in the literature over the power of this term for an infinite scattering volume, for a finite volume the appropriate term is unambiguous. At fusion-grade temperatures this can introduce non-negligible error. The results above were calculated with the correct power.

For incident light of arbitrary intensity and polarization, the scattered light can be expressed in terms of the Mueller matrix elements: for example, $S_0^{(s)} = M_{00} \cdot S_0^{(i)} + M_{01} \cdot S_1^{(i)}$. For fully polarized incident light, with $S_0^2 = S_1^2 + S_2^2 + S_3^2$, scattered light from a single electron remains completely polarized, but due to the nature of the electron distribution function, light scattered from many electrons will include photons of many different polarization states. This is described by the degree of polarization, $P = \sqrt{S_1^2 + S_2^2 + S_3^2}/S_0$, which can range from 0 to 1, and the degree of depolarization, D = 1 - P. The degree of depolarization should not be confused with the "depolarization factor" from the spectral decomposition approach mentioned above, which denotes a reduction in scattered intensity.

Diagnostic implications

From the theoretical results above, a few constraints on diagnostic design can be determined. Both scattering angle and incident laser polarization are important parameters for Thomson scattering diagnostics. Most diagnostics operate with scattering angles near $\theta = \pi/2$ but covering a wide range, while the LIDAR Thomson system proposed for ITER would operate near $\theta = \pi$ [13]. Universal use of linearly polarized incident light with the electric field aligned perpendicular to the scatter-


Figure 2.4: Degree of depolarization for linearly polarized incident light with $\psi = 0$.

ing plane originates in the simplicity offered to scattering spectrum calculation as well as optimization of scattered intensity for cold electrons.

Figure 2.4 shows the degree of depolarization across the full range of scattering angles and fusion relevant temperatures for linearly polarized light with $\psi = 0$. Depolarization is strongest near perpendicular scattering, although not exactly at $\theta = \pi/2$. The exact angle of maximum depolarization is temperature dependent, and these results are consistent with the findings of Ref. [9] showing maximum depolarization deviating slightly from $\pi/2$. However, this is only a local maximum due to the choice of linearly polarized light—the true maximum occurs for elliptically polarized light, see Figure 2.5(a).

Far away from $\pi/2$ scattering, the depolarization drops off rapidly. For both forward and backward scattering, the degree of depolarization is no more than a few percent for expected reactor temperatures. A polarization-based Thomson scattering technique would be highly unsuitable for diagnostics like the proposed ITER LIDAR system, while traditional Thomson scattering diagnostics near $\theta = \pi/2$ would offer greater depolarization with stronger temperature dependence.

Evaluating the effectiveness of different laser polarization parameters is slightly



Figure 2.5: (a) Depolarization and (b) predicted relative error in temperature measurement for 20 keV plasmas at $\pi/2$ scattering angle with varying incident laser polarization (ellipticity χ and orientation angle ψ).

more complicated. Some configurations offer strong degrees of depolarization, but weak scattered intensity. In Fig. 2.5(b), the predicted relative error in the temperature measurement is plotted against both the orientation angle and the ellipticity. This includes only Poisson statistics for the scattered photons and neglects background light; background signal is accounted for in the diagnostic simulations of the next section. The region near $\psi = \pi/2$ exemplifies the high depolarization, low scattered intensity trade off: large measurement errors limit diagnostic viability and the features are highly sensitive to electron temperature, making this section of parameter space one to be avoided.

The minimum error corresponds to circular polarization. Thomson scattering literature focuses almost exlusively on linear polarization, and even Ref. [9] is restricted in analysis to linearly polarized light. These results highlight the versatility of this approach and the need to consider all incident polarizations. However, it should be noted that, for the purposes of a Thomson polarization diagnostic, linearly polarized light with $\psi = 0$ can achieve error bars competetive with circularly



Figure 2.6: Polarization measurement design adapted from Ref. [8], with modifications for elliptically polarized incident light.

polarized light.

Diagnostic design and simulation

Using the insight developed above, the viability of a polarization-based Thomson scattering diagnostic can be evaluated by applying the theoretical results to the proposed design in Ref. [8]. The original design was intended for linearly polarized incident light with $\psi = \pi/4$, so the diagram in Figure 2.6 has been modified slightly to enable measurements with elliptically polarized light. The four Stokes components are related to six measureable intensities, although only four of the measureable quantities are independent. For the modified diagnostic proposal, three of the intensity measurements from the original design are chosen: without phase retardation, the intensity is measured after polarization selection at angles of 0°, 90°, and 135° relative to an electric field with $\psi = 0$. The fourth independent intensity measurement is obtained following phase retardation of $\pi/2$ and polarization selection at 135° (corresponding to right-hand circularly polarized light). The only modification to the original design requires removing the half-wave plate and

second Glan-Thompson prism, replacing them with the beamsplitter, quarter-wave plate, and appropriate polarizers in the lower right of the diagram. This would lead to a slightly more complicated mounting arangement, but allows measurements with elliptically polarized incident light. The Stokes vectors are calculated from the measured intensities using the following relations:

$$S_{0} = I_{0^{\circ}} + I_{90^{\circ}}$$

$$S_{1} = I_{0^{\circ}} - I_{90^{\circ}}$$

$$S_{2} = I_{0^{\circ}} + I_{90^{\circ}} - 2I_{135^{\circ}}$$

$$S_{3} = I_{0^{\circ}} + I_{90^{\circ}} - 2I_{135^{\circ}}^{\pi/2}$$
(2.7)

Note that $I_{\text{Stokes}} = |E|^2$ is proportional to the usual intensity definition $I_{\text{Poynting}} = \frac{1}{2} \epsilon_0 c |E|^2$.

The simulated diagnostic utilizes a laser capable of producing 5 J pulses at 1064 nm wavelength with an integration time of 10 ns. The beam waist is 20 mm and the scattering volume has length 55 mm, while the collection optics subtend a solid angle of 1.19 msr. The scattering angle is $\theta = \pi/2$ from a core location with variable electron temperature $T_e(0)$. The background light is assumed to be entirely bremsstrahlung, ignoring line radiation. To model the bremsstrahlung, the nonrelativistic approach in Ref. [1] is used. Relativistic effects on the bremsstrahlung spectrum are ignored as they account for less than a 10% effect at the temperatures and wavelengths used here [14]. The Gaunt factor, however, is approximated as 7. The electron temperature and density profiles are assumed to be parabolic, $T_e(r) = T_e(0) \cdot (1 - r^2)$, with core electron density of $1 \cdot 10^{20}$ cm⁻³. The background light is integrated along a 4 m line of sight and over wavelengths in the range 200-2000 nm to accomodate the full width of the scattered spectrum. These parameters are chosen for similarity with ITER.

The simulated diagnostic error bars are calculated for core temperatures ranging from 1 keV to 50 keV. Three cases are compared: $\chi = \pi/4$, (ψ,χ) = (0,0), and (ψ,χ) = ($\pi/6$,0). The circularly polarized light utilizes the full, four-component polarimeter shown, while the linear cases utilize reduced forms: measurement of only I_{0°}, I_{90°},

and I_{135° for the $(\psi, \chi) = (\pi/6, 0)$ case and further reduction to only I_{0° and I_{90° for the $(\psi, \chi) = (0, 0)$ case. The reduced forms of the polarimeter benefit from improved scattered signal amplitude on the measured channels.

The polarization measurement error, σ_P , is related to the error on each of the statistically independent intensity measurements, σ_{I_i} , in the standard manner:

$$\sigma_{\rm P}^2 = \sum_{\rm j} \left(\frac{\partial P}{\partial I_{\rm j}}\right)^2 \sigma_{I_{\rm j}}^2 \tag{2.8}$$

and the intensity measurement errors are determined by Poisson statistics on both the scattered laser signal and background bremsstrahlung. The $\partial P/\partial I_j$ terms evaluate to

$$\frac{\partial P}{\partial I_{0^{\circ}}} = \frac{1}{S_{0}} \left(-P + \frac{S_{1} + S_{2} + S_{3}}{PS_{0}} \right) \\
\frac{\partial P}{\partial I_{90^{\circ}}} = \frac{1}{S_{0}} \left(-P + \frac{-S_{1} + S_{2} + S_{3}}{PS_{0}} \right) \\
\frac{\partial P}{\partial I_{135^{\circ}}} = -\frac{2S_{2}}{PS_{0}^{2}} \\
\frac{\partial P}{\partial I_{135^{\circ}}} = -\frac{2S_{3}}{PS_{0}^{2}}$$
(2.9)

From the error in the polarization measurement, the relative error in the temperature measurement is

$$\frac{\sigma_{T_e}}{T_e} = \frac{\sigma_P}{\mu \frac{\partial P}{\partial \mu}}$$
(2.10)

The term $\partial P/\partial \mu$ is calculated numerically, as the analytical form is excessively complicated for arbitrary incident laser polarization.

The results are shown in Figure 2.7. Above 9 keV, all cases achieve error bars of less than 5%, and less than 2% above 23 keV, making them competitive with standard Thomson scattering diagnostics. The circular polarization case offers the best performance for the full, four-component polarimeter, and even outperforms the reduced three-component variation. However, the two-component polarimeter



Figure 2.7: Predicted error in temperature measurement for three implementations of the polarization diagnostic: circular, linear with $\psi = \pi/6$, and linear with $\psi = 0$.

achieves the best results across the full range of temperatures, with error bars below 1% at 50 keV. The flatness of the curves above 20 keV indicates that these diagnostics would be robust over a wide range of fusion-relevant temperatures.

Polarization over a narrow wavelength range

While the scattered spectra are very broad for fusion-grade plasmas, the results in Eqn. 2.5 require integration over all wavelengths. As indicated by the integration of background bremsstrahlung from 200 to 2000 nm in the estimate above, the need to measure the total scattered power leads to several technical challenges. A significant fraction of the bremsstrahlung background is radiated at shorter wavelengths, and such a wide window will also include substantial contributions from line radiation (especially H_{α}). For silicon APDs commonly used in TS polychromator systems, the response is typically low below 400 nm and above 1100 nm, cutting off the red-shifted portion of the spectrum. Additionally, most optical components do not

have uniform response over such a wide range. Frequency-doubled Nd:YAG lasers at 532 nm would produce narrower scattered spectra, but extending deeper into the UV where bremsstrahlung contributions are larger and APD responsivity is low. Choosing a laser with longer wavelength would broaden the spectrum further.

Realistic experimental constraints require accepting only scattered photons within a limited wavelength range. Spectral analysis simplifies evaluation of the feasibility of a polarimeter under these conditions, so the Stokes formalism is not used here. The analysis is resticted to the case of linear incident laser light with $\psi = 0$, where only the signals I_{0° and I_{90° need to be evaluated. A spectrum of the same form as Eqn. 2.4 is desired, with the factor q dependent on the polarization state of the scattered light. For I_{0° , the depolarization factor is calculated for scattered light with polarization parallel to the incident light. Rather than calculate the depolarization factor explicitly for scattered light polarizated perpendicular to the incident light, the total depolarization factor for light of all polarization states is calculated, and I_{90° is determined from the difference between the two depolarization factors. Following the approach in Ref. [8], the scattered spectrum is integrated separately for these two cases. From Ref. [1], the general expression for the scattered power is given by:

$$\frac{\mathrm{dP}}{\mathrm{d}\omega_{\mathrm{s}}} = n_{e}r_{e}^{2}\langle S_{\mathrm{i}}\rangle V\mathrm{d}\Omega \int |\Pi \cdot \hat{e}|^{2}f(\beta)\,\delta\left[c\left(\mathbf{k}_{\mathrm{s}}-\mathbf{k}_{\mathrm{i}}\right)\cdot\boldsymbol{\beta}-\left(\omega_{\mathrm{s}}-\omega_{\mathrm{i}}\right)\right]\mathrm{d}^{3}\boldsymbol{\beta} \qquad (2.11)$$

where $\Pi \cdot \hat{e}$ is a polarization tensor on the incident field, $\langle S_i \rangle$ is the incident power (Poynting), and f (β) is the electron distribution function. For clarity, P is reserved for degree of polarization while $dP/d\omega_s$ or $dP/d\lambda_s$ denote the power spectrum. For linearly polarized laser light (with $\psi = 0$) and scattered light selected with polarization parallel to the incident light, this spectrum has been well studied in the

literature. The (1,1) approximation published by Naito, et. al [15] is chosen here:

$$\begin{pmatrix} \frac{dP}{d\lambda_s} \end{pmatrix}_{\parallel} = S_{\omega} \cdot q_{\parallel}$$

$$S_{\omega} = \frac{n_e r_e^2 \langle S_i \rangle V d\Omega x^4}{2\lambda_i K_2(\mu) \sqrt{1 + x^2 - 2x \cos \theta}} \exp\left(-\mu \sqrt{\frac{1 + x^2 - 2x \cos \theta}{2x(1 - \cos \theta)}}\right)$$

$$q_{\parallel} = 1 - 4\eta \zeta \frac{2\zeta - \eta(2 - 3\zeta^2)}{2\zeta - \eta(2 - 15\zeta^2)}$$

$$(2.12)$$

where $x = \omega_s / \omega_i$ and

$$\zeta = zy$$

$$\eta = \frac{y}{\mu}$$
(2.13)

with

$$z = \sqrt{1 + \frac{(x-1)^2}{2x(1-\cos\theta)}}$$

$$y = \frac{1-\cos\theta}{\sqrt{z^2 + \sin^2\theta}}$$
(2.14)

The notation for x, y, z, and μ used here is adapted slightly from the ϵ , y, x, and α notation used in Ref. [15]. This is done simply for consistency with the rest of the notation used in this thesis.

For scattered light with all polarization states present, the form of the polarization tensor is [16]:

$$|\Pi \cdot \hat{\mathbf{e}}|^{2} = (1 - \beta^{2}) \frac{\omega_{s}^{2}}{\omega_{i}^{2}} \left[1 - (1 - \beta^{2}) \left\{ \frac{\beta_{e}(1 - \cos\theta)}{(1 - \beta_{s})(1 - \beta_{i})} \right\}^{2} \right]$$
(2.15)

This case is not well studied, so the scattering spectrum is also calculated in the form

$$\left(\frac{\mathrm{dP}}{\mathrm{d\lambda}_{\mathrm{s}}}\right)_{\mathrm{tot}} = S_{\omega} \cdot q_{\mathrm{tot}} \tag{2.16}$$



Figure 2.8: Predicted error in temperature measurement for different wavelength integration ranges: 200-2000 nm (red curve) to match the Stokes vector calculations, and several narrower ranges (black curves) to more accurately represent measurement conditions.

where q_{tot} is obtained by numerically integrating Eqn. 2.11 using the form of the polarization tensor given in Eqn. 2.15. From this, the measured signals are $I_{0^{\circ}} = \left(\frac{dP}{d\lambda_s}\right)_{\parallel}$ and $I_{90^{\circ}} = \left(\frac{dP}{d\lambda_s}\right)_{tot} - \left(\frac{dP}{d\lambda_s}\right)_{\parallel}$, and the polarization P can be evaluated by integration of the spectra over arbitrary wavelength ranges. For sufficiently wide wavelength windows, predictions with this approach agree with the Stokes vector approach. Figure 2.8 shows the comparison of several different windows. Reducing the range of integration to match APD sensitivity, and further reducing the range to cut down on bremsstrahlung are beneficial for diagnostic error bars. Although the windows reject significant numbers of scattered photons, they reject a greater fraction of the background. Windows of 100 nm or less lead to higher error at high T_e, but are still below 3%. Appropriate choice of window also reduces the minimum T_e at which such a diagnostic could operate.

Advantages and implementation challenges

A polarization-based diagnostic offers several advantages over the polychromator design common now. With a maximum of four channels per radial position, the polarization diagnostic could translate into cost savings in detector hardware and digitizer channels. The need for fewer measurement channels would be less operationally demanding, while fewer optics make for a simpler, more robust diagnostic. The two-component form of the polarimeter maximizes these advantages. The primary challenge to implementing a TS polarimeter is preserving the polarization state of the scattered light, which can potentially be altered by both the plasma, through Faraday rotation, and the collection optics.

While Faraday rotation of far-infrared laser light is regularly used as a diagnostic, for the shorter wavelength light commonly used in Thomson scattering the estimated effect of Faraday rotation is negligible. Overestimating the contribution with a 5 T magnetic field and a 4 m scattering path length parallel to the magnetic field, the Faraday effect rotates scattered light at 1064 nm by $5.96 \cdot 10^{-4}$ radians. Blueshifted light will rotate less, and the actual path for both incident and scattered light will mostly be perpendicular to the magnetic field. Even at a few keV, the perpendicular polarized Thomson scattered photons will dominate the Faraday rotation contribution. Cotton-Mouton effects on scattered ellipticity are estimated to be several orders of magnitude smaller.

The collection optics pose a greater challenge. While the mirror system proposed for ITER (or the lens systems on current devices) should not significantly alter polarization, the same is not true of the fibers used to transport the collected light to detectors. Most fibers do not preserve polarization; fibers capable of preserving polarzation in the wavelength range of interest are available, but expensive. Additionally, while the original design in Ref. [8] makes optional use of a half-wave plate to simplify the arrangement of detectors for the $I_{-45^{\circ}}$ and $I_{+45^{\circ}}$ components, the full, four-component polarization meter suggested here requires the use of a quarter-wave plate to measure the $I_{135^{\circ}}^{\pi/2}$ component of the scattered light. Currently available waveplates do not have a uniform response over the wide ranges of wave-

lengths needed, making the polarization meter unfeasible for circularly polarized light. While the three- and two-component polarimeters for linear incident light do not suffer from the non-uniform responses of the wave plates, even the available Glan-Thompson prisms for near-IR wavelengths do not have uniform response over thousands of nanometers.

Given these constraints, two possible implementations of a two-component Thomson polarimeter are suggested. With polarization-preserving fibers, existing polychromators could be modified with an additional channel to measure perpendicularly polarized light. At low T_e , the diagnostic would function as a polychromator, and at high T_e it could function as a polarimeter by summing the spectral bins for measurement of parallel polarized photons. Alternatively, the Glan-Thompson prisms could be mounted directly in the cassette to split the scattered light into parallel/perpendicular components before the fibers. This would double the number of fibers necessary but eliminate the need for specialized polarization preserving fibers.

It is fortuitous that the two-component polarimeter is both the most feasible option in light of the technical challenges and the best performing. Given the sub-percent error bars it is predicted to be capable of, even line radiation and APD response (quantum efficiency and additional noise enhancement factor) contributions to the errors should be well within the 10% specifications required for ITER. Since existing devices are capable of achieving sufficiently high electron temperature, experimental feasibility studies could happen in the near future.

2.3 TS hardware

The hardware for the Thomson scattering diagnostic can be divided into two groups: the laser system and beamline for generating and delivering the incident beam, and the light collection and detection system for measuring Thomson scattered photons. The Thomson diagnostic has been through significant upgrades over the course of this thesis work: the Spectron lasers have been realigned and readjusted for improved pulse shape and pulse energy stability, additional channels have been added to 15 of the 21 polychromators for improved spectral resolution, the APD amplifiers have been upgraded, and new digitizers have been installed for background light measurements. However, the basic functionality of the system remains largely the same: 2 J laser pulses at repetition rates up to 25 kHz, simultaneous scattered light detection from 21 radial positions in the 700 nm to 1100 nm wavelength range, and sufficient resolution for temperature measurements across the full range of temperatures seen in MST with error bars typically between 5 and 15 percent.

Laser system

The laser hardware consists of two commercially available Spectron SL858 Nd:YAG lasers with modified power supplies and Pockel cell drivers. The Spectron lasers produce nominal 2 J pulses at 1064 nm with a width of 9 ns (FWHM) [17]. Originally able to produce only one laser pulse per flashlamp pulse, the Pockel cell drivers were upgraded in 2007 to use Bergmann Messgeräte Entwicklung KG drivers (Part No. ds11d/KD*P). These drivers reduced electrical noise and enabled operation with multiple laser pulses per flashlamp pulse. The minimum time separation for this system, based on the physical characteristics of the Pockel cell, flashlamp and laser rod, is 80 μ s. In 2008, the power supplies were replaced with the QXF54 and Flash Lamp Power Supply Control System (FLPSCS) designed and built by the Physical Sciences Laboratory. The new power supplies and control system allow operation with flash lamp pulses of variable width at repetition rates up to 1 kHz [18].

The modified Spectron system, interleaving the pulses from each laser, can operate in continuous or burst operation as shown in Figure 2.9. In continuous operation, the maximum attainable repetition rate is 2 kHz for up to 15 ms (the heat load on the flash lamps establishes a limit of 15 pulses per laser). This mode of operation is frequently used for equilibrium temperature profile measurements, but is largely unsuitable for fluctuation measurements. In burst operation, as many as four laser pulses can be fired during a single flashlamp pulse for a single laser, yielding up to eight pulses at 25 kHz. These bursts can be repeated at a rate of 1

kHz for several milliseconds (again limited by the flash lamp heat load). While the burst mode is limited in duration (both the number of pulses in each burst and the total number of bursts), it provides much higher time resolution than continuous operation. All data taken for this thesis uses the burst mode of operation, either triple pulse (6 total pulses per burst for 5 bursts) or quad pulse (8 total pulses per burst for 3 bursts).



Figure 2.9: TS scattering operational modes: continuous (top) and burst (bottom). The burst mode shown is for quad-pulse, while triple-pulse operation would yield 5 bursts with a total of 30 pulses.

Beamline

The Thomson laser system is housed in a separate room from the MST machine area, requiring a 15 m travel path before the laser photons reach the scattering volume in the plasma. The beamline consists of five turning mirrors, a focusing lens, two brewster windows and terminates in a beam dump outside of the vacuum vessel [17]. The focusing lens reduces the beam waist to guarantee that the laser is within the scattering volume. Although the two lasers are spatially separated over most of the beamline (partly as an unavoidable consequence of using two lasers and partly to avoid unnecessary damage to the mirrors), the paths converge inside the vacuum vessel so that both lasers fall within the scattering volume. The exact spatial separation between them, however, is difficult to measure and they are assumed to be close enough within the scattering volume that measurements of temperature and density are at the same location.

At each turning mirror, a camera images light passing through the mirror to determine the location of each laser on the mirror. The combined system of cameras and mirrors is controllable through a LabView interface (both automatic and manual). The adjustable mirrors facilitate alignment of the laser beam, as well as maintaining alignment through a run day. The beam line is sufficiently long that thermal expansion and contraction of the building has a noticeable effect on laser alignment over the course of a single day (especially in the summer). Adjustment of the mirrors from shot to shot prevents alignment drift and reduction in scattered signal intensity.

Collection and detection

Located at 20° poloidal and 222° toroidal, the Thomson collection optics consist of a seven-element lens system. During use, the lens system is inserted through a 4.5 inch port so that the front element is flush with the outer wall of the vacuum vessel. The port size is the largest allowable on MST, due to generation of field errors at the edge. A boron-nitride limiter at the edge of the port protects the lens from plasma interaction, but the front element still requires regular replacement. When not in use, the retracted optics are protected by a gate valve.

The lens system images the central, vertical chord of the laser beam onto an image plane which coincides with the front faces of 23 fiber bundles. The radial resolution of the images is ~1 cm in the core and ~2 cm at the edge, and the transmission of the lens system is ~85% from 700-1100 nm. The fibers transport the scattered photons back to the laser room, where 21 polychromators are rack mounted behind screens to prevent stray laser light in the room from contaminating the scattered signal. The polychromators, General Atomics model No. GAPB-1064-4-1K [19], separate the scattered light into multiple spectral bins between 1065 nm and 700 nm. Fifteen of the polychromators were upgraded from 2011-2012 to utilize six spectral bins,

while the remaining six polychromators have eight spectral bins (see Figure 2.16). The light is binned with notch filters, which pass light in a narrow band. Light accepted by a filter is focused onto an avalanche photodiode (APD) (EG&G part No. C30956E [20]), while the rejected light is reflected to the remaining notch filters and detectors.

The photons incident on the APD surface generate photoelectrons, governed by the quantum efficiency, η , of the photodiode. The photoelectrons are accelerated by electric fields within the diode, and through the avalanche region generate additional electrons with an average multiplication factor, M. The avalanche process also increases the signal noise: while the shot noise of the incident scattered photons is a Poisson process (\sqrt{N}), the avalanche multiplication contributes additional noise beyond Poisson statistics. The noise increases to \sqrt{FN} , where F is the noise enhancement factor describing the additional avalanche noise (F \ge 1). The APD output is a current signal, which is converted to voltage by an amplifier, with gain G, designed by General Atomics [21]. 84 of the amplifiers are an early generation design from General Atomics, while an additional 54 amplifiers have been constructed in-house from the same design. The newer amplifiers have only minor corrections to the bias resistors for improved stability with newer op-amps whose design specs have changed since the 1990s.

Because the APD gain is temperature dependent, the polychromators are watercooled to provide stable operating temperatures. During a run day, small temperature drifts can be tolerated as the gain drifts only 2-3% per °F. However, during calibrations, the tolerance for temperature drifts is much lower and the APD temperature must be within 0.4 °F over a 40-60 minute period.

Each amplifier gives two output signals: a DC voltage signal and a delay-line subtracted 'AC' signal. Typical DC and AC voltage traces are shown in Figure 2.10. All 138 DC signals are digitized at 10 ns time resolution over a 2V full-scale range with 16 bit resolution (Struck SIS3302). The 2 V full scale covers the entire linear operating range of the APD/amplifiers and allows digitization of the full signal—both scattered laser light and background plasma light. The negative-going pulse in the DC signal at ~1300 ns is characteristic of the APD response to Thomson scattered

laser light: the rapid increase in number of photons at the detector drives significant photoelectric current and a more negative signal, but due to the short duration of the laser pulse the APD current signal quickly returns to the background level (at the APD-amplifier time constant of ~40 ns). The AC signals are digitized at 1 ns time resolution over a much smaller full scale, typically 200 mV, with 8 bit resolution (Acqiris DC270). Due to the delay-line subtraction, the initial, negative-going pulse in the output is followed ~100 ns later by a positive-going pulse with marginally reduced amplitude. This results in the bipolar waveform shown. The advantage of the delay-line subtraction on the AC signals is that the slowly varying background is largely removed from the signal, leaving only the pulsed signal from the scattered laser light. The full scale can then be set to match the laser pulse height without regard to the background photons. Although the Acqiris has only 8 bit resolution compared to the 16 bit resolution of the Struck, the bit noise of the integrated scattered signal is minimized on the Acqiris with the lowest full-scale setting (50 mV) and 1 ns time resolution.



Figure 2.10: Signal with time changing background, measured with both DC and delay-line subtracted AC. Data from shot 1140122040, Poly 10 Channel 3, with r/a = 0.14 at time 24.551 ms

2.4 Calculating electron temperature

Using the recorded signals, an electron temperature can be inferred for each laser pulse at each radial position. First, for each spectral 'bin' curve fitting to the recorded signals distinguishes between the background and scattered contributions to the measured light. The measured values of scattered light are compared to a model signal based on the temperature and density dependent scattering spectrum from Section 2.1. From the comparison of measured to modeled signal, the probability that the modeled temperature and density accurately represent the plasma values is computed; the temperature and density are then inferred from the maximum of the probability distribution.

Bayesian statistics provide the foundation for the probability calculations used in plasma parameter inferrence. This approach has two significant advantages. First, Bayesian probability lends a useful framework for combining information from all spectral channels of the TS diagnostic (and from multiple diagnostics for flucutuation measurements discussed later). Secondly, it allows a straightforward propagation of errors. For an introduction to Bayesian statistics see [22], or [23] for an in-depth review of the applications of Bayesian inferrence across a widerange of physical fields. Bayesian analysis has been successfully applied to the MST Thomson scattering diagnostic [24] as well as TS diagnostics on other devices [25]. Bayesian analysis also plays a crucial role in 'data fusion' and Integrated Data Analysis [26].

Determining the measured signal

The curve fits to the digitized signals consist of a polynomial background (up to order 2 given the ~0.6 μ s window used) and a characteristic pulse representing the scattered signal. See Figure 2.11. The characteristic pulse is the known response of the APD/amplifiers to the Spectron laser pulses. It has been measured by directing stray laser light into the daily calibration inputs for each of the polychromator channels and averaging over multiple pulses. The amplitude of the characteristic pulse fits to the signal yields the total integrated scattered signal (in Volt-seconds).

Using a characteristic pulse rather than simply integrating the signal leads to an improved signal-to-noise ratio [27].



Figure 2.11: Voltage data with background polynomial fit and characteristic pulse fit. Data from shot 1130904063 and Poly 14, Channel 2, at r/a = 0.106 and time 23.245 ms

Bayesian analysis

A model signal is then constructed, using the known characteristics of the APD/amplifiers and polychromators. The total scattered signal is the product of the scattered spectrum with the polychromator notch filter transmission, the APD quantum efficiency and avalanche multiplication, and the amplifier gain. The transmission of the collection lens and fiber bundles also determines the scattered signal intensity reaching the detectors; however, calibrating for this transmission is not currently feasible and, to lowest order, only affects the density measurement, so these are not accounted for in the model signal. The noise in the signal is modeled as enhanced Poisson noise, using the APD noise enhancement factor. The model signal S_{model} (in Volt-seconds) is given by:

$$S_{model} = \int S_{\omega}(\lambda, T_e, n_e) G M \eta(\lambda) T(\lambda) d\lambda$$
(2.17)

where S_{ω} is the scattered spectrum (photons/nm) at the specified plasma conditions. The amplifier gain (G) is expressed in Volt·seconds/photon, while the avalanche multiplication factor (M), quantum efficiency (η), and the transmission (T) are unitless. The noise is

$$\sigma = \sqrt{\mathsf{GMF} \cdot \mathsf{S}_{\mathsf{measured}}} \tag{2.18}$$

where $S_{measured}$ is also in Volt-seconds and the noise enhancement factor (F) is unitless.

Bayes' Theorem relates the probability that a model or inference (X) is true given the data (D)—a difficult quantity to determine but one that is very interesting—to the probability that the data is true given the model (a much easier quantity to calculate). Also important is additional background information (I).

$$P(X|D, I) = \frac{P(D|X, I)P(X|I)}{P(D|I)}$$
(2.19)

P(D|X, I) is the likelihood, representing the probability that the data is true given the model. P(X|I) is the prior probability and represents information already known (i.e. - the temperature must be positive, finite, and presumably within the range of measurement for the diagnostic). P(D|I) is known as the evidence and is important for model comparison. When dealing with parameter estimation using a single model, however, its only purpose is as a normalization coefficient. P(X|D, I), called the posterior probability, represents the full knowledge of the inference X incorporating the results of the measurement.

Applying Bayes' theorem to Thomson scattering measurements, the probability that the plasma has temperature T_e and density n_e given the measured signal D

with error σ can be written:

$$P(T_e, n_e | D, \sigma, I) = \frac{P(D|T_e, n_e, \sigma, I)P(T_e, n_e|I)}{P(D|I)}$$
(2.20)

Since P(D|I) is only important for normalization, it can be neglected. $P(T_e, n_e|I)$ is assumed to be uniform over the measurement range (~10-5000 eV). Finally, assuming that the signals are independent and Gaussian distributed, the likelihood reduces to the usual, χ^2 representation and the posterior can be written:

$$P(T_e, n_e | D, \sigma, I) = \frac{1}{\prod_{j=1}^{N_D} \sqrt{2\pi}\sigma_j} \exp(-\frac{1}{2}\chi^2)$$
(2.21)

$$\chi^{2} = \sum_{j=1}^{N_{D}} \frac{[S_{APD,j} - S_{model,j}]^{2}}{\sigma_{j}^{2}}$$
(2.22)

where j indicates channel number and N_D is the total number of channels for the polychromator. This leads to a two-dimensional probability distribution as shown in Figure 2.12, and the maximum probability indicates the most likely T_e , n_e combination. To obtain one-dimensional distributions, one parameter is treated as a 'nuisance' parameter and marginalized out via integration. For example, marginalizing density:

$$P(D|T_e) = \int P(D|T_e, n_e) dn_e$$
(2.23)

and the error is estimated from the width (1/e) of the one-dimensional probability distribution.

2.5 Calibration for fluctuation measurements

Accurate temperature measurements require a complete characterization of the detector response to the scattered laser light. This is especially true for temperature fluctuation measurements, where fluctuations of only a few eV or less may lay



Figure 2.12: Two-dimensional probability grid, with marginalized one-dimensional PDFs. Data from shot 1130904063 and Poly 14, at r/a = 0.106 and time 23.245 ms

buried under equilibrium temperatures of several hundred eV and a much larger contribution from shot noise. As discussed above, the model signal depends on the spectral transmission of the polychromators and the gain of the APD/amplifiers. Those two pieces of information are sufficient to determine the relative signal levels on each channel for a given temperature. Additionally, measurement of the APD/amplifier noise enhancement is necessary for accurate treatment of the photon noise in the measured signal. The calibration of the detectors is consequently done in two phases: simultaneous measurement of the Spectral transmission of the fully assembled polychromators. The procedures outlined below follow the techniques developed in previous work [28] for calibration at a single wavelength. These techniques have been adapted for calibration at multiple wavelengths, and an in-depth discussion of the technical issues is provided separately [29, 30].

APD calibration at a single wavelength

The APDs to be calibrated (as many as five with current hardware) are mounted on an aluminum cooling block alongside a reference detector. The cooling block mimics the polychromator mounts and maintains constant temperature during the calibration process. The reference detector is another APD/amplifier with known gain; previously calibrated in-house by comparison to a commercially calibrated InGaAs photodiode, the current reference detector was calibrated directly by the Canadian National Research Council Institute for National Measurement Standards (NRC-INMS) over a range of wavelengths (700-1100 nm). The measured spectral responsivity is shown in Figure 2.13. A pulsed, near-infrared LED close to the laser wavelength (Roithner LED1050-35K42 with centroid at 1022 nm) acts as a source of photons, which then illuminate all the APDs via an integrating sphere and fiber optics. The LED is driven with an OmniPulse PLDD-50-SP compact current source to obtain short pulses (~13 ns FWHM) at a 1 kHz repetition rate, mimicking the laser light for similarity to operating conditions. Measurements of APD/amplifier gain roll-off with pulse width indicate that this is not a significant effect, however. To achieve sufficient statistics, 100,000 pulses from the LED are recorded simultaneously on each APD. This is determined largely by resolution requirements for the noise enhancement.

Calculation of the APD/amplifier gain ($G_{APD} = G_{amp}M\eta$) is relatively straightforward. With the known response of the reference APD, the total number of photons in each LED pulse can be calculated, and the gain of the other APDs is determined by the mean integrated signal:

$$G_{APD} = \frac{S_{APD}}{S_{ref}} L_i G_{ref}$$
(2.24)

where S_{APD} , S_{ref} represent the mean signals of the APD and reference detector, L_i is the measured ratio of fiber transmission for the i-th position of the APD relative to the reference detector's fiber, and G_{ref} is the gain of the reference detector.

The noise enhancement is determined from the signal-to-noise ratio. Both the



Figure 2.13: Spectral responsivity of the reference APD used for calibrations.

pulsed signal and the dark signal (APD signal prior to the LED pulse with no incident photons), are integrated over a 320 ns window. Sample pulsed and dark signals are shown in Figure 2.14, along with histograms of the integral values over all 100,000 LED pulses. From the variance in the dark (σ_{dark}^2) and illuminated (σ_{light}^2) signals, the noise enhancement is:

$$SNR = \sqrt{\frac{\eta N}{F}}$$
(2.25)

$$\frac{F}{\eta} = N_{APD} \frac{\sigma_{light}^2 - \sigma_{dark}^2}{S_{APD}^2}$$
(2.26)

Due to the small variation in the signal (~3%) and the low signal amplitudes typically used for calibration (~60-90 mV to mimic maximum scattered signal intensity), a large number of pulses are required to obtain reasonable error bars on the noise enhancement measurement (10-15%). The quoted noise enhancement for the APDs used is ~5, and the expected quantum efficiency near the LED wavelength is ~65%, consistent with typical measured F/η values of ~7.3.

The light source output must be highly consistent for these results to be accu-



Figure 2.14: Dark (black) and pulsed (red) signals for a single diode pulse (left) and histograms of the integrated signals after 100,000 pulses (right). The contribution due to dark current has been subtracted out.

rate—variations in LED intensity can contribute to the pulsed signal integral and skew the noise enhancement measurements to larger values. The LEDs used in the calibration have been measured with a fast photodiode to determine the pulser variability, and are within limits for output variation contributions to be less than 10-15% of the measured value.

APD calibration at multiple wavelengths

Following the procedure outlined above, obtaining a calibration for the APD/amplifiers that is valid across the range of wavelengths monitored by the polychromators is largely a matter of acquiring and integrating additional LEDs into the setup. This is not as easy as it seems, given the tight constraints on LED pulse width, output variability, and the desirability of a narrow output spectrum. Four LEDs have passed these minimum requirements and are currently in use (Osram SFH4860 at 660 nm, Optek OP230WPS at 848 nm, Osram SFH4545 at 948 nm, Roithner LED1050-35K42 at 1022 nm). The LEDs are each pulsed at 1 kHz, as before, but with a 5 μ s delay between each LED. The long delay allows the APD signal to return to the dark value before the next pulse arrives, but makes the time window containing all four pulses prohibitively long. Instead, each LED trigger also triggers a shorter digitizer segment (640 ns) containing only the pulse for that particular LED. The signal and dark integrals are 320 ns long, as before. The pulse delay and individual segment approach improves the amount of time required to calibrate each group of APD/amplifiers, but the length of a run is still increased in comparison to previous calibrations (from 15-20 minutes to 45-50 minutes). Considering that each group has half again as many APDs to calibrate at four times as many wavelengths, this is not an unreasonable increase in time. However, maintaining constant temperature over this time period does become more difficult (although still achievable).

Using the measured spectral response of the reference APD and other detectors measured by NRC-INMS as a 'characteristic' response, the measured APD gain at each LED wavelength can be fit with the characteristic wavelength dependence. This yields a single value, the amplitude of the fit, to represent the gain for each APD

$$G_{APD,fit} = G_{amp} M \eta(\lambda_{peak})$$
(2.27)

where λ_{peak} is the wavelength of maximum APD response. The wavelength dependence of the characteristic response can then be folded into the transmission function

$$R_{\text{poly}}(\lambda) = T_{\text{filter}}(\lambda) \cdot \eta_{\text{APD}}(\lambda)$$
(2.28)

While the gain is strongly wavelength dependent (Figure 2.13) due to the APD quantum efficiency, the amplifier gain and the noise enhancement factor are nominally independent of the incident photon wavelength. The effective noise enhancement calculated above, F/η , is wavelength dependent due to the quantum efficiency. Since the wavelength dependence of G_{APD} should be dominated by quantum efficiency, $G_{APD}F/\eta = G_{amp}MF$ should be largely independent of wavelength. Comparison of this parameter for the reference APD calculated from all of the calibration runs that it was used for yields a roughly constant value for all four LEDs, shown in Figure 2.15.



Figure 2.15: The gain-noise product, GMF, is largely independent of wavelength. The mean value is indicated by the dashed line.

Having a wavelength independent noise parameter greatly simplifies calibration requirements. The necessity of recording 100,000 pulses at a given wavelength is dictated by statistical requirements. However, if $G_{\alpha mp}MF$ only needs to be measured once rather than over a range of wavelengths, the time required for a given set of APDs to be calibrated can be significantly reduced without sacrificing quality.

Polychromator spectral calibration

This stage of the calibration is performed with the APD/amplifiers mounted on the polychromators as during normal operation. A lamp, acting as a white light source, shines through a monochromator (Acton SpectraPro 500i) set to a variable wavelength with the output coupled to a fiber optic. The monochromator output is fed alternately to the reference APD (still mounted on the cooling block) and to a polychromator to be calibrated. The monochromator is scanned from 700 nm to 1100 nm in 1 nm increments. At each wavelength, the DC output of the APD/amplifier is digitized over a 2.52 μ s time period. This is repeated 100 times, and the average signal over all 100 iterations is recorded before proceeding to the next wavelength. Once past the laser line, a shutter is closed manually on the light source so that the values recorded between roughly 1080 nm and 1100 nm can be averaged to determine the dark voltage. The measured signals from the polychromator APDs are divided by the reference APD signal to eliminate variations due to the lamp spectrum and monochromator transmission. The fiber optic transmission is not identical for both setups, but the difference is assumed to be negligible. Repeating back-to-back calibrations for different polychromators allows comparison of the reference APD signal over time to verify that the lamp output is not changing (or that the APD temperatures are constant). Typical transmission functions for 4-, 6-, and 8-channel polychromators are shown in Figure 2.16.

2.6 Correlating TS fluctuation measurements with other diagnostics

Because TS Spectrons are limited to 25 kHz operation for bursts of only 6-8 pulses (and even Fast TS, with 75 kHz resolution, can only do about 15 pulses), direct Fourier transforms of the signals for fluctuation analysis are of limited value. The fact that the tearing modes are above the Nyquist limit of the Spectron lasers makes Te fluctuation analysis even more formidable. However, through judicious use of information from other diagnostics with sufficient temporal resolution and a



Figure 2.16: The polychromator transmission functions for the 6-channel polychromators (top) with the old, 4-channel functions in red, and the 8-channel polychromators (bottom).

Bayesian statistical framework, correlating electron temperature fluctuations with mode activity in MST is still possible.

The pioneering work in this area was performed by Hillary Stephens, and interested readers should refer to her thesis for initial correlation of temperature fluctuations with tearing modes [24]. A brief outline of the technique is provided below, with additional modifications to obtain both amplitude and phase of the temperature fluctuations.

Modeling electron temperature fluctuations

Since the Thomson scattering diagnostic does not have sufficient temporal resolution for direct measurement of temperature fluctuations correlated with tearing modes or higher frequency phenomena, additional information must be incorporated into the analysis. The results obtained in [24] and the results presented in this thesis all incorporate information about the phase of modes measured by coils in the magnetic array. In practice, though, any diagnostic which can provide information about the phase of a mode could be used.

Using temperature data with phase information from the toroidal array or other diagnostic, measured temperature fluctuations can be modeled relatively simply as

$$\mathsf{T}_e = \mathsf{T}_{e,0} + \tilde{\mathsf{T}}_{e,n} \cos \zeta_n \tag{2.29}$$

where $T_{e,0}$ is the equilibrium temperature, $\tilde{T}_{e,n}$ is the amplitude of temperature fluctuations correlated with a particular mode, and ζ_n is the phase of the mode. Since the magnetic array measures mode phase relative to (ϕ , θ) = (0°,241°), far from the Thomson measurement chord, the phase must be corrected to account for this shift.

$$\zeta_n = \zeta_{n,array} + n\phi_{TS} + m(\theta_{TS} - 241^\circ)$$
(2.30)

A single "event" consists of a burst of Thomson laser pulses, typically spanning ~200 μ s, plus toroidal array phase data. An example is shown in Figure 2.17. A model waveform with arbitrary $T_{e,0}$ and \tilde{T}_e is overplotted, and the probability that these parameters fit the data is expressed as:

$$P(T_{e,0}, \tilde{T}_{e,n} | T_e(t), \sigma, I) = \frac{1}{\prod_{j=1}^{N_D} \sqrt{2\pi}\sigma_j} \exp(-\frac{1}{2}\chi^2)$$
(2.31)

$$\chi^{2} = \sum_{j=1}^{N_{D}} \frac{[T_{e,measured}(t_{j}) - T_{e,model}(t_{j})]^{2}}{\sigma_{j}^{2}}$$
(2.32)

This model is deceptively simple—clearly, multiple sources of real temperature fluctuations are present (especially from the many tearing modes present in MST) as well as fluctuations due to photon statistics and noise in the detectors/amplifiers. A model containing only the equilibrium and a single source of fluctuations cannot account for the full complexity of temperature behavior. Even with phase information from another diagnostic, for a single shot some of the apparent fluctuation amplitude will be contamination from other sources. However, each of these other sources will contribute fluctuation power partially out of phase with the intended



Figure 2.17: Measured T_e from shot 1121012032, Poly 9 at z/a=0.149 and 24 ms. The black, dashed line indicates the average T_e value across the burst and the red, dashed line represent an arbitrary fluctuation in phase with the n=5 mode measured at the wall.

mode. So long as the noise contributions have random phase offset relative to the measured mode phase, over a large enough ensemble of shots/events, the contributions from these other sources will average to zero.

By forward modeling with Bayesian analysis, the probability that the fluctuation model fits data measured for a single event can be calculated. The result is a twodimensional probability grid over $T_{e,0}$ and $\tilde{T}_{e,n}$. This is shown in the plot below for a single event. Typically, only one of the parameters is of interest, in which case the remaining parameter can be marginalized by integration to obtain a onedimensional probability distribution. The probability distributions for both $T_{e,0}$ and \tilde{T}_e obtained this way are also shown in Figure 2.18.

Equipped with single-event probability distributions for the parameter of interest, ensemble analysis of many events consists of straightforward multiplication of the probability distributions to obtain a total PDF. The evolution of the total PDF with additional events is shown in Figure 2.19 below. For only a few events, the uncertainty in the data produces broad probability distributions corresponding to



Figure 2.18: Two-dimensional probability grid for the equilibrium and fluctuation amplitude, with one-dimensional marginalized PDFs. Data from shot 1121012032, Poly 9 at z/a=0.149 and 24 ms

large error bars on the fluctuation amplitude. With sufficiently large ensembles, though, PDFs with narrow widths are achievable and small amplitude fluctuations are resolvable.

$$P(\tilde{T}_{e,n}|T_{e,ensemble},\sigma,I) = \prod_{j}^{shots} P(\tilde{T}_{e,n}|T_{e}^{j}(t),\sigma,I)$$
(2.33)

The importance of phase

The form of model given above assumes that the measured temperature fluctuations are completely in phase with the measured magnetic perturbations at the wall. This is not always the case, however, and a few different effects can contribute to the measured phase of the mode. Even for fluctuations that are locally in phase



Figure 2.19: Evolution of probability distribution with increase in number of measured events. Figure from [24].

with a particular mode, the toroidal geometry can introduce an apparent phase shift relative to the measurements at the wall. Additionally, correlation between the temperature fluctuations and the radial component of the magnetic perturbations can lead to a real phase shift.

The first effect is due to the outward, Shafranov shift of the flux surfaces in MST. Figure 2.20 shows the flux surfaces for a) cylindrical geometry with no Shafranov shift and b) toroidal geometry with non-zero Shafranov shift. For the cylindrical case, the magnetic axis and the geometric axis are the same. As a result, the flux surfaces are concentric circles. The TS laser chord, which passes vertically through the geometric center, intersects every flux surface at 90° poloidal (above the midplane) and 270° poloidal (below the midplane). For fluctuation measurements at a distance z below the midplane, if the magnetic perturbation is global with no



Figure 2.20: Flux surfaces in a) cylindrical geometry, where the magnetic and geometric axis align, and b) toroidal geometry, where the Shafranov shift of the magnetic axis moves the flux surfaces outward and produces an apparent phase shift for TS measurements along a vertical chord.

radial phase dependence (i.e. - no shear effects or other distortion of the mode), then the local phase $\zeta(z) = n\phi + m\theta$ matches the phase at the wall $\zeta(a)$.

For toroidal geometry, though, the magnetic axis is shifted outward from the geometric axis. The flux surfaces, while still approximately circular, are no longer concentric; flux surfaces closest to the magnetic axis are shifted the most, while flux

surfaces near the edge are not significantly shifted. As a result, the vertical chord through the geometric axis no longer intersects the flux surfaces at 90° and 270° poloidal. Focusing on the lower half of the chord (below the midplane), the TS laser path intersects the flux surfaces at poloidal angles less than 270°. At a distance z below the midplane, where the laser path intersects a surface with radius ρ and Shafranov shift Δ , the shift in poloidal angle is given by:

$$\delta_{\rm S} = \tan^{-1}\left(\frac{\Delta}{z}\right) \tag{2.34}$$

The poloidal angle at the measurement location is $\theta(z) = 270^{\circ} - \delta_{\rm S}(z)$ and the local phase is $\zeta(z) = \zeta(a) - m\delta_{\rm S}(z)$. If the temperature fluctuations are completely in phase with the magnetic perturbations, an apparent phase shift of $m\delta_{\rm S}$ will be observed. At the geometric axis, with z = 0, the laser path is tangential to the flux surface and the phase shift is $\pi/2$.

Additionally, since the radial component of the tearing mode magnetic perturbation is $\pi/2$ out of phase with the poloidal component, any correlation of the temperature fluctuations in phase with \tilde{B}_r will introduce a phase shift away from the phase ζ of \tilde{B}_{θ} . While correlated electron temperature fluctuations in phase with with \tilde{B}_r have not yet been observed in standard discharges, Chapter 5 discusses the first observations of \tilde{T}_e in phase with \tilde{B}_r in non-reversed discharges. Distinguishing such effects is important for diagnosis of pressure contributions to both the stress tensor (momentum transport) and the kinetic dynamo (current transport).

For these reasons, it is important to resolve both the fluctuation amplitude and phase. Therefore, an additional parameter must be added to the model above to account for phase effects beyond the different diagnostic positions.

$$T_e = T_{e,0} + \tilde{T}_{e,n} \cos\left(\zeta_n + \delta\right) \tag{2.35}$$

In this case, δ is a fixed phase offset. The correlation analysis is performed as before, and repeated for multiple values of δ . For example, the resulting correlated temperature fluctuation amplitudes versus phase are shown in Figure 2.21 for two

different radial locations. The range of phase values covers half a period, from $-\pi/2$ to $\pi/2$ in steps of $\pi/8$, and the behavior of the fluctuation amplitudes is well-fit by a sinusoid as shown.



Figure 2.21: Phase scan for two different radial positions, with cosine fits. Temperature fluctuations are correlated with the n=5, \tilde{b}_{θ} signals at the wall.

If the real temperature fluctuations have the form A exp $\{i\zeta_n\}$ while the model fluctuations have the same spatial structure and frequency (n, m, ω) but are offset by a constant phase, exp $\{i(\zeta_n - \delta)\}$, the correlated fluctuation amplitude is given by the inner product of the real fluctuations with the model:

$$\langle A \exp\{i\zeta_n\} | \exp\{i(\zeta_n - \delta)\} \rangle = A \exp\{i\delta\}$$
(2.36)

The real part of this is just A cos δ , consistent with the results of the phase scan in Figure 2.21. Since the phase offset and fluctuation amplitude are both folded into the correlated amplitude, two correlations at different phases are required to extract this information. Let $\langle X | \exp \{i(\zeta_n - \delta)\}\rangle$ denote the ensemble averaged correlation of a measured signal, X, with a model signal. Then, in analogy with the pseudospectral techniques developed for probe measurements [31], the two orthogonal correlation amplitudes are:

$$T = \langle \tilde{T}_{e} | \exp\{i\zeta_{n}\} \rangle$$

$$T^{\dagger} = \langle \tilde{T}_{e} | \exp\{i(\zeta_{n} - \pi/2)\} \rangle$$
(2.37)

From these correlation amplitudes, the total fluctuation amplitude is

$$\tilde{\mathsf{T}}_e = \sqrt{\mathsf{T}^2 + (\mathsf{T}^\dagger)^2} \tag{2.38}$$

and the phase offset from \tilde{b}_{θ} is

$$\delta = \tan^{-1} \left(\mathsf{T}^{\dagger} / \mathsf{T} \right) \tag{2.39}$$

The errors in the fluctuation amplitude and phase offset are easily determined by propagation of the uncertainties in the correlated amplitudes.

For all of the measurements presented in subsequent chapters, the phase of correlated fluctuations is corrected for the Shafranov shift. Equilibrium reconstructions obtained from MSTFit for appropriate plasma conditions are ensemble averaged to obtain the mean value of the Shafranov shift Δ as a function of the effective radial coordinate ρ . From these values, the phase shift δ_S is calculated at each Thomson measurement location along the z-axis. The contribution $m\delta_S(z)$ is then subtracted from the phase δ calculated from T and T[†].

2.7 Magnetic coil arrays

On MST, a toroidal array of magnetic coil triplets is mounted on the inner surface of the vacuum vessel. Each triplet consists of orthogonal sensors to measure the toroidal, poloidal, and radial components (B_{ϕ} , B_{θ} , B_{τ}) of the edge magnetic field. The triplets are all at the same poloidal angle (241°) and equally spaced toroidally around the machine. Although there are 64 triplets in the array, only 32 are used for the analysis in this thesis. Of these 32 triplets, only the poloidal coils are used.
The signal from each poloidal coil is split and digitized twice. One of the signals is integrated first and then digitized using the D-TACQ digitizer at 200 kHz. The other signal is digitized without integration on the TR1612 digitizers at 3 MHz time resolution.

After each plasma discharge, the integrated B_{θ} signals are automatically analysed with a Fourier spatial mode decomposition [32]. The poloidal mode number m cannot be resolved by the toroidal array coils since they are all at the same poloidal angle, but the toroidal mode number can be resolved (with 32 coils) for $|n| \leq 16$. From the Fourier decomposition, the amplitude, phase, and velocity of each mode are calculated and recorded.

The integrated B_{θ} signals provide sufficient time resolution to measure the tearing mode activity and are used exclusively in the next chapter for measurements of correlated electron temperature. However, for higher-frequency phenomena the unintegrated poloidal coil signals must be used. The energetic particle modes discussed in Chapter 6 cover a range of frequencies roughly from 50 kHz to 200 kHz, well within the Nyquist limit for the unintegrated signals, and have typical duration of 60-100 μ s, within the time resolution of the unintegrated signals. To correlate temperature fluctuations with these modes, the unintegrated poloidal coil signals (B_{θ}) are Fourier decomposed spatially following the procedure in Ref. [33], which is equivalent to the spatial decomposition of the integrated signals. At each time point, the array of coil signals is decomposed into both sine and cosine functions of argument $2\pi i x[n]/L$. Here, $L = 360^{\circ}$ describes the total toroidal span of the array in degrees and the x[n] values represent the toroidal locations of each coil in degrees. The indices i range from zero to half the number of coils (the Nyquist limit) and correspond to the toroidal mode numbers, i.e. $n_i = i$. This spatial transform is computed at each time point, and from the resulting time series of sine and cosine terms the amplitude $\dot{B}_{\theta,n}$ and phase $\zeta_{\theta,n}$ as functions of time for each mode are determined. For modes above the tearing mode frequencies the sine and cosine time series are first bandpass-filtered to select the desired frequency range; the mode amplitude and phase are then constructed from the filtered time series.

2.8 References

- [1] I. H. Hutchinson. *Principles of Plasma Diagnostics*. Cambridge University Press, Cambridge, UK, 2 edition, 2002.
- J. Sheffield, D. Froula, S.H. Glenzer, and N.C. Luhmann. *Plasma Scattering of Electromagnetic Radiation: Theory and Measurement Techniques*. Elsevier Science, 2010. ISBN 9780080952031.
- [3] T. P. Hughes. A new method for the determination of plasma electron temperature and density from Thomson scattering of an optical maser beam. *Nature*, 194(4825):268, 1962. doi: http://dx.doi.org/10.1038/194268b0.
- Kelly Warner and Gary M. Hieftje. Thomson scattering from analytical plasmas. *Spectrochim. Acta, Part B*, 57(2):201, 2002. doi: http://dx.doi.org/10.1016/ S0584-8547(01)00372-X.
- [5] V. A. Zhuravlev and G. D. Petrov. Scattering of radiation by finite volumes of relativistic plasma streams. *Sov. J. Plasma Phys.*, 5(1):3, 1979.
- [6] A.C. Selden. Simple analytic form of the relativistic Thomson scattering spectrum. *Phys. Lett. A*, 79(5-6):405, 1980. doi: http://dx.doi.org/10.1016/ 0375-9601(80)90276-5.
- [7] A.C. Selden. Simple analytic form of the relativistic Thomson scattering spectrum. Culham Laboratory Report CLM-R220, January 1982.
- [8] F. Orsitto and N. Tartoni. Proposal for a new electron temperature diagnostic for fusion reactors. *Rev. Sci. Instrum.*, 70(1):798, 1999. doi: 10.1063/1.1149414.
- [9] S. E. Segre and V. Zanza. Polarization of radiation in incoherent Thomson scattering by high temperature plasma. *Phys. Plasmas*, 7(6):2677, 2000. doi: 10.1063/1.874110.

- [10] E. Parke, V. V. Mirnov, and D. J. Den Hartog. A polarization-based Thomson scattering technique for burning plasmas. *JINST*, 9(02):C02030, 2014. doi: doi:10.1088/1748-0221/9/02/C02030.
- [11] V.V. Mirnov, D.L. Brower, D.J. Den Hartog, W.X. Ding, J. Duff, and E. Parke. Electron kinetic effects on interferometry, polarimetry and Thomson scattering in burning plasmas. In *Proceedings of the 24th IAEA Fusion Energy Conference*, San Diego, USA, October 2012.
- [12] V.V. Mirnov, D.L. Brower, D.J. Den Hartog, W.X. Ding, J. Duff, and E. Parke. Electron kinetic effects on optical diagnostics in fusion plasmas. In *Proceedings of the International Conference on Fusion Reactor Diagnostics*, Varenna, Italy, September 2013.
- [13] G. A. Naylor, R. Scannell, M. Beurskens, M. J. Walsh, I. Pastor, A. J. H. Donné, B. Snijders, W. Biel, B. Meszaros, L. Giudicotti, R. Pasqualotto, and L. Marot. The ITER Thomson scattering core LIDAR diagnostic. *JINST*, 7(03):C03043, 2012. doi: doi:10.1088/1748-0221/7/03/C03043.
- [14] Chris Quigg. Relativistic correction to plasma bremsstrahlung. *Phys. Fluids*, 11(2):461, 1968. doi: http://dx.doi.org/10.1063/1.1691936.
- [15] O. Naito, H. Yoshida, and T. Matoba. Analytic formula for fully relativistic Thomson scattering spectrum. *Phys. Fluids B*, 5(11):4256, 1993. doi: http: //dx.doi.org/10.1063/1.860593.
- [16] J. H. Williamson and M. E. Clarke. Construction of electron distribution functions from laser scattering spectra. J. Plasma Phys., 6:211, 1971. doi: doi:10.1017/S0022377800025794.
- [17] J. A. Reusch, M. T. Borchardt, D. J. Den Hartog, A. F. Falkowski, D. J. Holly, R. O'Connell, and H. D. Stephens. Multipoint Thomson scattering diagnostic for the Madison Symmetric Torus reversed-field pinch. *Rev. Sci. Instrum.*, 79 (10):10E733, 2008. doi: 10.1063/1.2956742.

- [18] D. J. Den Hartog, J. R. Ambuel, M. T. Borchardt, J. A. Reusch, P. E. Robl, and Y. M. Yang. Pulse-burst operation of standard Nd:YAG lasers. *JPCS*, 227(1): 012023, 2010.
- [19] T. N. Carlstrom, J.C. DeBoo, R. Evanko, C. M. Greenfield, C. L. Hsieh, R. T. Snider, and P. Trost. A compact, low cost, seven channel polychromator for Thomson scattering measurements. *Rev. Sci. Instrum.*, 61(10):2858, 1990. doi: 10.1063/1.1141806.
- [20] EG&G Canada, Ltd. Large area long wavelength enhanced silicon avalanche photodiodes for general-purpose applications. Data Sheet C30956E, January 1991.
- [21] C. L. Hsieh, J. Haskovec, T. N. Carlstrom, J. C. DeBoo, C. M. Greenfield, R. T. Snider, and P. Trost. Silicon avalanche photodiode detector circuit for Nd:YAG laser scattering. *Rev. Sci. Instrum.*, 61(10):2855, 1990. doi: 10.1063/1.1141805.
- [22] D.S. Sivia and J. Skilling. *Data analysis: a Bayesian tutorial*. Oxford science publications. Oxford University Press, 2006. ISBN 9780198568315.
- [23] Udo von Toussaint. Bayesian inference in physics. *Rev. Mod. Phys.*, 83(3):943, 2011. doi: 10.1103/RevModPhys.83.943.
- [24] H. D. Stephens. Electron Temperature Structures Associated With Magnetic Tearing Modes in the Madison Symmetric Torus. PhD thesis, University of Wisconsin-Madison, 2010.
- [25] R. Fischer, C. Wendland, A. Dinklage, S. Gori, V. Dose, and the W7-AS team. Thomson scattering analysis with the Bayesian probability theory. *Plasma Phys. Control. Fusion*, 44(8):1501, 2002.
- [26] R. Fischer, A. Dinklage, and E. Pasch. Bayesian modelling of fusion diagnostics. *Plasma Phys. Control. Fusion*, 45(7):1095, 2003.
- [27] R. O'Connell, D. J. Den Hartog, M. T. Borchardt, D. J. Holly, J. A. Reusch, and H. D. Stephens. Optimizing a Thomson scattering diagnostic for fast

dynamics and high background. *Rev. Sci. Instrum.*, 79(10):10E735, 2008. doi: http://dx.doi.org/10.1063/1.2957844.

- [28] H. D. Stephens, M. T. Borchardt, D. J. Den Hartog, A. F. Falkowski, D. J. Holly, R. O'Connell, and J. A. Reusch. Calibration of a Thomson scattering diagnostic for fluctuation measurements. *Rev. Sci. Instrum.*, 79(10):10E734, 2008. doi: 10.1063/1.2964229.
- [29] L. A. Morton and E. Parke. Thomson scattering calibration report. Technical report, Madison Symmetric Torus, Dept. of Physics, University of Wisconsin-Madison, July 2013.
- [30] L. A. Morton, E. Parke, and D. J. Den Hartog. Detailed modeling of the statistical uncertainty of Thomson scattering measurements. *JINST*, 8(11): C11003, 2013. doi: http://dx.doi.org/10.1088/1748-0221/8/11/C11003.
- [31] T. D. Tharp. *Measurements of Nonlinear Hall-driven Reconnection in the Reversed Field Pinch*. PhD thesis, University of Wisconsin-Madison, 2010.
- [32] D. Craig. Magnetic mode analysis in MST. Technical report, Madison Symmetric Torus, Dept. of Physics, University of Wisconsin-Madison, May 2005.
- [33] J. J. Koliner. *Neutral Beam Excitation of Alfvén Continua in the Madison Symmetric Torus Reversed Field Pinch*. PhD thesis, University of Wisconsin-Madison, 2013.

Chapter 3

Tearing Modes and Fast Ions in the RFP

The relaxation process described in Chapter 1 is driven by a class of resistive MHD instabilities known as tearing modes. Gradients in the current profile drive nonlinear growth of core-resonant tearing modes with poloidal mode number m = 1. These modes couple with edge-resonant m = 0 modes and drive significant particle, heat, current, and momentum transport. Given the key role tearing modes play in RFP confinement, knowledge of their structure and dynamics is critically important. A number of schemes have been developed to mitigate and control tearing modes in the RFP. Recent results demonstrating suppression of core modes with neutral beam injection offer an additional method for stabilization of MHD activity, similar to results observed in tokamaks. The nature of fast-ion interaction with tearing modes in the RFP is not well understood, however. Electron temperature fluctuations have proven to be a useful tool for non-perturbative diagnosis of tearing modes; the behavior of correlated fluctuations in neutral beam heated plasmas may help in the search to identify the mechanism for mode suppression.

Section 3.1 provides a brief overview of the formation of tearing modes through magnetic reconnection. Section 3.2 describes the diagnosis of tearing modes based on edge magnetic signals and the empirical process for distinguishing between m

= 0 and m = 1 modes. Characteristic temperature perturbations associated with profile flattening due to tearing mode islands are described in Section 3.3. The description of temperature fluctuations due to island flattening serves two purposes here: justification of the fluctuation model described in the previous chapter and establishing the framework for identification of rational surface locations in the next chapter. Measurements of mode suppression in MST are given in Section 3.4, while a discussion of the theory behind MHD stabilization by fast ions is covered in Section 3.5. Several mechanisms for stabilization are described here with a large body of tokamak literature serving as the backdrop. Only one mechanism—modification of the current profile and safety factor—is addressed in this thesis, however.

3.1 Origin and characteristics of tearing modes

Early work on the effects of finite conductivity led to the description of new classes of resistive instabilites including the tearing mode [1]. In an infinitely conductive plasma, the magnetic field lines are 'frozen' into the fluid—as the fluid elements move in the plasma, the field lines move with them without breaking. Finite plasma resistivity allows magnetic field lines to reconnect and alter the magnetic topology, a process which occurs in many laboratory and astrophysical plasmas.

The equilibrium field for an RFP, described in Chapter 1, twists helically around the magnetic axis. The bending of the field lines is given by the safety factor, q:

$$q = \frac{rB_{\phi}}{RB_{\theta}} \tag{3.1}$$

where r is the minor radius and R is the major radius. Magnetic fluctuations are resonant when the wave vector is perpendicular to the magnetic field:

$$\vec{k} \cdot \vec{B} = 0 \tag{3.2}$$

or

$$\frac{\mathrm{mB}_{\theta}}{\mathrm{r}} + \frac{\mathrm{nB}_{\Phi}}{\mathrm{R}} = 0 \tag{3.3}$$

where m and n are the poloidal/toroidal wavenumbers. Combining the resonance condition with Equation 3.1, this can be re-written in terms of the safety factor:

$$q(\mathbf{r}) = -\frac{\mathbf{m}}{\mathbf{n}} \tag{3.4}$$

At rational values of the safety factor, then, fluctuations in the magnetic field become resonant. A plot of the safety factor in Figure 3.1 shows that at the edge of MST, q(a) < 0 for typical discharge conditions. For non-reversed plasmas, $q(a) \ge 0$, although q(a) = 0 is most commonly used. Nominally, an infinite number of rational values of q are present (in reversed discharges, there are an infinite number on either side of the reversal surface). However, the modes with the largest amplitude tend to have the smallest wave numbers. For MST, these are the m = 0,1 modes. The m = 0 modes are all resonant at the reversal surface (marginally resonant for non-reversed plasmas with q(a) = 0), with typical toroidal wavenumbers n = 1-4. The m = 1 modes are resonant in the core with typical |n| = 5-8. Note that Eqn. 3.4 implies that, in the core with q > 0 and m = 1, n < 0. Previous work has confirmed the helicity of these modes, but for simplicity the toroidal mode number will always be positive throughout this thesis as there is no room for confusion regarding modes with opposite helicity.

Gradients in the current density profile provide a source of free energy that allows the resonant modes to grow. The growth of these modes causes the magnetic field lines to tear and reconnect to form magnetic islands, giving the tearing mode its name. This is shown in Figure 3.2. The islands cause field lines inside the rational surface at r_s , where $q(r_s) = -\frac{m}{n}$, to connect with field lines outside the rational surface. This provides a mechanism for particles and heat to move from the core of the plasma outward and can lead to significant transport. The island width is determined by both the amplitude of the radial component of the magnetic perturbation ($\tilde{B_r}$) and the gradient of q at the resonant surface (which is related to the magnetic shear):

$$w = 4\sqrt{\frac{r_{s}\tilde{B_{r}}}{B_{\theta}nq'}}$$
(3.5)



Figure 3.1: Safety factor, q, versus radius. m = 1 modes are resonant in the core while m = 0 modes are resonant at the reversal surface, where q = 0. Figure from Ref. [2].

In MST, the islands are generally large enough to overlap, creating stochastic fields and significant particle and heat transport [3, 4]. For reversed-field pinches, this leads to generally poor energy and confinement times compared to other devices. Typical particle and energy confinement times for standard MST discharges are $\sim 0.5 - 1.0 \text{ ms} [5, 6]$.

3.2 Magnetic diagnosis of tearing modes

Although the tearing modes are resonant only at a particular rational surface, the magnetic perturbations are global in nature. Since the amplitude of the tearing modes is non-zero at the wall, edge magnetic coils can be used to diagnose internal tearing modes without resorting to more perturbative probe techniques.

Although no Fourier decomposition can distinguish between tearing modes with different m number using only the toroidal array data, empirically the m =



Figure 3.2: A sheared equilibrium configuration (top). Magnetic reconnection leads to field lines inside the rational surface connecting with field lines outside the rational surface and the formation of magnetic island structures (bottom).

0 and m = 1 modes can be differentiated based on n number [7]. The magnetic fluctuations at the wall due to m = 0 modes are predominately toroidal, with negligible contributions to the poloidal component. For m = 1 modes, the fluctuations have both toroidal and poloidal components. Therefore, the m = 1 modes can be identified from the B₀ coil measurements; $|n| \ge 6$ modes are predominantly m = 1. Since the safety factor does not reach $q_0 = 0.25$ in MST discharges, modes with m = 1

1, $|n| \leq 4$ are not resonant. The $|n| \leq 4$ modes are predominantly m = 0 and can be identified from the B_{ϕ} coil measurements. The |n| = 5 mode can be either m = 1 if $q_0 > 0.2$ or m = 0 if it is resonant at the reversal surface.

For non-reversed plasmas, $q_0 > 0.2$, typically. In reversed discharges, the qprofile evolves over the course of a sawtooth cycle. Safety factor profile evolution through a sawtooth is shown in Figure 3.3, obtained from an ensemble average of equilibrium reconstructions for several thousand sawtooth events [8]. Immediately after a sawtooth event, current redistribution brings q_0 above 0.2 and the (m,n) =(1,5) mode is temporarily resonant. As the plasma recovers, however, the current profile becomes more strongly peaked on axis and q_0 drops below 0.2, removing the (1,5) resonant surface and quenching that mode. For both reversal conditions, the (0,5) mode amplitude is relatively small, so the n = 5 signal is predominantly m = 1 except later in the reversed sawtooth cycle when the (1,5) mode is quenched.



Figure 3.3: Equilibrium reconstruction of q-profile evolution over the course of a sawtooth cycle: radial profile (left) and value on-axis (right). Figure from Ref [8].

3.3 Temperature flattening and fluctuations

Due to the reconnected field lines associated with tearing modes, large islands lead to rapid heat transport. Discharges with tearing mode activity in a variety of devices display a characteristic flattening of the electron temperature profile in the vicinity of the tearing mode rational surface: tokamaks [9], stellarators [10], and the reversed-field pinch [2]. See Figure 3.4 for a typical profile in the RFP—equilibrium $T_{e,0}$ values are plotted with $\tilde{T}_{e,6}$ fluctuations superimposed such that the island O-point is on the right side of the profile and the X-point is on the left [2]. On the core side of the island, the temperature is reduced, while the temperature on the outer side is elevated, producing a plateau centered on the rational surface. No such plateau is observed at the X-point.



Figure 3.4: Island flattening associated with the n = 6 tearing mode in MST: equilibrium profile (right) and fluctuation amplitude (left). Figure adapted from Ref. [2].

The tearing mode correlated temperature fluctuations measured in MST exhibit standard structure frequently observed in island-flattened profiles. A characteristic temperature perturbation profile is shown in Figure 3.5 in the vicinity of the rational surface for an island with complete flattening. Empirical models of the observed temperature flattening have been established [11, 12]. Combining a description of the perturbed magnetic flux due to an island with the heat transport equation, these



Figure 3.5: Characteristic temperature fluctuation profile (left) for magnetic islands with complete flattening of the equilibrium profile (right). The vertical dashed lines indicate the rational surface location.

models predict perturbations to the temperature profile for both thin islands (only partial temperature flattening) and large islands (complete temperature flattening). The resulting temperature fluctuations can be expanded in a Fourier series of spatial harmonics. The harmonic content of the temperature fluctuations is determined by the degree of flattening.

\tilde{T}_e models for magnetic islands

The helical temperature fluctuation model developed in Ref. [11] is summarized below, followed by a brief discussion of physically relevant parameters that can be extracted from measurements of temperature fluctuations across an island structure. The perturbed flux $\psi(r, \theta, \phi, t)$ due to the magnetic island can be separated into a component describing the radial structure and a component describing the dependence on helical angle: $\psi(r, \theta, \phi, t) = \psi(r) \cos \zeta$, where the helical phase angle $\zeta = m\theta - n\phi - \omega t$ and ω is the rotation frequency. For tearing instabilities, $\psi(r) \approx \Psi$ near the rational surface and the normalized, perturbed flux surfaces have the form

$$\Omega = 8\frac{x^2}{W^2} + \cos\zeta \tag{3.6}$$

where $x = r - r_s$ and W is the island width. Ω is a scalar quantity that is constant along a flux surface, making it a useful flux surface 'label'.

The heat transport is given by $\vec{q} = -\kappa_{\parallel}\nabla_{\parallel}T - \kappa_{\perp}\nabla_{\perp}T$, where \vec{q} is the heat flux, $\kappa_{\parallel} / \kappa_{\perp}$ are the parallel/perpendicular thermal conductivities, and $\nabla_{\parallel} / \nabla_{\perp}$ are the gradients parallel/perpendicular to the magnetic field. In the absence of heat sources or sinks, $\nabla \cdot \vec{q} = 0$, yielding

$$\kappa_{\parallel} \nabla_{\parallel}^2 \mathsf{T} + \kappa_{\perp} \nabla_{\perp}^2 \mathsf{T} = 0 \tag{3.7}$$

Near the magnetic island, combining Eq. 3.6 and Eq. 3.7, the heat diffusion satisfies

$$\left(\frac{W}{2W_{c}}\right)^{2}\frac{\partial}{\partial\zeta}\sqrt{\Omega-\cos\zeta}\frac{\partial\tilde{T}}{\partial\zeta}+\frac{\partial}{\partial\Omega}\sqrt{\Omega-\cos\zeta}\frac{\partial\tilde{T}}{\partial\Omega}=0$$
(3.8)

where $T(r, \zeta) = T_0(r_s) + \tilde{T}(x, \zeta)$, with T_0 the unperturbed equilibrium value at the rational surface. Note that $\tilde{T}(x, \zeta)$ includes both the perturbation due to island flattening as well as the Taylor expansion of the equilibrium profile around the rational surface. W_c defines a characteristic island width for flattening of the temperature profile:

$$W_{\rm c} = \left(\frac{\kappa_{\perp}}{\kappa_{\parallel}}\right)^{1/4} \left(\frac{8R_0r_s}{s_sn}\right)^{1/2} \tag{3.9}$$

where the magnetic shear at the rational surface is $s_s = \frac{r_s}{q(r_s)} \frac{dq}{dr} \Big|_{r_s}$. Far from the rational surface, the temperature is a function of the perturbed flux surfaces due to the high parallel thermal conductivity. Close to the rational surface, the temperature remains a function of the island surfaces for large islands with $W \gg W_c$, but is not a flux surface function for thin islands with $W \ll W_c$.

The solutions to this equation can be expressed as a Fourier series:

$$\delta T(x,\zeta) = \sum_{\nu} \delta T_{\nu}(x) \cos \nu \zeta$$
(3.10)

where $\delta T(x, \zeta) = T(x, \zeta) - T_0(x)$. Unlike $\tilde{T}(x, \zeta)$ above, $\delta T(x, \zeta)$ here is the true helical perturbation due to the island, with the full equilibrium profile subtracted out.

The functions δT_{ν} are antisymmetric, crossing zero at the rational surface. Near the rational surface, $\delta T_{\nu}(x) \propto x^3$ for all ν . The first harmonic dominates fluctuations outside the island (and inside the island for thin islands). For large islands with complete flattening, harmonics with $\nu \ge 2$ can contribute significantly, but only inside the separatrix. Due to the dominance of the first harmonic, ECE measurements of tearing mode temperature fluctuations in tokamaks frequently rely on the first harmonic contribution alone. Likewise, the model for temperature fluctuation correlations developed in Chapter 2 for Thomson scattering measurements, with $T_e = T_{e,0} + \tilde{T}_e \cos(\zeta + \delta)$, depends on the dominance of the first harmonic.

Based on these models, temperature fluctuations correlated with a tearing mode can yield measurements of several important quantities. From the zero-crossing in amplitude, the rational surface location can be determined. From the width of the region between the two peaks in amplitude, the island width can also be determined. Additionally, since the fluctuation amplitudes outside the island depend on the mode eigenfunction structure, the tearing mode stability can also be extracted from temperature fluctuations. Due to the current sheet at the rational surface, the magnetic perturbation is continuous across the rational surface but the derivative is not (due to the constant- ψ approximation). The linear stability parameter Δ' , which characterizes the 'outer region' solution of the ideal MHD equations outside the resistive layer at the rational surface, is related to this discontinuity:

$$\Delta' = \frac{1}{\psi} \frac{\mathrm{d}\psi}{\mathrm{d}x} \Big|_{\mathbf{r}_{s}^{-}}^{\mathbf{r}_{s}^{+}}$$
(3.11)

For $\Delta' > 0$, the island growth rate is positive and the mode is unstable, while the mode is stable for $\Delta' < 0$. For cases where the temperature fluctuations accurately represent the perturbed flux surfaces, small differences in the shape of the magnetic surfaces on either side of the island can be discerned from high resolution temperature fluctuation measurements. These differences are crucial to Δ' and estimation of the stability parameter from ECE temperature fluctuation measurements has been used on TFTR to confirm measurements of classically stable but neoclassically

unstable tearing modes [13].

q-profile constraint for equilibrium reconstruction

Typically, motional Stark effect (MSE) or polarimetry measurements are used to constrain the q-profile. The generally good agreement between island correlated electron temperature fluctuations and measured q-profiles has been used to suggest q-profile constraint through this method as well [12], although for tokamaks coreresonant tearing modes are undesirable for operation and generally only a few are present simultaneously. Pioneering work on JT-60A utilized ECE measurements of island zero-crossings to identify the rational surface location and steer current drive for tearing mode suppression [14, 15]. This technique has been applied on other devices, including DIII-D [16], where comparison of real-time zero-crossing measurements to q-profile reconstructions with MSE demonstrated not only the accuracy of rational surface location but also the potential for improved performance with lower noise. In the RFP, where tearing modes are present in large numbers and are fundamental to standard discharges, the opportunity for q-profile constraint via island correlated temperature fluctuations is promising.

There are some challenges and complications to this approach that warrant attention. First, asymmetric islands have been observed in other devices [17], leading to discrepancy in the zero-crossing estimate obtained by the first and second spatial harmonics. Additionally, theoretical results predict spatial asymmetry due to corrections to the constant- Ψ approximation and higher order derivatives of the equilibrium flux as well as phase distortions due to flow shear and temperature gradients across the island [18]. Despite this, the first spatial harmonic zero-crossing can correspond well with MSE constrained reconstructions even for islands with significant asymmetry [16]. The n = 6 islands measured in MST are relatively symmetric, although the island structures measured for higher n number modes can be more heavily distorted.

Furthermore, while electron temperature fluctuations correlated with tearing modes in MST show the characteristic profile flattening expected from tokamak

results, electron density fluctuations do not. Equilibrium density profiles in the core of standard discharges are relatively flat, however. Finally, the measured mode widths obtained through temperature fluctuations are much smaller than the widths predicted from Eqn. 3.5 [2]. The predicted island widths suffered some uncertainty in the estimation of \tilde{B}_r and $q'(r_s)$, but the stochasticity of overlapping islands also suggests that temperature fluctuations correlated with the remnant island structures will underestimate the full island width.

3.4 Fast ions in the RFP

Fast-ion confinement

Despite the stochastic fields present in standard and non-reversed MST discharges, which dominate the bulk plasma transport properties, fast-ion confinement is largely insensitive to the turbulent field. Measurements of fast-ion confinement times have been performed by observing the decay in measured neutron flux following turn-off of the neutral beam. Typical fast-ion confinement times for standard discharges are 10-20 ms, while discharges with reduced stochasticity (due to current profile control) can achieve confinement > 30 ms [19]. These confinement times are more than an order of magnitude greater than the bulk confinement, and are consistent with purely classical slowing of the fast ions.

Computation of fast particle orbits (using the RIO code) shows that the ion orbits have a different rotational transform from the magnetic field lines [19]. In analogy to the magnetic safety factor, an 'ion guiding center (IGC) safety factor' can be defined: $q_{fi} = rv_{\phi}/Rv_{\theta}$, with v_{ϕ} and v_{θ} the toroidal and poloidal guiding center velocities. The IGC safety factor and the magnetic safety factor are not equal, as seen in Figure 3.6. A similar IGC island 'width' can be defined: $w_{fi} = 4\sqrt{\frac{rv_r}{v_{\theta}nq'_{fi}}}$. Not only do the core-most guiding center islands show reduced overlap, but the region near the magnetic axis is resonance-free for typical MST conditions.

Results of additional modeling with the TRANSP code [20] indicate that the fast ions are core-localized with high density [21]. The distribution shown in Figure 3.7



Figure 3.6: Safety factor for fast-ion guiding center (red) vs safety factor for magnetic field lines (black).

is strongly peaked, both spatially in the core and in velocity space—the fast ions are predominantly high-pitch $(v_{\parallel}/|v|)$. The modeled fast-ion density routinely reaches 15% of the bulk electron density, and has been observed as high as 25%. These transport calculations do not account for fast-ion losses due to transport during bursts of high-frequency instabilities (see Chapter 6), however, and experimental fast-ion densities are estimated to be closer to 8% of n_e .

Measured effects on tearing modes

During neutral beam injection, bulk heating has been observed during discharges with reduced stochasticity [22]. The beam also exerts a torque on the plasma, increasing rotation speed. The line-integrated flow velocity has been observed to increase by as much as ~75% [23]. Tearing modes of all n numbers, which rotate with the plasma, are observed to increase in velocity. As in tokamaks, fast ions in the reversed-field pinch have been observed to induce both stabilizing and destabilizing effects.

Figure 3.8 shows typical effects of neutral beam injection on a core kink-tearing



Figure 3.7: TRANSP results for neutral beam injection of hydrogen into RFP plasmas. Fast ions are core-localized with high pitch. The hydrogen ions are also super-Alfvénic. Figure courtesy of Jay Anderson.

mode during non-reversed discharges. Shortly after the neutral beam turns on, the amplitude of the core-most mode (in this case the n = 5 mode) reduces by ~60% relative to a non-beam-heated plasma. The reduction persists for the duration of neutral beam injection, ceasing only after the neutral beam has turned off. The ramp-down and restoration of the mode amplitude are consistent with the rise and decay of the neutron flux.

Mode amplitude reduction is observed only for the core-most mode; outer modes are not significantly reduced in amplitude, as shown in Figure 3.9. Mode suppression occurs in non-reversed, standard, and enhanced confinement (PPCD) discharges with neutral beam heating, although the modes affected are not the same: the n = 5 mode is core resonant for non-reversed and standard discharges but not PPCD discharges, leaving the n = 6 mode closest to the core. Furthermore, mode suppression occurs only for co-current beam injection. Counter-current injection studies do not show appreciable mode suppression, but fast ions are not as well confined in this case.



Figure 3.8: Ensemble averages of a) plasma current and neutral beam power, b) electron density and neutron flux, c) n = 5 mode amplitude with (red) and without (black) neutral beam, and d) average suppression factor. Figure from Ref. [21].

3.5 Stabilization of MHD activity by fast ions: theoretical predictions

Stabilization of MHD modes due to fast ions has been studied in tokamaks for several decades. Typical sources of fast-ion populations in high temperature plasmas include ion-cyclotron resonance heating (ICRH) and neutral beam injection, while fusion reactions will produce significant populations of energetic alpha particles for reactors. Two primary mechanisms for stabilization of MHD activity have been identified: fast-ion pressure and fast-ion current drive. Pressure induced stabilization is only achievable with sufficiently energetic particles, and is relevant to fusion produced alphas. Current drive stabilization, while typically observed with high-energy ICRH produced ions, is not subject to the same energy constraint but does require adequate current profile shaping. Several other effects which may be



Figure 3.9: Suppression factor vs toroidal mode number for neutral beam heated non-reversed plasmas (red) where the n = 5 mode is core-most and deeply reversed PPCD plasmas (blue) where the n = 6 mode is core-most. Outer modes show little to no suppression. Figure from Ref. [21]

relevant are also discussed. This section concludes with an evaluation of possible mechanisms which may explain the results observed on MST.

Fast-ion pressure

Following the discovery of fast-ion induced destabilization of fishbone modes in tokamaks, initial observations of MHD stabilization were obtained via ion cyclotron resonance heating [24]. The ICRH generated fast ions stabilized the m = n = 1 mode responsibile for sawteeth in tokamaks, leading to longer sawtooth periods and, in some cases, dramatic reduction of m = 1 activity for extended periods of time. Subsequent theoretical investigations, summarized in [25], identified fast-ion pressure effects as the mechanism.

In tokamaks, the magnetic field varies like 1/R. This leads to mirroring and produces a significant population of 'trapped' particles (as opposed to 'passing'), even for high energy ions created via ICRH or neutral beam injection. Due to magnetic drifts, the trapped particles precess toroidally. The third adiabatic invariant, Φ , to leading order corresponds to the poloidal magnetic flux through the area

defined by precession of the trapped fast-ion-orbit guiding centers. For sufficiently energetic particles, the bounce frequency associated with the trapped orbit is much greater than the MHD wave frequencies and Φ is conserved.

The trapped fast ions spend more time in regions with bad toroidal curvature and should be destabilizing. However, enforcing conservation of Φ leads to the possibility that electromagnetic perturbations due to the MHD modes, through momentum balance, do work against the fast-ion pressure. If the fast ions are represented as current loops, adiabatic variations in B cause the current loops to expand or contract to conserve the flux. In the presence of fast-ion density gradients, expansion or contraction of the current loops changes the internal energy of the fast ions and does work with or against the fast-ion pressure. Although discussed initially in the context of the m = n = 1 mode, this physical picture can be applied more generally to modes with arbitrary m and n, and the conditions for stabilization in a tokamak are:

- Safety factor: q must be monotonic and less than one on axis
- Core-localization: fast-ion density must be peaked near the axis $(\partial n_{fi}/\partial r < 0)$
- Positive drift: the average magnetic drift is in the positive direction (parallel to B)

As noted in [25], these conditions matched those found in ICRH heating of tokamak plasmas and explained observed suppression of MHD modes. Sawtooth stabilization has been observed with ICRH, but the fast-ion pressure can also destabilize MHD modes [26]. With appropriate choice of plasma conditions, destabilization of either sawteeth, fishbones, or both, is predicted. Decreased sawtooth periods and fishbone destabilization with ICRH have been observed in JET [27], consistent with the predictions of [26].

While ICRH typically produces particles with energies in the MeV range, neutral beam injection is generally limited to tens or hundreds of keV. For neutral beam injection, the conservation of Φ is only marginally satisfied in high temperature

plasmas. Nonetheless, sawtooth stabilization due to fast-ion pressure with NBI has been observed [28].

Fast-ion current drive

If ICRH power is deposited off-axis, the core-localization condition is not met and the fast-ion perturbed energy is not effective for stabilization. ICRH driven fast-ion current, however, can still stabilize MHD activity. If the k_{\parallel} spectrum of the launched wave is asymmetric, minority ions moving in one direction will preferentially gain perpendicular energy from the wave. As a result, the minorityion velocity increases and the collisionality decreases, establishing a net toroidal drift of the minority ions. Due to conservation of momentum, the background ions drift in the opposite direction and partially offset the minority-ion current. Electrons are dragged collisionally by both background and minority ions, also modifying the current drive. Trapping of both the minority ions and the electrons further complicates the description, but under appropriate conditions ICRH drives net current. Due to the resonant condition, $\omega - \omega_{ci} = k_{\parallel} v_{\parallel,i}$, the sign of the driven current changes across the cyclotron layer.

This dipolar current can either flatten or steepen the current density profile, leading to locally reduced or enhanced shear. Tuning ICRH to deposit near the q = 1 rational surface alters the stability of the m = n = 1 mode. Experiments in JET have successfully stabilized and destabilized sawteeth with ICRH shear control [29].

Other effects

The effects of circulating particles on tearing modes in tokamaks have also been considered [30]. The guiding center motion of energetic particles results from both field-aligned velocity and magnetic drifts. Perturbations associated with the tearing modes create islands in the fast-ion velocity field, similar to the magnetic islands and shifted radially due to the drift velocity. If the velocity islands are shifted by an amount greater than the magnetic island width, near the rational surface the

perturbations result in deformed constant-density contours. In the presence of a fast-ion density gradient, the fast-ion density varies along the rational surface and produces a net parallel current. The sign of the current is independent of the direction of fast-ion motion; instead the sign of the density gradient determines whether the additional current stabilizes or destabilizes the mode. A fast-ion density profile peaked outside the tearing mode rational surface is predicted to be stabilizing. Due to the opposite sign of dq/dr in RFPs compared to tokamaks, the dependence on $\partial n_{fi}/\partial r$ described in Ref. [30] would also change sign: a distribution that is peaked on-axis is predicted to stabilize tearing modes in the RFP.

Alternatively, an evaluation of the tearing stability factor including circulating energetic particle contributions in Ref. [31] found both the fast-ion density gradient and direction of circulation to be important. For a fast-ion distribution with energy of a few hundred keV and negative density gradient, co-current injection can stabilize tearing modes while counter-current injection destabilizes them.

For energetic particles, the Larmor radius can be quite large (for typical MST discharges the Larmor radii of NBI fast ions are approximately 10% of the minor radius). While most work describing fast-ion effects on MHD stability neglects the effect of particles with large gyroradii, recent results suggest that finite-Larmor radius (FLR) effects may be important. In the asymptotic limit of strong FLR effects, a fluid description is sufficient to capture the relevant physics without solving the drift kinetic equation [32]. Quasi-neutrality requires that the electron density balance the bulk and fast-ion densities: $n_e = n_{i,bulk} + n_{i,fast}$. However, in the limit where fast ions do not respond to $\vec{E} \times \vec{B}$ drifts, the bulk ions do not completely compensate the current due to electron drift. Accounting for this modified current leads to a tearing stability factor that depends on the fast-ion density gradient at the rational surface. Estimates of the critical gradient necessary for stabilization are well below predicted gradients for neutral beam injection on MST. The results of Ref. [32] are qualitatively consistent with the results of Ref. [33], which suggest that FLR effects due to an anistropic fast-ion distribution can strongly alter the stability of resistive MHD modes.

Predictions for fast ions in the RFP

A number of potential explanations exist for fast-ion stabilization of MHD activity in MST. Unfortunately, a proper evaluation of most of these effects is beyond the scope of this work. First, measurement of the stability parameter for the suppressed modes is not feasible at present. Secondly, the fast-ion population is not well diagnosed. Finally, measurements of q_0 are not available, either. Nonetheless, some light can be still be shed on this issue.

Unlike the bulk particles, the trapped fraction for NBI fast ions on MST is predicted to be low. While orbit modeling suggests that neutral beam injection deposits some ions into trapped orbits, the majority of fast ions are expected to be passing [34]. The trapped particle effects identified by Ref [25], involving the conservation of the third adiabatic invariant, are not likely to play a significant role for the RFP.

NBI appears to have a subtle effect on the current profile, however. While net current drive has not yet been observed, the fast ions appear to redistribute the current. This can have an important effect on the q-profile, and thus mode stability. For non-reversed and standard discharges, where the n = 5 rational surface is close to the magnetic axis, small changes in r_s can have a significant impact on the mode growth rate and saturated amplitude. A change in the value of q_0 by only a few percent can potentially remove the resonant surface from the plasma entirely. Although direct measurement of q_0 is not available at present, q-profile constraint in the mid-radius through tearing mode correlated electron temperature fluctuations is a feasible alternative for diagnosing current redistribution. This is the goal of the next chapter.

3.6 References

 Harold P. Furth, John Killeen, and Marshall N. Rosenbluth. Finite-resistivity instabilities of a sheet pinch. *Phys. Fluids*, 6(4):459, 1963. doi: http://dx.doi. org/10.1063/1.1706761.

- [2] H. D. Stephens. Electron Temperature Structures Associated With Magnetic Tearing Modes in the Madison Symmetric Torus. PhD thesis, University of Wisconsin-Madison, 2010.
- [3] A. B. Rechester and M. N. Rosenbluth. Electron heat transport in a tokamak with destroyed magnetic surfaces. *Phys. Rev. Lett.*, 40(1):38, 1978. doi: 10.1103/ PhysRevLett.40.38.
- [4] B. V. Chirikov. A universal instability of many-dimensional oscillator systems. *Phys. Rep.*, 52(5):263, 1979. doi: http://dx.doi.org/10.1016/0370-1573(79) 90023-1.
- S. Hokin, A. Almagri, S. Assadi, J. Beckstead, G. Chartas, N. Crocker, M. Cudzinovic, D. Den Hartog, R. Dexter, D. Holly, S. Prager, T. Rempel, J. Sarff, E. Scime, W. Shen, C. Spragins, C. Sprott, G. Starr, M. Stoneking, C. Watts, and R. Nebel. Global confinement and discrete dynamo activity in the MST reversed-field pinch. *Phys. Fluids B*, 3(8):2241, 1991. doi: http://dx.doi.org/10.1063/1.859642.
- [6] N. E. Lanier, D. Craig, J. K. Anderson, T. M. Biewer, B. E. Chapman, D. J. Den Hartog, C. B. Forest, S. C. Prager, D. L. Brower, and Y. Jiang. Control of density fluctuations and electron transport in the reversed-field pinch. *Phys. Rev. Lett.*, 85(10):2120, 2000. doi: 10.1103/PhysRevLett.85.2120.
- [7] S.-H. Choi. *Origin of Tearing Mode Excitation in the MST Reversed Field Pinch*. PhD thesis, University of Wisconsin-Madison, 2006.
- [8] J. A. Reusch. Measured and Simulated Electron Thermal Transport in the Madison Symmetric Torus Reversed Field Pinch. PhD thesis, University of Wisconsin-Madison, 2011.
- [9] M.F.F. Nave, A.W. Edwards, K. Hirsch, M. Hugon, A. Jacchia, E. Lazzaro, H. Salzmann, and P. Smeulders. Observation of MHD structures in JET temperature profiles. *Nucl. Fusion*, 32(5):825, 1992.

- [10] R. Jaenicke and W VII-A Team. Effect of tearing modes on temperature and density profiles and on the perpendicular transport in the W VII-A stellarator. *Nucl. Fusion*, 28(10):1737, 1988.
- [11] Richard Fitzpatrick. Helical temperature perturbations associated with tearing modes in tokamak plasmas. *Phys. Plasmas*, 2(3):825, 1995. doi: http://dx.doi. org/10.1063/1.871434.
- [12] J. Berrino, E. Lazzaro, S. Cirant, G. D'Antona, F. Gandini, E. Minardi, and G. Granucci. Electron cyclotron emission temperature fluctuations associated with magnetic islands and real-time identification and control system. *Nucl. Fusion*, 45(11):1350, 2005.
- [13] C. Ren, J. D. Callen, T. A. Gianakon, C. C. Hegna, Z. Chang, E. D. Fredrickson, K. M. McGuire, G. Taylor, and M. C. Zarnstorff. Measuring Δ' from electron temperature fluctuations in the Tokamak Fusion Test Reactor. *Phys. Plasmas*, 5 (2):450, 1998. doi: http://dx.doi.org/10.1063/1.872729.
- [14] A. Isayama, Y. Kamada, S. Ide, K. Hamamatsu, T. Oikawa, T. Suzuki, Y. Neyatani, T. Ozeki, Y. Ikeda, K. Kajiwara, and the JT-60 team. Complete stabilization of a tearing mode in steady state high-β_p H-mode discharges by the first harmonic electron cyclotron heating/current drive on JT-60U. *Plasma Phys. Control. Fusion*, 42(12):L37, 2000.
- [15] A. Isayama, Y. Kamada, N. Hayashi, T. Suzuki, T. Oikawa, T. Fujita, T. Fukuda, S. Ide, H. Takenaga, K. Ushigusa, T. Ozeki, Y. Ikeda, N. Umeda, H. Yamada, M. Isobe, Y. Narushima, K. Ikeda, S. Sakakibara, K. Yamazaki, K. Nagasaki, and the JT-60 Team. Achievement of high fusion triple product, steady-state sustainment and real-time NTM stabilization in high-β_p ELMy H-mode discharges in JT-60U. *Nucl. Fusion*, 43(10):1272, 2003.
- [16] Y. S. Park and A. S. Welander. Real-time determination of magnetic island location for neoclassical tearing mode control in DIII-D. *Plasma Phys. Control. Fusion*, 48(9):1447, 2006.

- [17] J. P. Meskat, H. Zohm, G. Gantenbein, S. Günter, M. Maraschek, W. Suttrop, Q. Yu, and ASDEX Upgrade Team. Analysis of the structure of neoclassical tearing modes in ASDEX upgrade. *Plasma Phys. Control. Fusion*, 43(10):1325, 2001.
- [18] D. De Lazzari and E. Westerhof. The role of asymmetries in the growth and suppression of neoclassical tearing modes. *Plasma Phys. Control. Fusion*, 53(3): 035020, 2011.
- [19] G. Fiksel, B. Hudson, D. J. Den Hartog, R. M. Magee, R. O'Connell, S. C. Prager, A. D. Beklemishev, V. I. Davydenko, A. A. Ivanov, and Yu. A. Tsidulko. Observation of weak impact of a stochastic magnetic field on fast-ion confinement. *Phys. Rev. Lett.*, 95(12):125001, 2005. doi: 10.1103/PhysRevLett.95.125001.
- [20] R. J Goldston, D. C McCune, H. H Towner, S. L Davis, R. J Hawryluk, and G. L Schmidt. New techniques for calculating heat and particle source rates due to neutral beam injection in axisymmetric tokamaks. *J. Comput. Phys.*, 43(1):61, 1981. doi: http://dx.doi.org/10.1016/0021-9991(81)90111-X.
- [21] J. K. Anderson, A. F. Almagri, D. J. Den Hartog, S. Eilerman, C. B. Forest, J. J. Koliner, V. V. Mirnov, L. A. Morton, M. D. Nornberg, E. Parke, J. A. Reusch, J. S. Sarff, J. Waksman, V. Belykh, V. I. Davydenko, A. A. Ivanov, S. V. Polosatkin, Y. A. Tsidulko, L. Lin, D. Liu, G. Fiksel, H. Sakakita, D. A. Spong, and J. Titus. Fast ion confinement and stability in a neutral beam injected reversed field pinch. *Phys. Plasmas*, 20(5):056102, 2013. doi: 10.1063/1.4801749.
- [22] J. Waksman, J. K. Anderson, M. D. Nornberg, E. Parke, J. A. Reusch, D. Liu, G. Fiksel, V. I. Davydenko, A. A. Ivanov, N. Stupishin, P. P. Deichuli, and H. Sakakita. Neutral beam heating of a RFP plasma in MST. *Phys. Plasmas*, 19 (12):122505, 2012. doi: http://dx.doi.org/10.1063/1.4772763.
- [23] Tom Dobbins. The effects of fast ions on tearing modes and momentum transport around reconnection events in a RFP. Senior Honors Thesis, University of Wisconsin-Madison, 2013.

- [24] D. J. Campbell, D. F. H. Start, J. A. Wesson, D. V. Bartlett, V. P. Bhatnagar, M. Bures, J. G. Cordey, G. A. Cottrell, P. A. Dupperex, A. W. Edwards, C. D. Challis, C. Gormezano, C. W. Gowers, R. S. Granetz, J. H. Hammen, T. Hellsten, J. Jacquinot, E. Lazzaro, P. J. Lomas, N. Lopes Cardozo, P. Mantica, J. A. Snipes, D. Stork, P. E. Stott, P. R. Thomas, E. Thompson, K. Thomsen, and G. Tonetti. Stabilization of sawteeth with additional heating in the JET tokamak. *Phys. Rev. Lett.*, 60(21):2148, 1988. doi: 10.1103/PhysRevLett.60.2148.
- [25] F. Porcelli. Fast particle stabilisation. *Plasma Phys. Control. Fusion*, 33(13):1601, 1991.
- [26] R.B. White. *Theory of Tokamak Plasmas*. North-Holland, 1989. ISBN 9780444874818.
- [27] M. J. Mantsinen, S. Sharapov, B. Alper, A. Gondhalekar, and D. C. McDonald. A new type of MHD activity in JET ICRF-only discharges with high fast-ion energy contents. *Plasma Phys. Control. Fusion*, 42(12):1291, 2000.
- [28] C. Angioni, A. Pochelon, N. N. Gorelenkov, K. G. McClements, O. Sauter, R. V. Budny, P. C. de Vries, D. F. Howell, M. Mantsinen, M. F. F. Nave, S. E. Sharapov, and contributors to the EFDA-JET Workprogramme. Neutral beam stabilization of sawtooth oscillations in JET. *Plasma Phys. Control. Fusion*, 44(2): 205, 2002.
- [29] V.P. Bhatnagar, D.F.H. Start, J. Jacquinot, F. Chaland, A. Cherubini, and F. Porcelli. Local magnetic shear control in a tokamak via fast wave minority ion current drive: theory and experiments in JET. *Nucl. Fusion*, 34(12):1579, 1994.
- [30] Chris C. Hegna and A. Bhattacharjee. Suppression of magnetic islands by energetic ions in toroidal plasmas. *Phys. Fluids B*, 2(8):1804, 1990. doi: http: //dx.doi.org/10.1063/1.859452.
- [31] Huishan Cai, Shaojie Wang, Yinfeng Xu, Jintao Cao, and Ding Li. Influence of energetic ions on tearing modes. *Phys. Rev. Lett.*, 106(7):075002, 2011. doi: 10.1103/PhysRevLett.106.075002.

- [32] V. V. Mirnov, F. Ebrahimi, C. C. Kim, J. R. King, M. C. Miller, J. A. Reusch, J. S. Sarff, D. D. Schnack, C. R. Sovinec, and T. D. Tharp. Effects of toroidal geometry and FLR nonlocality of fast ions on tearing modes in reversed field pinch. In *Proceedings of the 23rd IAEA Fusion Energy Conference*, pages THS/P5– 11, Daejeon, Republic of Korea, October 2010.
- [33] V. A. Svidzinski and S. C. Prager. Effects of particles with large gyroradii on resistive magnetohydrodynamic stability. *Phys. Plasmas*, 11(3):980, 2004. doi: http://dx.doi.org/10.1063/1.1643058.
- [34] J. L. Clark, W. Capecchi, J. Egedal, and J. K. Anderson. Fast ion orbit topology in the reversed-field pinch. *Bull. Am. Phys. Soc.*, 58(16):CP8.73, 2013.

Chapter 4

Safety Factor Modification with NBI

A wide variety of potential mechanisms for stabilization of MHD activity were described in Chapter 3. Given the diversity of mechanisms and the limited diagnostic coverage of fast ions in MST, a full evaluation of each (or any) mechanism is beyond the scope of this work. However, tearing mode correlated temperature fluctuations, in combination with the island model developed in Chapter 3, offer a unique opportunity to diagnose the safety factor in the core of MST discharges. Measurements of rational surface locations in previous work have successfully distinguished dynamic changes in the equilibrium profile over the course of the sawtooth cycle in MST. This provides a sensitive indicator for changes in the current profile and magnetic shear due to the fast-ion population. A full assessment of mode stability due to these changes is also beyond the scope of this work, but the feasibility of stabilization through current profile modification can still be assessed.

In Section 4.1, the experimental conditions for measurements in non-reversed plasmas are described along with the correlated temperature fluctuations. Changes to mode structure are described along with observed changes to the equilibrium q-profile. Section 4.2 outlines the process of equilibrium reconstruction using MSTFit and the effectiveness of rational surface constraints obtained via Thomson scattering. The results of equilibrium reconstruction for non-reversed discharges are given in Section 4.3. Differences of only a few percent due to NBI are resolved for both the safety factor and current density on-axis. These changes result in steeper

current gradients and lower shear; the n = 5 rational surface, which appears only marginally resonant without NBI, is clearly removed from the plasma during NBI. Rational surface measurements in reversed discharges are described in Section 4.4 and compared to the results for non-reversed discharges. Safety factor modification is not observed within the error bars, indicating significantly reduced NBI current drive for reversed discharges. These observations suggest that NBI current drive and the resulting safety factor modification may be inadequate to explain MHD stabilization on their own; alternative mechanisms such as FLR effects may be more consistent with the observed behavior. This is discussed in the conclusion along with suggestions for future research.

4.1 Correlated T_e fluctuations

All measurements in this chapter, except where noted, were taken in non-reversed plasmas with $I_p = 300$ kA and $n_e \sim 1.0 \cdot 10^{13}$ cm⁻³. Ensembles with NBI (177 shots) and without NBI (180 shots) were collected over several days, with the discharges on each day alternating between on/off shots for greater similarity between ensembles. For discharges with NBI, the beam was operated with the standard majority hydrogen mix (~97% H, 3% D) at full power (25 kV, 50 kA). The beam was turned on 12 ms after the start of the discharge (just before the flattop in the current) until 35 ms after the start of the discharge (approximately the end of the flattop). The Bt-Programable Power Supply (Bt-PPS) was also used for toroidal field control rather than the legacy system, but the plasma performance and beam operation are not significantly altered by Bt-PPS.

Thomson data were taken with the Spectron lasers in triple-pulse mode; the first burst occured at 22 ms and the last burst occured at 26 ms on all shots. For beam on shots where the neutral beam arced during operation (leading to early termination of the beam), only TS bursts prior to the arc were accepted into the ensemble. During correlation analysis, the bursts were filtered based on plasma conditions, with the relevant parameters consisting of I_p , line-integrated n_e , core-average T_e , and m = 0 amplitude. The line-integrated density signals were primarily obtained

from FIR, specifically chord P06, which is closest to the magnetic axis for these discharges. However, for discharges without FIR data of adequate quality, the CO_2 interferometer was used. Due to lack of an absolute calibration for Thomson scattering density measurements, the relative n_e obtained from fits to TS data was not used. The core-average T_e value was obtained from the average value of TS data points with z/a < 0.38 (also filtered for data quality). The m = 0 amplitude was used to filter events based on temporal proximity to small bursts (~40 G) of (m,n) = (0,1) activity, which happen sporadically due to the marginal resonance of the m = 0 modes at the edge. Using a time window of ± 0.15 ms, each laser pulse in the burst was filtered independently, with the (0,1) amplitude required to be less than 5 G within the window. Additionally, due to the variation in amplitude and velocity between modes with different toroidal number, filters for these parameters were applied individually for each mode. The mode velocity and amplitude filters were intentionally set to be weak, only eliminating outliers.

Correlation analysis was performed for modes n = 5-8. Tables 4.1–4.4 summarize the maximum/minimum filter values for each parameter as well as the average and standard deviation of the parameter over the NBI On/Off ensembles. The fluctuation amplitudes and phases relative to \tilde{B}_{θ} measured at the wall are shown in Figures 4.1,4.2, 4.5, and 4.6. Black/red curves correspond to plasmas without/with neutral beam injection. The changes in the measured structures are summarized below.

	NBI Off, 222 events				NBI On, 223 events			
Parameter	Max	Min	Mean	Std. Dev.	Max	Min	Mean	Std. Dev.
I _p (kA)	312	302	306.8	2.3	312	302	307.1	2.1
F	NA	NA	-5.8E-4	9.3E-4	NA	NA	-6.2E-4	9.8E-4
$n_e (10^{13} \text{ cm}^{-3})$	1.15	0.85	0.96	0.07	1.15	0.85	0.99	0.07
$T_{e,core}$ (eV)	265	195	228.3	17.6	265	195	228.2	20.4
v_5 (kps)	32	17	24.8	3.2	51	24	36.7	5.0
$\tilde{B}_{\theta,5}(\mathfrak{a})$ (G)	18	4.5	10.0	2.6	11.5	1.8	6.1	1.5

Table 4.1: n = 5 Filters and Parameter Values

	NBI Off, 222 events				NBI On, 193 events			
Parameter	Max	Min	Mean	Std. Dev.	Max	Min	Mean	Std. Dev.
I _p (kA)	312	302	306.8	2.3	312	302	307.2	2.1
F	NA	NA	-5.8E-4	9.4E-4	NA	NA	-6.8E-4	9.9E-4
$n_e (10^{13} \text{ cm}^{-3})$	1.15	0.85	0.96	0.07	1.15	0.85	0.99	0.07
$T_{e,core}$ (eV)	265	195	228.3	17.7	265	195	227.3	20.3
v_6 (kps)	28	13	21.6	3.2	45	25	31.6	3.3
$\tilde{B}_{\theta,6}(a)$ (G)	14	2	5.4	1.7	14	2	6.1	1.7

Table 4.2: n = 6 Filters and Parameter Values

Table 4.3: n = 7 Filters and Parameter Values

	NBI Off, 204 events				NBI On, 198 events			
Parameter	Max	Min	Mean	Std. Dev.	Max	Min	Mean	Std. Dev.
I _p (kA)	312	302	306.8	2.3	312	302	307.1	2.1
F	NA	NA	-6.4E-4	9.2E-4	NA	NA	-6.8E-4	9.8E-4
$n_e (10^{13} \text{ cm}^{-3})$	1.15	0.85	0.96	0.07	1.15	0.85	0.99	0.07
$T_{e,core}$ (eV)	265	195	227.6	17.5	265	195	226.9	20.2
v_7 (kps)	28	13	19.8	3.1	36	19	27.1	3.9
$\tilde{B}_{\theta,7}(\mathfrak{a})$ (G)	7	1	3.7	1.1	8	1	3.7	1.1

Table 4.4: n = 8 Filters and Parameter Values

	NBI Off, 206 events				NBI On, 211 events			
Parameter	Max	Min	Mean	Std. Dev.	Max	Min	Mean	Std. Dev.
I _p (kA)	312	302	306.9	2.3	312	302	307.2	2.1
F	NA	NA	-6.1E-4	9.1E-4	NA	NA	-6.3E-4	9.9E-4
$n_e (10^{13} \text{ cm}^{-3})$	1.15	0.85	0.96	0.07	1.15	0.85	0.99	0.07
$T_{e,core}$ (eV)	265	195	227.8	17.7	265	195	227.7	20.2
v_8 (kps)	26	11	18.2	2.9	33	16	23.8	3.9
$\tilde{B}_{\theta,8}(a)$ (G)	6	1	2.6	0.9	6	1	2.6	0.9



Figure 4.1: Amplitude (top) and phase (bottom) of T_e fluctuations correlated with n = 5 edge magnetic signals without NBI (black) and with NBI (red).

Suppression of the n = 5 **mode**

For the n = 5 mode, the most significant effect of neutral beam injection is reduction of the temperature fluctuation amplitude by $45\pm10\%$. This is comparable to the reduction of mode amplitude at the wall by $39\pm2\%$. The centroid of the observed structure is approximately z = 0.1090 ± 0.0021 m without NBI and z = 0.0965 ± 1000

0.0035 m with NBI. The Shafranov shift makes resolution of the full structure impossible, however, so the behavior of n = 5 correlated T_e fluctuations near the magnetic axis is not clear. While the observed shift of the centroid (~1.3 ± 0.4 cm) is statistically significant, it is not clear how much meaning this has without information about the T_e fluctuation amplitude near the axis. Estimates of the location of the n = 5 current perturbation from polarimetry also suggest inward movement [1]. The width of the structure may also change, although the inward movement of the peak and the Shafranov shift obscure this. For the NBI on case, the full-width at half maximum cannot be calculated. Finally, minimal change is observed in the phase of the correlated temperature fluctuations.

Given that the mode amplitude at the wall reduces by 39% with NBI, the lack of more strongly apparent changes in the width or location of the peak in n = 5 amplitude is difficult to interpret. The tearing mode island width (Eq. 3.5) depends on both the radial perturbation amplitude and the local magnetic shear. While the reduction in mode amplitude at the wall suggests that the island width should be reduced by nearly $\sqrt{2}$, reduced shear could compensate (the impact on q is discussed later in Section 4.3). However, if the island width is unchanged, then the temperature fluctuation amplitude should be similarly unchanged. Due to the stochastic nature of the core, a sufficiently high degree of stochasticity may lead to an apparent width that does not depend strongly on mode amplitude.

As an additional complication the exact nature of the n = 5 mode is in doubt. While the $n \ge 6$ modes all exhibit standard tearing parity in temperature fluctuation structure, the n = 5 mode does not. Across the full width of the observed structure, the mode is completely in phase with \tilde{B}_{θ} measured at the wall. No phase flip is observed, and only the single peak in temperature amplitude is visible. These results are consistent with those found in Ref. [2].

The Shafranov shift leaves open the possibility that the phase flip in temperature fluctuations is simply beyond the view of the vertical Thomson scattering axis. The observed peak, however, is relatively broad and flat-topped; the structure is not well fit by the tearing mode island fluctuation models of the previous chapter. For an island near the magnetic axis, significant deformation may occur. The X-point
may disappear altogether, significantly altering the topology of the perturbation. Alternatively, due to the low shear near the magnetic axis, kink modes may be unstable. While a resistive kink may still form an island structure, an ideal kink or single-helical-axis-like state may produce the observed fluctuation profiles. The structure observed in Ref. [2] was estimated to be consistent with a helical perturbation to the magnetic axis. These observations have motivated the identification of the n = 5 mode as 'kink-tearing' throughout this thesis in order to distinguish it from the other tearing modes. However, more work is necessary to clarify the nature of this mode.

Due to the uncertainty in the identity of the n = 5 mode and the complications involved in analyzing the changes in the observed structure of the n = 5 mode, no further use of these results is made here. The remainder of this chapter will focus on the modes with observed tearing parity and the identification of rational surfaces.

Effects of NBI on the n = 6 **tearing mode**

In an interesting twist, the amplitude of the n = 6 tearing mode actually increases slightly with NBI (~13 ± 3%). The plasma conditions for these discharges are at slightly higher current and density than the discharges for which possible suppression of the n = 6 mode was reported in Chapter 3. The bulk plasma may have a strong impact on the beam profile, so variation of these parameters may have a significant impact on the fast-ion distribution. The q-profile modification discussed later may play a role in destabilizing the n = 6 mode. From the peak amplitude of the temperature structures, modest enhancement is also observed on the outer edge, but this is not statistically significant and may simply be a result of island movement. Examination of the peak to peak region suggests that the width of the mode may decrease, as the number of radial positions inside the island decreases by one. Furthermore, from the amplitudes and the location of the phase flip, the island location appears to change.

In order to quantify these changes, a simple island flattening model is applied.



Figure 4.2: Amplitude (top) and phase (bottom) of T_e fluctuations correlated with n = 6 edge magnetic signals without NBI (black) and with NBI (red). The phase flip across the rational surface is accounted for in the amplitude plot by a change in sign.

Consistent with the analysis performed in Ref. [3], the complete island flattening model developed in Ref. [4] for diagnosis of island stability is adapted here. This version of the model neglects the higher order terms used in Ref. [4] which describe small effects on the temperature fluctuation structure: the ratio of $B_{\theta}^{\prime\prime}$ to B_{θ}^{\prime} (where

the ' denotes the radial derivative) and the linear stability parameter Δ' . The temperature fluctuations reduce to the form $\frac{dT_e}{dx}\Delta x$, where dT_e/dx represents a local temperature gradient across flux surfaces and Δx represents the radial range of flux surfaces sampled by a fixed scattering volume as the tearing mode island rotates past the measurement location. As in Chapter 3, the radial coordinate $x = r - r_s$.



Figure 4.3: Island flux surfaces, with gray region denoting the flux surfaces with complete temperature flattening. Vertical lines indicate the different flux surfaces sampled by two fixed scattering volumes: x_{out} , which always samples flux surfaces outside the separatrix, and x_{in} , which crosses the separatrix.

The extreme values of the flux surfaces (and therefore temperatures) sampled at a fixed volume are obtained alternately when the sampling point is at the helical angle of $\zeta = \pm \pi$ (the island O-point) or the helical angle $\zeta = 0$ (the X-point). For a volume that samples flux surfaces at the radial coordinate x, the flux surface crossing this volume at the X-point has radial coordinate d(x) at the O-point (using the notation of Ref. [4]). See Figure 4.3. The amplitude of the temperature fluctuations is then $|dT/dx|_{\pm} \cdot |d(x) - x|$, where dT/dx represents the gradient in the perturbed temperature profile just outside the island at the O-point. The subscript \pm indicates the dependence on the sign of x. Using the island flux surface description given by Eq. 3.6, d(x) is expressed as:

$$d(x) = \pm \sqrt{\frac{w^2}{4} + x^2}$$
(4.1)

and, from Ref. [4], the linear gradient at the O-point is given by:

$$\left(\frac{\mathrm{dT}}{\mathrm{dx}}\right)_{\pm} = A\left(1 \mp \frac{\pi w}{8\mathrm{r}_{\mathrm{s}}}\right) \tag{4.2}$$

with *w* the full island width and A a constant. For a scattering volume that samples only flux surfaces outside the separatrix, this gives a fluctuation amplitude of

$$\left|\tilde{\mathsf{T}}_{e}\right| = \left|\mathsf{A}\left(1 \mp \frac{\pi w}{8\mathsf{r}_{s}}\right)\right| \cdot \left|\pm \sqrt{\frac{w^{2}}{4} + x^{2}} - x\right|$$
(4.3)

For a scattering volume which samples flux surfaces both inside and outside the separatrix, this must be modified slightly. Inside the separatrix, complete flattening of the temperature profile means dT/dx = 0 and none of the flux surfaces sampled contribute to the fluctuation amplitude. Only the region between d(x) and the edge of the island, $x = \pm w/2$, contributes to the fluctuation amplitude:

$$\left|\tilde{\mathsf{T}}_{e}\right| = \left|\mathsf{A}\left(1 \mp \frac{\pi w}{8\mathsf{r}_{s}}\right)\right| \cdot \left|\sqrt{\frac{w^{2}}{4} + x^{2}} - \frac{w}{2}\right| \tag{4.4}$$

This yields a model in three parameters (temperature gradient, island width, and rational surface location) which can be fit to the Thomson scattering fluctuation measurements. Only radial locations near the island with sufficient fluctuation power were selected for fitting. Since the fluctuations are measured over a finite volume, the modeled fluctuation profile is averaged across each scattering volume to determine the predicted fluctuation amplitude. This effect turns out to be negligible, however, as seen from the modeled profiles below. Using the Gaussian approximations outlined in Chapter 2, Bayesian statistics again provides an

estimator of the probability that a particular set of parameters fits the data

$$P(A, w, r_s | \tilde{T}_e, \sigma) = \frac{1}{\prod_j \sigma_{T_{e,j}}} \exp\left(-\frac{1}{2}\chi^2\right)$$
(4.5)

$$\chi^{2} = \sum_{j} \frac{\left[\tilde{T}_{e,measured} - \tilde{T}_{e,model}\right]^{2}}{\sigma^{2}_{T_{e,j}}}$$
(4.6)

This probability distribution is calculated over a 3D grid sufficiently large to capture the full width of the probability distribution. Each parameter is marginalized to acquire three probability distributions of dimension one, and the most likely value and the error (1/e width of the distribution) are calculated for each. The most likely fluctuation profiles with/out NBI are shown in Figure 4.4, and the values and errors for each parameter are listed in Table 4.5.

	NBI Off		NBI On	
Parameter	Value	Error	Value	Error
A (eV/m)	510	120	880	185
w (cm)	7.3	1.0	5.4	0.6
$z_{\rm s}$ (cm)	18.79	0.54	17.62	0.29

Table 4.5: Island Model Fit Parameters

The amplitude of the temperature gradient increases with NBI. Approximately 10 eV of core heating is observed during neutral beam injection, and this may explain the steepening of the gradient. Due to small systematic errors, some corrugation is present in the equilibrium profiles, however. This complicates comparison of the gradient values obtained from the fits to the observed equilibrium gradients, but the fit coefficients for the gradient are generally consistent with the equilibrium data.

The island width appears to decrease by \sim 35% with NBI. This is consistent with the observation made earlier that the number of radial positions within the island decreases. This is not consistent with the increase in amplitude of the magnetic perturbation at the wall, however. While the error bars obscure the difference in the change in island width, the possible reduction in *W* suggests that multiple



Figure 4.4: Helical model fits to n = 6 temperature fluctuations without NBI (top) and with NBI (bottom). Measured fluctuations in the vicinity of the island (red) plotted with most likely helical model (blue) and predicted volume-averaged fluctuations (green).

mechanisms may be working against each other—for example, changes in the current profile may contribute to additional instability while the increase in flow shear may inhibit the island size. An increase in stochasticity may also produce a smaller remnant island.

Lastly, the rational surface is observed to shift inward by $\sim 1.1 \pm 0.6$ cm. This is a strong indication that the fast-ion population is altering the current profile, and consequently the shear. The ability to resolve such a shift represents a unique opportunity for Thomson scattering to constrain the magnetic field profiles. In combination with estimates of the n = 7 and n = 8 rational surface locations, these measurements are used in the next section to constrain equilibrium fits.

A note on the phase of temperature fluctuations

From Fig. 4.2, the n = 6 fluctuation phase relative to \tilde{B}_{θ} at the wall for plasmas without NBI clearly deviates from the expected 0 or π relationship expected of tearing modes. This is a critical feature that is explored in Chapter 5 in the context of current transport and the dynamo. For the island flattening model applied here, though, one possible implication of the phase shift is that the parallel heat conductivity may not be high enough to justify the use of this model. For an equilibrium with intact flux surfaces, this would be a serious concern; in these RFP discharges, however, stochasticity destroys the flux surfaces. The observed structures are merely remnant islands-stochasticity weakens the correlation of temperature fluctuations with magnetic perturbations outside the surviving island regions. In the stochastic region between islands, the isothermal surfaces result from a superposition of perturbations due to many modes with different n value. Unlike a system with intact flux surfaces, where the isothermal contours correspond to magnetic perturbations due to a single mode and temperature fluctuations are therefore expected to have phase of 0 or π relative to B_{θ}, in a stochastic region such a restriction on the phase is not necessarily guaranteed. In fact, the radial locations with a significant phase shift are all located in the stochastic region between the n = 6 and n = 5 remnant structures. For locations that predominantly sample the remnant n = 6 island or locations edge-ward of the island, no significant phase shift is observed. The island flattening model is assumed to still be valid for this case.



Figure 4.5: Amplitude (left) and phase (right) of T_e fluctuations correlated with n = 7 edge magnetic signals without NBI (black) and with NBI (red). The phase flip across the rational surface is accounted for in the amplitude plot by a change in sign.

Effects of NBI on other modes

The n = 7 and 8 modes do not exhibit any observable change in amplitude at the wall or temperature fluctuation amplitude. Due to the small size of the remnant

islands for these modes, only a few radial positions are observed to be within the islands. The amplitudes of the island structures are also small, making it difficult to distinguish between the island features and the extended radial structure present.

The location of the phase flip for these modes is clearly resolvable and may shift with NBI, although the error in the rational surface estimate discussed below prevents a definitive statement. Unlike the n = 6 mode, the rational surfaces for the n = 7 and 8 appear to move outward. The opposing movement of these modes is another indicator that current drive on axis is countered by current redistribution in the mid-radius. Since the temperature fluctuation model for island flattening is not easily applied to these modes, linear interpolation is used instead. Interpolation is performed between the two points on either side of the phase flip. The error in the linear fit is inferred based on the error in the fluctuation amplitude. While the scattering volume width contributes to measurement uncertainty, the effect of volume averaging on the helical fits to the n = 6 mode was shown to be negligible. Volume averaging is likewise assumed to have a negligible effect on the linear interpolation so it was not included in the analysis. Including the scattering volume width in estimates of the uncertainty leads to error bars that are unphysically large and not supported by close examination of the temperature structures. The values for these measurements, along with the n = 6 rational surface locations, are summarized in Table 4.6.

Lastly, while the n = 5 measurements showed no change in mode phase, in addition to the rational surface shifts for $n \ge 6$, the phase behavior outside of the remnant islands also changes. For plasmas without NBI, the phases of the temperature fluctuations core-ward of each island show significant offset from the phase of \tilde{B}_{θ} , indicating correlation with \tilde{B}_r . During NBI, the phase offset is eliminated for each mode, implying elimination of $\tilde{T}_{e,r}$. The impact of temperature fluctuations correlated with radial magnetic perturbations is discussed in Chapter 5.



Figure 4.6: Amplitude (left) and phase (right) of T_e fluctuations correlated with n = 8 edge magnetic signals without NBI (black) and with NBI (red). The phase flip across the rational surface is accounted for in the amplitude plot by a change in sign.

4.2 Rational surface constraint with tearing modes

The rational surface location measurements are summarized in Table 4.6 and Figure 4.7 below. The correlated temperature fluctuations afford millimeter resolution of the rational surface locations. Identification of multiple rational surfaces offers a constraint on both the value of q in the mid-radius and the gradient in q. The distance between the n = 6 and n = 7 modes, for example, increases from ~5 cm to over 7 cm with NBI. Flattening of the safety factor is clearly resolvable, strongly constraining the equilibrium profiles. Due to the limited set of internal diagnostics available on MST, the addition of rational surface measurements from Thomson scattering expands diagnostic coverage in a significant way. Dynamic changes in the q-profile were measured via temperature fluctuations in Ref. [2]. The rational surface measurements here further demonstrate the sensitivity of Thomson scattering and motivate the use of rational surfaces as a constraint in equilibrium reconstruction techniques.

	NBI Off		NBI On	
Mode	Value	Error	Value	Error
z_{6} (cm)	18.79	0.54	17.62	0.29
<i>z</i> ₇ (cm)	23.73	0.64	25.12	0.84
<i>z</i> ₈ (cm)	27.42	0.55	28.22	0.44

Table 4.6: Rational surface locations

Polarimetry measurements [1] suggest that the current density in the core increases by 25% \pm 15% with neutral beam injection. The total current does not increase, however, suggesting current redistribution leading to a profile that is more peaked on axis. This redistribution would reduce the safety factor in the core while increasing it in the mid-radius, flattening the q-profile. The observed shift in rational surfaces obtained from correlated temperature fluctuations is consistent with these results. Given the proximity of q₀ to a value of 0.2, small changes in the q-profile may have significant implications for n = 5 stability. A reduction of q₀ by only 5–10% may be sufficient to remove the n = 5 rational surface from the plasma.

The equilibrium reconstruction routine MSTFit [5, 6] has been modified to accept rational surface measurements obtained via Thomson scattering. These measurements can better constrain equilibria for an independent verification of current redistribution as well as diagnosis of q-profile modification. The salient aspects



Figure 4.7: Rational surface locations for the n = 6, 7 and 8 modes with (red) and without (black) NBI.

of equilibrium reconstruction are discussed below, followed by an analysis of the effectiveness of Thomson rational surface measurements as an equilibrium constraint. Reconstructed profiles are ensemble averaged over the entire dataset used for temperature fluctuation measurements; the results are discussed in Section 4.3.

MSTFit equilibrium reconstruction

The following discussion is informed by the description of MSTFit provided in Ref. [5].

Combining Maxwell's equations with the radial force balance $\vec{J} \times \vec{B} = \nabla P$ results in the description of a toroidal equilibrium obtained by Grad and Shafranov [7, 8]. Assuming axisymmetry (the toroidal derivative $d/d\phi$ vanishes), the equilibrium can be written in the form:

$$\frac{\Delta^*\psi}{4\pi^2} = -\mu_0 R^2 \frac{dP}{d\psi} - \frac{1}{2} \frac{dF^2}{d\psi}$$
(4.7)

where both F and P are functions of the poloidal flux, ψ ; $P = P(\psi)$ represents the

plasma pressure and $F = F(\psi) = RB_{\phi}$ corresponds to the total poloidal current inside the radius R. $F(\psi)$ here should not be confused with the reversal parameter; the unfortunate overlap in variables is due to historical convention. The elliptical operator Δ^* on the poloidal flux corresponds to $\Delta^*\psi = R^2\nabla \cdot (\frac{1}{R^2}\nabla\psi)$. The equilibrium magnetic field is $\vec{B} = \frac{1}{2\pi R}\nabla\psi \times \hat{\phi} + \frac{F}{R}\hat{\phi}$ and the toroidal component of the equilibrium current is:

$$J_{\Phi} = \frac{2\pi FF'}{\mu_0 R} + 2\pi RP' \tag{4.8}$$

This expression for the toroidal current forms the basis of equilibrium reconstruction with MSTFit, and solutions are determined by both F and P. If J_{ϕ} is specified, then a consistent solution to Maxwell's equations and the force balance can be determined and the remaining profiles calculated. For example, the poloidal current and total parallel current are:

$$J_{\parallel} = \frac{2\pi}{|B|} \left(\frac{F'B^2}{\mu_0} + FP' \right)$$

$$J_{\theta} = \frac{2\pi}{\mu_0} F'B_{\theta}$$
(4.9)

Although both P and F can be treated as free parameters, typically P is well constrained by the data. Electron density information is provided by either CO₂ or FIR interferometry, while electron temperature profiles are provided by Thomson scattering. Ion temperature measurements can be provided by Rutherford scattering or CHERS, although the assumption $T_i \approx T_e$ is frequently substituted for experimental measurements. Regardless, for most MST plasmas, P and P' do not contribute significantly to the equilibrium.

As a result, F provides the most significant constraint for MST equilibria. In MSTFit, F' is treated as the free function and F is obtained via integration. F is directly constrained by the total toroidal flux, the toroidal field at the wall, and the value of B on axis. While these constraints are sufficient to capture the coarse features of the current profile, F' accounts for more subtle effects. In order to obtain a constraint on F', typically both $J_{\Phi}(0)$, from polarimetry, and $B_{\Phi}(0)$, from MSE,

are needed. Alternatively, the on-axis value of the safety factor can also provide a constraint on F':

$$q(0) = \frac{2B_{\Phi}(0)}{\mu_0 R J_{\Phi}(0)} = \frac{1}{\pi R F'(0)}$$
(4.10)

Without direct measurement of q_0 , previous work [5] has suggested using core, m = 1 mode activity as a proxy. While the Thomson rational surface measurements do not enable measurement of q_0 and the n = 6 rational surface is not close to the magnetic axis, mid-radius rational surface constraint provides an effective alternative.

The measurement of multiple rational surfaces constrains both B_{ϕ} and B_{θ} as well as the gradients in these parameters. Given the error bars attainable with Thomson scattering, this strongly constrains both F and F' in the core. Despite the distance of the n = 6 mode from the magnetic axis, reasonable on-axis constraint of the B and J profiles is still feasible. The increased leverage from F' constraint compared to pressure constraint from equilibrium temperature profiles is a major improvement for the Thomson scattering diagnostic.

A basis function expansion of the F' profile is used, which is equivalent to splining. F' is required to be positive definite and zero at the boundary (the current must be zero at the edge). The splines utilize M ordered pairs, referred to here as knots, with the first value representing the knot location and the second value representing the value of F' at the knot. The knot locations may be fixed or allowed to vary; this is determined by the user, along with the total number of knots. Knots with fixed locations contribute M - 1 free parameters to the fit, while knots with dynamic locations contribute 2M - 3 free parameters. The on-axis and boundary knots are always fixed, while the interior knots may be dynamic.

The process for obtaining a solution is iterative. An initial guess for the equilibrium is determined using a Modified Polynomial Function model (a variation of the Bessel Function Model). Across a 2-D triangular mesh grid of a circular cross section, consistent current and flux values are calculated. From these profiles, predicted signal values are computed and compared to the measured signals. Following a non-linear least squares minimization via a downhill simplex algorithm, the free parameters are adjusted and the process repeated until the most likely equilibrium is obtained. Once the equilibrium has been determined, relevant quantities like J and B are flux-surface averaged to reduce the equilibrium to one dimension. The effective minor radius is given by:

$$\rho_{\rm V} = \sqrt{\frac{V_{\rm \psi}}{2\pi^2 R}} \tag{4.11}$$

where V_{ψ} is the volume of the flux surface ψ .

In order to accomodate TS rational surface measurements, the calculation of χ^2 in the comparison of predicted signals to measured data was adapted to include flux surface averaging of the safety factor. Since the TS measurements are ensemble averages of fluctuations correlated with rotating tearing modes, flux surface averaged q values are the most appropriate comparison rather than local q values. The local values are less computationally expensive, however, and adding flux surface averging to each iteration adds significantly (although not prohibitively) to the time required to achieve an equilibrium solution. The flux surface averaged values of q are projected onto the vertical TS chord for direct comparison with the zero-crossing measurements in Table 4.6.

Effectiveness of TS rational surface constraints

The effectiveness of TS rational surface constraints on MSTFit equilibria was evaluated in Ref. [9]. The results indicated strong constraints with a few beneficial outcomes.

First, the complexity of the model for F' that could be justified by the available data increased with TS rational surface information. The reduced χ^2 value is given by:

$$\chi_{\nu}^{2} = \frac{1}{\nu} \sum_{i} \frac{(D_{i} - M_{i})^{2}}{\sigma_{i}^{2}}$$
(4.12)

where the D_i are the data with experimental uncertainty σ_i and M_i are the predicted signals. ν represents the number of degrees of freedom, equivalent to the number of independent signals being fit minus the number of free parameters in the model. Comparison of χ^2_{ν} between different models provides a useful framework for determining whether or not the addition of free parameters is justifiable. χ^2_{ν} must decrease by at least 15% to warrant an additional parameter [10]. Without TS data and only edge constraints, the maximum number of knots available for F' was three (using a free-knot model). The addition of TS data made fits with four knots feasible. Since the rational surface measurements add valuable constraint on the equilibria in the under-diagnosed core region, the increase in model complexity with Thomson scattering is not surprising.

Additionally, the TS rational surface measurements reduced the incidence of unphysical profiles. Without significant core constraint, reconstructed profiles can frequently yield safety factors that are flat across much of the core and too low for even the n = 6 mode to be resonant. These unphysical values were more likely to happen with models utilizing too many free parameters. The addition of TS rational surface measurements yielded more physical profiles with the on-axis value above 1/6 and closer to the expected 1/5 resonance. Even for models with more free parameters than justifiable, Thomson rational surface measurements reduced the rate at which unphysical profiles occured.

MSE data was not available for this ensemble, but interferometry/polarimetry data was. Reconstructions with TS data only were compared to reconstructions with FIR data only using two different approaches. First, each discharge was constrained only by the FIR data available at the TS time points for that discharge. In this case, the polarimetry data did not significantly constrain the equilibrium on its own. Due to the convolution of density and magnetic field in the polarimetry data, reconstruction can vary two different profiles to reduce χ^2 . The net effect is to dilute the impact of polarimetry data on the B and J profiles, so single-shot polarimetry did not provide significant equilbrium constraint. Fits using Thomson scattering rational surfaces were dominated by the small errors in the measurements, so comparison between these two cases was not meaningful.

To obtain a stronger constraint, the interferometry/polarimetry data were ensemble averaged across TS time points from all discharges. This significantly reduced

the error bars and put the polarimetry data on more equal footing with TS rational surface measurements. However, even in this case the fits yielded unrealistically low values of q. Unfortunately, this means that an independent verfication of the equilibrium profiles generated using Thomson rational surfaces is not available at the present. Probe studies with the Deep Insertion Probe are capable of measurements up to 30 cm from the wall of MST at low current and may provide an alternative verification technique.

4.3 Current redistribution and q-profile modification

Using the measured rational surface locations for the n = 6-8 modes with and without NBI, equilibrium reconstruction was performed for all time points where the plasma conditions and n = 5 activity at the wall met the restrictions listed in Table 4.1. Reconstructed profiles with $q_0 < 1/6$ were rejected as unphysical. Interestingly, this did not occur for any of the NBI on equilibria; only a small number of the equilibria for NBI off exhibited such behavior. The equilibria that passed these filters were averaged to obtain the mean profiles shown in the figures below. The error in the reconstruction was estimated as the standard deviation of each parameter over the ensemble of fits.

Shown in Figure 4.8 are reconstructed profiles for the total magnetic field and the toroidal and poloidal components for the ensemble without NBI. The constraints from the measured rational surface locations strongly impact the resulting profiles, with error bars for B_{tot} less than 1% across the minor radius. Error bars for B_{θ} were less than 3% across the minor radius, and less than 1% in the core for B_{ϕ} (the relative error in B_{ϕ} was large near q = 0 at the edge). Similar profiles are obtained with the ensemble for plasmas with NBI. B_{tot} and B_{ϕ} are largely unaffected, but B_{θ} shows a slight increase in slope in the core. This is connected to the changes in the current profile shown in Figure 4.9.

Although the sign convention on MST typically defines the current as negative as it flows opposite the magnetic field, the absolute values are plotted here for simplicity. Consistent with the estimates from polarimetry, the on-axis current



Figure 4.8: Reconstructed profiles for B_{tot} (blue), B_{ϕ} (red), and B_{θ} (green) without NBI.



Figure 4.9: Reconstructed parallel current profiles without NBI (black) and with NBI (red).

density increases with NBI. This increase is offset by a reduction in current density in the mid-radius. The net change in current with NBI ($J_{\parallel,on}$ - $J_{\parallel,off}$) is plotted

in Figure 4.10. The change in on-axis density corresponds to an increase of $7.9 \pm 5.1\%$. As a result of redistribution, the current gradient near the magnetic axis increases by nearly a factor of three, while the gradient near the n = 6 rational surface increases by ~20–30%. Near the n = 7 and 8 rational surfaces, the inferred gradient remains roughly the same or decreases slightly. The increased gradient is expected to contribute to additional instability and may explain the increase in amplitude of the n = 6 mode at the wall. The increased gradient would oppose the observed reduction in n = 5 mode amplitude, however, assuming the mode is resistive rather than ideal. This is further complicated by the removal of the rational surface from the plasma.



Figure 4.10: Change in parallel current with NBI. Current density increases on-axis and in the edge, but decreases in the mid-radius.

Reconstructed q-profiles, Figure 4.11, indicate that the observed flattening in the mid-radius corresponds to a reduction of q_0 by 0.013 ± 0.007 . Even without NBI, the predicted profiles indicate marginal resonance of the n = 5 mode. With NBI, the rational surface is predicted to be removed from the plasma. Nominally, the loss of the rational surface should damp the n = 5 mode. In practice, however, perturbations with n = 5 structure may still persist while q_0 is in the vicinity of 1/5.

Understanding how strongly the n = 5 mode responds to the inferred change in q_0 is a more complicated question that requires a better understanding of the nature of the n = 5 mode itself. However, measurements of q-profile modification in reversed discharges, described in the next section, cast doubt on this mechanism's role in mode stabilization.



Figure 4.11: Safety factor without NBI (black) and with NBI (red).

4.4 Comparison to reversed discharges

Measurements of tearing mode activity were made in somewhat similar reversed discharges. The plasma current was the same, 300 kA, and the reversal parameter was F = -0.2. Confinement during quiescent periods in reversed plasmas is slightly better than in non-reversed discharges, yielding higher overall temperatures. The density was also lower in the standard discharges. This complicates any direct comparison of the effects of fast ions with the F = 0 discharges discussed previously. The ensemble at these conditions was also not as extensive as the ensemble used for non-reversed discharges (119 shots without NBI, 108 shots with NBI). Combined

with the dynamic nature of the profiles over the course of a sawtooth period, the resolution of these measurements is not as high.

At sawtooth events, fast-ion confinement is severely degraded and the population rapidly transported out of the plasma. During the quiescent period following a sawtooth events, the fast-ion population recovers over a period of 2–3 ms. This corresponds roughly to the duration of the n = 5 mode. As a result, at the beginning of the sawtooth cycle the amplitude of the mode is largely unaffected. Modest reduction in mode amplitude is observed as the fast-ion population is restored; the suppression factor increases over this time period, reaching a maximum of ~45%, until the n = 5 terminates. This behavior is observed in both the edge magnetic signals, see Figure 4.12, and the temperature fluctuations, Figure 4.13.



Figure 4.12: Ensemble averaged edge B_{θ} signals for n = 5 mode in reversed discharges without (black) and with (red) NBI. The marginal increase in amplitude with NBI around 3.0–3.5 ms is due to bursting activity discussed in Chapter 6.

In addition to the reduction in mode amplitude, the fast-ion population also leads to an earlier termination of the n = 5 mode. The persistence of the n = 5mode is observed to depend on bulk plasma density, as shown in Figure 4.14, with higher density corresponding to longer average lifetimes. The presence of fast ions



Figure 4.13: n = 5 correlated \tilde{T}_e in reversed discharges without (blue) and with (red) NBI.

may alter this relationship, with the time-of-death depending less strongly on bulk density in plasmas with NBI. The range of operational densities is limited, however, so the effect is not clear. Overall, the lifespan of the n = 5 mode is reduced by ~0.5–1.5 ms with NBI.

In order to examine the q-profile effects, fluctuation measurements were binned into windows 0.75 ms wide, and several windows were selected from 2.0 ms to 4.25 ms after the sawtooth. This covers roughly the full range of times for which the n = 5 mode persists in these plasmas. Applying the same island flattening temperature fluctuation model to n = 6 correlated fluctuations, the island width and rational surface location were determined for each window. The evolution of these parameters is shown in Figure 4.15. The island width is generally observed to increase with time after the sawtooth, consistent with the increase in magnetic amplitude measured at the edge. No significant change is observed in the island width with NBI, although, interestingly, the width during the 3.5 to 4.25 ms window may be somewhat larger with the beam. The difference is not statistically significant, however.



Figure 4.14: Time of death (relative to sawtooth time) for n = 5 mode in reversed discharges as a function of density: without NBI (black) and with NBI (red).

The rational surface locations drift inward slightly over time, consistent with the evolving q-profile as the current density steepens in the core. While the rational surface is consistently measured to be ~0.3 cm inward with neutral beam injection, the difference between beam on and beam off is not distinguishable within the error bars. It may be the case that the modification of the q-profile is more subtle than the resolution afforded by this measurement. However, the n = 6 mode is closer to the magnetic axis in these discharges than in non-reversed discharges. As a consequence, it should be more sensitive to changes in q₀. Furthermore, the energetic particle modes, which are sensitive to the density gradient of the fast-ion population, generally become active within 2–3 ms following a sawtooth crash. This implies that the fast-ion density gradient achieves roughly equivalent values to F = 0 plasmas during the selected windows. The small shift in n = 6 rational surface locations indicates that the core-current drive observed in non-reversed plasmas is significantly reduced if not eliminated for reversed discharges.



Figure 4.15: Inferred n = 6 island width (top) and rational surface location along vertical Thomson chord (bottom) in reversed discharges: without NBI (black) and with NBI (red).

4.5 Discussion

Although an absence of MSE data for these discharges makes diagnosis of q_0 difficult, Thomson scattering measurements of rational surfaces in the mid-radius provide adequate constraint for equilibrium reconstruction to distinguish small

differences in inferred profiles. There are a few important caveats, however. For the fast-ion densities predicted by TRANSP modeling, the pressure term in equilibrium reconstruction may be non-negligible. Diagnosis of the fast-ion distribution is not currently adequate to provide a significant constraint on the pressure profile. This leaves a critical gap in understanding MHD stabilization mechanisms. Furthermore, the n = 5 mode, which is most strongly impacted by the presence of fast ions, is not well understood. Given these contraints, making a definitive statement about the mechanism for fast-ion suppression of MHD activity in MST is not feasible. But the measurements presented here suggest that one candidate for suppression, current modification, is inadequate to explain observations. Furthermore, the data provide a few hints regarding the nature of the n = 5 mode.

Inferred current profiles and safety factors for non-reversed discharges indicate redistribution of current which reduces q_0 and magnetic shear. The n = 5 mode is predicted to be marginally resonant in non-beam heated plasmas, with the fast-ion driven current removing the rational surface from the plasma. For reversed plasmas, however, mode suppression is observed with minimal changes in the q-profile. The lack of significant changes in the q-profile indicates a strong reduction in current drive and opens another mystery altogether. TRANSP is unable to accurately model reversed discharges due to the q = 0 surface within the plasma, so predicted profiles for fast-ion density may only be valid in non-reversed discharges. Effects due to trapping of energetic particles may also reduce current drive. Alternatively, the RFP dynamo in reversed discharges may provide a stronger profile-flattening mechanism. Regardless, the observation of n = 5 suppression in varying plasma conditions without consistent current profile modification suggests that this mechanism may not be responsible, or at least not solely responsible, for observed mode suppression.

Additionally, changes in the equilibrium current and safety factor indicate that the n = 6 mode in non-reversed discharges lies in a region of reduced shear and enhanced current gradient. The observed increase in mode amplitude at the wall appears to be consistent with these predictions. However, possible reduction in island width, coupled with previous observations of possible suppression of n = 6 activity under slightly different plasma conditions, complicate this picture. Multiple mechanisms, including flow shear, finite Larmor radius effects, and velocity space islands, may be affecting MHD activity. The $n \ge 6$ modes are better understood than the n = 5, exhibiting typical tearing parity in temperature fluctuation structure. Modeling the island parameters, particularly the width, in combination with improved spatial resolution or measurement accuracy, may provide a valuable tool for evaluating competing mechanisms.

Finally, the identity of the n = 5 mode deserves heightened scrutiny. Previous estimates show that observed n = 5 temperature fluctuation structures are consistent with helical perturbations of the magnetic axis. Sufficiently low shear in the core may give rise to kink activity. In reversed plasmas, the dependence of the n = 5 time-of-death on bulk density suggests that pressure may play a role in driving observed activity. While multiple mechanisms may be necessary to explain mode suppression in non-reversed plasmas, the suppression of n = 5 activity in the presence of increased current gradients also suggests an ideal rather than resistive mode. The marginal resonance of the mode in F = 0 plasmas without NBI, coupled with the persistence of n = 5 activity in beam-heated plasmas with q_0 below 0.2, further suggest that non-resonant kink predictions warrant attention. Non-resonant kink modes have been implicated in NSTX discharges with low shear and q_0 just above unity [11]. Inferred values of q_0 for NBI heated discharges are sufficiently close to resonance (approximately 5% below 0.2) to support similar activity.

4.6 References

- [1] L. Lin, W. X. Ding, D. L. Brower, J. J. Koliner, S. Eilerman, J. Reusch, J. K. Anderson, A. F. Almagri, B. E. Chapman, M. D. Nornberg, J. S. Sarff, J. Waksman, and D. Liu. Energetic-particle-driven instabilities and their effect on fast ions in a reversed field pinch. *Bull. Am. Phys. Soc.*, 57(12):JP8.137, 2012.
- [2] H. D. Stephens. Electron Temperature Structures Associated With Magnetic Tearing

Modes in the Madison Symmetric Torus. PhD thesis, University of Wisconsin-Madison, 2010.

- [3] Richard Fitzpatrick. Helical temperature perturbations associated with tearing modes in tokamak plasmas. *Phys. Plasmas*, 2(3):825, 1995. doi: http://dx.doi. org/10.1063/1.871434.
- [4] C. Ren, J. D. Callen, T. A. Gianakon, C. C. Hegna, Z. Chang, E. D. Fredrickson, K. M. McGuire, G. Taylor, and M. C. Zarnstorff. Measuring Δ' from electron temperature fluctuations in the Tokamak Fusion Test Reactor. *Phys. Plasmas*, 5 (2):450, 1998. doi: http://dx.doi.org/10.1063/1.872729.
- [5] J. K. Anderson. *Measurement of the electric resistivity profile in the Madison Symmetric Torus*. PhD thesis, University of Wisconsin-Madison, 2001.
- [6] J.K. Anderson, C.B. Forest, T.M. Biewer, J.S. Sarff, and J.C. Wright. Equilibrium reconstruction in the Madison Symmetric Torus reversed field pinch. *Nucl. Fusion*, 44(1):162, 2004. doi: http://dx.doi.org/10.1088/0029-5515/44/1/018.
- [7] Harold Grad. Toroidal containment of a plasma. *Phys. Fluids*, 10(1):137, 1967.
 doi: http://dx.doi.org/10.1063/1.1761965.
- [8] V. D. Shafranov. Plasma equilibrium in a magnetic field. In M. A. Leontovich, editor, *Reviews of Plasma Physics*, volume 2, page 103. Consultants Bureau, New York, 1966.
- C. A. Johnson. Implementing rational surface locations measured from Thomson scattering into MSTFit. Senior Thesis, University of Wisconsin-Madison, 2014.
- [10] P.R. Bevington and D.K. Robinson. *Data reduction and error analysis for the physical sciences*. McGraw-Hill Higher Education. McGraw-Hill, 2003. ISBN 9780072472271.
- [11] J. A. Breslau, M. S. Chance, J. Chen, G. Y. Fu, S. Gerhardt, N. Gorelenkov, S. C. Jardin, and J. Manickam. Onset and saturation of a non-resonant internal

mode in NSTX and implications for AT modes in ITER. *Nucl. Fusion*, 51(6): 063027, 2011. doi: http://dx.doi.org/10.1088/0029-5515/51/6/063027.

Chapter 5

Current Transport and the Dynamo

As discussed in the introduction, ohmic current drive sustains current in the RFP longer than resistive diffusion times. The discrete relaxation events known as saw-teeth play a crucial role in sustaining the RFP and are driven by non-linear growth of m = 1 modes. However, in reversed discharges the dynamo is active throughout the entire sawtooth cycle, including the quiescent period between sawteeth. A full description of the RFP dynamo has been historically challenging. Multiple sources have been identified as contributing to the dynamo, and description of these sources spans single- and two- fluid MHD as well as kinetic theory. Kinetic effects were initially discounted as inadequate to fully explain the observed dynamo, but the evolution of diagnostic capabilities has recently spurred renewed interest in these kinetic effects. Core dynamo measurements with sufficient resolution have demonstrated non-negligible emf generated due to density fluctuation driven kinetic dynamo terms.

The measurement of both amplitude and phase of temperature fluctuations described in Chapter 2, combined with large ensembles, has permitted the first measurements of core electron temperature fluctuations in phase with radial magnetic perturbations as shown in Chapter 4. These measurements are for non-reversed discharges, however; no such correlation has been observed in reversed discharges to date. While Hall dynamo effects have been measured with polarimetry during weak, m = 0 bursts in F = 0 plasmas, the quiescent periods of these discharges are

not as well studied and may not require dynamo action. A paramagnetic pinch [1], for example, could yield a non-reversed configuration without dynamo. The correlation of pressure fluctuations with radial magnetic perturbations leads to current transport due to radial streaming of field-aligned current, $\Gamma = \langle \tilde{p}_{e,\parallel} \tilde{B}_r \rangle$ as determined by previous work. This current transport has also been identified as a source of dynamo emf, $\vec{E}_{kin} = \frac{1}{en_e B_0} \nabla \cdot \langle \tilde{p}_{e,\parallel} \tilde{B}_r \rangle$. The correlated product $\Pi = \frac{1}{en_e B_0} \langle \tilde{p}_{e,\parallel} \tilde{B}_r \rangle$ can be separated into contributions from density fluctuations and temperature fluctuations. The measured product $\Pi_T = \frac{1}{eB_0} \langle \tilde{T}_e \tilde{B}_r \rangle$ is, therefore, discussed in the context of the RFP dynamo despite the lack of reversal.

Radial magnetic perturbations cannot be directly diagnosed by the edge coil arrays. In Section 5.1, the expected phase difference between \tilde{B}_r and \tilde{B}_{θ} is calculated and this prediction is compared to a wide range of experimental measurements in MST and other devices as well as numerical simulations. The modeled radial profiles of \tilde{B}_r are given in Section 5.2. Current transport due to the n = 5-8 tearing/kink-tearing modes is discussed in Section 5.3, where the net transport due to these modes is consistent with zero. Placing these measurements in the context of the RFP dynamo, a broad overview of the dynamo is provided in Section 5.4. The kinetic dynamo is outlined in Section 5.5, where previous work on radial streaming due to stochasticity is summarized and supplemented with a discussion of the importance of anisotropy for the kinetic dynamo. The difference in pressure fluctuations perpendicular and parallel to the equilibrium magnetic field is found to be critical to the kinetic dynamo. The chapter concludes with a discussion of the relative importance of thermal contributions to the kinetic dynamo, the impact of neutral beam injection, and suggestions for future research.

5.1 The phase of \tilde{B}_r

Since B_r goes to zero at the wall, direct diagnosis is difficult. An analytical calculation in cylindrical geometry indicates that the phase of \tilde{B}_r is shifted by $\pi/2$ radians from the phase of \tilde{B}_{θ} . This is born out by both probe measurements and numerical simulations.

Due to the (approximately) perfectly conducting boundary provided by the 5 cm thick aluminum wall of MST, two boundary conditions can be applied. The first, as indicated above, is the elimination of radial fluctuations at the wall:

$$\tilde{B}_{r}(a,\theta) = 0 \tag{5.1}$$

Additionally, due to graphite limiters at the wall which extend approximately 1 cm from the surface, a thin vacuum region exists between the plasma and the wall. The current must also go to zero, providing the second boundary condition:

$$\tilde{j}_{r}(a,\theta) = 0 \tag{5.2}$$

Applying Ampere's Law, where the fluctuations in the j-th component of B have the form

$$\tilde{B}_{j} = \left| \tilde{B}_{j} \right| \exp\left(im\theta + in\phi + i\delta_{j} \right)$$
(5.3)

yields the following relationship between \tilde{B}_{θ} and \tilde{B}_{ϕ} :

$$\mu_{0}\tilde{j}_{r}(a) = \hat{r} \cdot (\nabla \times \tilde{B})_{r=a}$$

$$= \hat{r} \cdot \hat{r} \left[\frac{1}{r} \frac{\partial \tilde{B}_{\phi}}{\partial \theta} - \frac{1}{R} \frac{\partial \tilde{B}_{\theta}}{\partial \phi} \right]_{r=a}$$

$$= \frac{\left| \tilde{B}_{\phi} \right|}{a} ime^{i(m\theta + n\phi + \delta_{\phi})} - \frac{\left| \tilde{B}_{\theta} \right|}{R} ine^{i(m\theta + n\phi + \delta_{\theta})}$$

$$= 0$$

$$(5.4)$$

Solving this for the relative amplitudes:

$$\frac{\left|\tilde{B}_{\Phi}\right|}{\left|\tilde{B}_{\theta}\right|} = \frac{an}{Rm}$$
(5.5)

and phases:

$$\delta_{\Phi} = \delta_{\theta} \tag{5.6}$$

So \tilde{B}_{θ} and \tilde{B}_{φ} are in phase with each other and their amplitudes scale according to

product of the aspect ratio and the ratio of the toroidal to poloidal mode numbers.

For the relationship between \tilde{B}_{θ} and \tilde{B}_{r} , the divergence of B and the boundary conditions yield:

$$(\nabla \cdot \tilde{B})_{r=a} = \left[\frac{1}{r} \frac{\partial}{\partial r} (r \tilde{B}_{r}) + \frac{1}{r} \frac{\partial \tilde{B}_{\theta}}{\partial \theta} + \frac{1}{R} \frac{\partial \tilde{B}_{\phi}}{\partial \phi} \right]_{r=a}$$

$$= \frac{\partial |\tilde{B}_{r}|}{\partial r} e^{i(m\theta + n\phi + \delta_{r})} + \frac{im |\tilde{B}_{\theta}|}{a} e^{i(m\theta + n\phi + \delta_{\theta})} + \frac{in |\tilde{B}_{\phi}|}{R} e^{i(m\theta + n\phi + \delta_{\phi})}$$

$$= \frac{\partial |\tilde{B}_{r}|}{\partial r} e^{i(m\theta + n\phi + \delta_{r})} + \frac{im |\tilde{B}_{\theta}|}{a} \left(1 + \frac{a^{2}n^{2}}{R^{2}m^{2}} \right) e^{i(m\theta + n\phi + \delta_{\theta})}$$

$$= 0$$

$$(5.7)$$

Expanding the exponentials into real and imaginary components, the phase relationship can be reduced to

$$\tan\left(\delta_{\theta} + \frac{\pi}{2}\right) = \tan\left(\delta_{r}\right)$$
$$\delta_{\theta} + \frac{\pi}{2} = \delta_{r} + l\pi$$
(5.8)

for integer values of l. Eqn. 5.7 then becomes

$$\frac{\partial \left|\tilde{B}_{r}\right|}{\partial r} = -\frac{m\left|\tilde{B}_{\theta}\right|}{a} \left(1 + \frac{\left|\tilde{B}_{\phi}\right|^{2}}{\left|\tilde{B}_{\theta}\right|^{2}}\right) e^{il\pi}$$
(5.9)

The exponential term must be real and positive, restricting l to even integer values and yielding the final relation between the phase of \tilde{B}_{θ} and \tilde{B}_{r} :

$$\delta_{\theta} + \frac{\pi}{2} = \delta_{r} \tag{5.10}$$

Therefore, the radial magnetic perturbation associated with the tearing modes leads the poloidal perturbation by $\pi/2$. This calculation was performed in [2], but due to a sign error [3], the relationship was reversed.

Ref. [2] also measured the phase of the radial perturbation for the n = 6 mode relative to the phase of \tilde{B}_{θ} at the wall. A scan of probe insertion depths indicates

that the mode phase varies significantly with radial location near the wall, but inside r/a values of ~0.8, the phase difference is relatively constant and approximately $\pi/2$. Unfortunately, the probe measurements also involve a sign error [3]; while the cylindrical prediction and probe measurements reported in [2] were consistent regarding relative phase, correcting for the sign error shows that the radial component leads the poloidal component as predicted above.

Similar measurements were made for multiple m = 1 modes in Ref. [4], although the maximum probe insertion depths were shallower than those for Ref. [2]. The reported poloidal components for the n = 5 and n = 6 modes had primarily real amplitudes, while the radial components were primarily imaginary. Again, this indicates a phase shift of $\pi/2$. The results are determined in a right hand coordinate system; taking into account the handedness of the coordinates and the signs of the poloidal/radial components shows that the radial perturbations for these modes also lead the poloidal perturbations. At $r/a\sim0.8$, the phase difference is within ~0.14 rad (8°) of $\pi/2$ for both modes. The n = 7 and n = 8 modes measured in Ref. [4] are somewhat more complicated. Like the n = 5 and 6 modes, the radial perturbations lead the poloidal perturbations. However, the phase offset also varies strongly with radial location and does not appear to converge toward $\pi/2$ as quickly as observed for the n = 5 and 6 modes. For the n = 7 mode, the relative phase at $r/a \sim 0.8$ has a discrepancy of approximately 0.47 rad (27°) from the predicted value. The n = 8 mode exhibits an even larger discrepancy of 0.68 rad (39°). Toroidal effects were not accounted for, though, and may play a significant role. The radial components were calculated using the measured poloidal and toroidal components in combination with the divergence free condition: $\nabla \cdot \mathbf{B} = 0$. A cylindrical expression was used for the divergence, but recent work indicates that the toroidal array on MST may be situated near the location of strongest toroidicity induced effects [5].

Early probe measurements in the edge of MST [6] are consistent with the $\pi/2$ phase shift, as are probe measurements across the full minor radius of HBTX1A [7]. The $\pi/2$ phase shift predicted by single-fluid MHD is born out by numerical calculations, for example with NIMROD [8]. Two-fluid MHD effects lead to a small deviation from $\pi/2$ which is critical for the Hall dynamo and also observed

numerically [8]. Some care must be taken when comparing to numerical predictions, though, as the current and toroidal magnetic field in typical MST operation are anti-parallel while numerical simulations frequently utilize parallel current.

The Thomson scattering measurements are all within r/a = 0.8, and the strongest effects of temperature fluctuations correlated with \tilde{B}_r are all within r/a = 0.4. Therefore, a constant phase shift of $\pi/2$ between \tilde{B}_r and \tilde{B}_{θ} is assumed at all radial locations.

5.2 Radial profiles for \tilde{B}_r



Figure 5.1: Model profiles for \tilde{B}_r . The amplitude is flat inside the rational surface, and decays linearly with radius outside the rational surface. Amplitudes are scaled empirically based on \tilde{B}_{θ} at the wall.

In addition to the phase, the radial profile of the amplitude of \tilde{B}_r must also be determined for an accurate estimate of current transport. Previous work has established an empirical scaling between \tilde{B}_{θ} at the wall and \tilde{B}_r in the core [9]. Radial profiles of \tilde{B}_r predicted by DEBS simulations were normalized to FIR interferometry/polarimetry measurements for multiple n numbers. These values were then compared to B_{θ} amplitudes measured at the wall to obtain an n-dependent scaling factor. A second order polynomial fit to the scaling factors versus n was used to determine the core amplitude.

The profiles are assumed to be flat inside the rational surface and decay linearly from the rational surface to the edge, see Figure 5.1 below. The rational surface measurements inferred earlier from the temperature fluctuation structures are used here. Although somewhat coarse, this model is largely consistent with the behavior observed in simulations for non-linear tearing modes and captures the general features of the \tilde{B}_r profiles (simulations of linear modes display stronger variation in r).

5.3 Temperature fluctuation driven current transport

The correlated product $\Pi_T = \frac{1}{eB_0} \langle \tilde{T}_e \tilde{B}_r \rangle$ is plotted in Figures 5.2-5.5 below for the n = 5-8 modes. The dominant contribution is due to n = 6 correlated fluctuations. These fluctuations drive net outward current transport, but are largely localized to the core. In fact, the peak in the current transport profile is a few centimeters core-ward of the n = 6 island. Each of the radial locations where Π_T can be distinguished from zero samples the stochastic region along the core-ward side of the remnant island. The locations that primarily sample the remnant island itself or the stochastic region along the edge-ward side have Π_T consistent with zero. For the remaining modes, individual structure is difficult to identify due to small fluctuation amplitudes. Due to the uncertainty in the correlated transport and the relatively coarse radial resolution of ~1 cm, evaluating the divergence of each mode individually is not particularly illuminating. It is worth noting, though, that for the n = 6 mode, the order of a few V/m. This is a substantial contribution for a single tearing modes.

However, from the sign of the measured phase differences shown in Chapter 4 and the sign of the estimated current transport, the n = 5, 7, and 8 modes are observed to counteract the current transport due to the n = 6 mode. Summing the contributions due to the n = 5, 7, and 8 modes makes this clearer, as seen in



Figure 5.2: Correlated product Π_T for n = 5 fluctuations.



Figure 5.3: Correlated product Π_T for n = 6 fluctuations.

Figure 5.6 below. The summed transport terms contribute to a significant peak coinciding with the peak due to the n = 6 mode, but opposite in sign. The opposing transport drive due to the different modes appears to be largely balanced. This can be seen in Figure 5.7, where the contributions for all four modes are summed.


Figure 5.4: Correlated product Π_T for n = 7 fluctuations.



Figure 5.5: Correlated product Π_T for n = 8 fluctuations.

The combined transport terms are consistent with zero, suggesting that thermal contributions to the kinetic dynamo are negligible.

From a linear fit to the summed transport quantities, a gradient in the current transport term of approximately 0.066 V/m may still be possible. Taking the



Figure 5.6: Correlated product Π_T for n = 6 fluctuations versus the sum of contributions to Π_T from n = 5, 7, and 8 fluctuations.



Figure 5.7: Total correlated product Π_T for n = 5-8 fluctuations. The opposing contributions from different modes largely cancel. The red dashed line is obtained through linear regression and has slope of -0.066 V/m.

divergence of a linear profile with slope m: $\frac{1}{r}\frac{\partial}{\partial r}(r \cdot mr) = 2m$. Such a profile

would yield a uniform electric field of approximately -0.132 V/m across most of the minor radius. The density fluctuation contributions, $\frac{T_e}{en_eB_0}\nabla \cdot \langle \tilde{n}_e\tilde{B}_r \rangle$, have been measured to be on the order of 0.1 V/m in reversed discharges, as discussed in Section 5.5. The measurements presented here do not permit resolution of a dynamo electric field that small; measurements with improved errors are needed to determine whether or not thermal contributions are negligible relative to the density fluctuation terms. Modes with $n \ge 9$, which have not been measured here, may also provide an important contribution. In the region outside $r/a \sim 0.5$, the correlated temperature fluctuations are much lower in amplitude and the deviation of the fluctuation phase from \tilde{b}_{θ} is much smaller. Effects due to toroidicity can have a significant impact for these conditions and should not be ignored.

5.4 The RFP dynamo

The work presented so far has been focused exclusively on experimental observations of the correlated product $\langle \tilde{T}_e \tilde{B}_r \rangle$ and associated current transport. In order to place these measurements in the larger context of the RFP dynamo, a broad outline of the dynamo is presented here. This motivates a discussion of the kinetic dynamo in the following section.

An inductively applied toroidal electric field is responsible for current drive in MST. The applied field E_{ϕ} aligns with the magnetic field in the core to drive parallel current, and this current generates the poloidal magnetic field. In the RFP edge, where B_{ϕ} reduces to zero and reverses sign, E_{ϕ} is primarily perpendicular to the magnetic field and cannot drive parallel current. Measured current profiles do not reverse sign with B_{ϕ} , however. Without additional current drive, resistive decay of poloidal current in the edge would only be balanced by the decay of toroidal flux. This suggests that the standard (parallel) Ohm's law valid in tokamaks, $E_{\parallel} = \eta J_{\parallel}$, is inadequate for the RFP. The difference between the inductive electric field and the current is known as the RFP dynamo. This dynamo acts to flatten the current profile by reducing core current and increasing edge current, as shown in Figure 5.8.



Figure 5.8: Reconstructed profiles for parallel electric field and current from Ref. [10]. The electric field does not balance ηJ_{\parallel} and other dynamo terms are required.

A more generalized Ohm's law is provided by two-fluid MHD:

$$\eta \vec{J} - \vec{E} = \vec{v} \times \vec{B} - \frac{1}{n_e e} \vec{j} \times \vec{B} + \frac{\nabla p_e}{n_e e} - \frac{m_e}{e^2 n_e} \frac{\partial J}{\partial t}$$
(5.11)

The $\partial J/\partial t$ term, representing the electron inertia, is generally small and neglected here. Separating each term into a mean and spatially varying component, the mean parallel emf can be expressed in terms of the products of correlated fluctuations:

$$\eta \vec{J}_{\parallel} - \vec{E}_{\parallel} = \langle \tilde{v} \times \tilde{B} \rangle_{\parallel} - \frac{1}{n_{e}e} \langle \tilde{j} \times \tilde{B} \rangle_{\parallel}$$
(5.12)

The pressure gradient term appearing in the generalized Ohm's law does not contribute to the mean parallel emf (to lowest order) as ∇p is perpendicular to B (this neglects Biermann battery-like effects). The additional terms, $\tilde{v} \times \tilde{B}$ and $\tilde{j} \times \tilde{B}$, compensate for the difference between electric field and current density profiles. They constitute the RFP dynamo, and each term is described below.

The $\tilde{v} \times \tilde{B}$ contribution is known as the MHD dynamo. This dynamo term has long been known to be a significant drive of parallel emf. First predicted in MHD simulations [11, 12], the MHD dynamo was initially measured in the RFP edge

using Langmuir probes, under the assumption that $\vec{E} \times \vec{B}$ drifts dominated the velocity fluctuations [13]. Subsequent measurements of impurity ion flow, in the edge using an optical probe [14] and in the core using line-integrated Doppler spectroscopy [15], confirmed the role of the MHD dynamo in balancing Ohm's law. Localized core measurements using charge exchange recombination spectroscopy of C⁺⁶ ions have also demonstrated the importance of the MHD dynamo [10].

While these measurements show MHD dynamo terms to be significant, they also show that the MHD dynamo alone cannot balance Ohm's law across the entire minor radius at all times. Probe measurements of $\tilde{v} \times \tilde{B}$ showed that m = 0 flows in the edge were only able to balance Ohm's law outside the reversal surface; inside the reversal surface the MHD dynamo term was negligible. In the core, the total parallel emf due to the MHD dynamo has only been determined on-axis with CHERS. The measured MHD dynamo is sufficient to balance Ohm's law during the quiescent periods between sawteeth in reversed discharges; while the MHD dynamo is also significant during sawtooth events, other terms may be needed to balance Ohm's law. In the mid-radius, local measurements with CHERS have only been able to resolve the poloidal flow contribution to the dynamo (leading to a toroidal emf). Significant emf is observed in the mid-radius due to poloidal flow, but a large fraction of this is perpendicular rather than parallel. The line-integrated Doppler spectroscopy measurements suggest that radial velocity fluctuations do not play a significant role in the core MHD dynamo; no local measurement has been made of either radial or toroidal velocity fluctuation contributions in the mid-radius, however.

The j × B contribution is known as the Hall dynamo. Initial measurements of the Hall dynamo in the edge showed a non-zero, but weak emf. Subsequent measurements of the Hall dynamo in the core utilizing FIR interferometry/polarimetry found that, while the generated emf was small during quiescent periods, the dominant, n = 6 tearing mode drove a significant emf during sawtooth events [16, 17]. The field was comparable in magnitude to the parallel electric field (~50 V/m) and sufficient to balance Ohm's law, but largely localized near the rational surface. Also of significance, nonlinear coupling was found to be an important aspect of the Hall

dynamo. In F = 0 plasmas, where the q = 0 surface is removed from the plasma volume, the observed Hall dynamo was weaker due to reduced coupling between the n = 6 mode and other m = 1 modes through the m = 0 edge resonant modes. Probe measurements of the Hall dynamo inside the reversal surface show similar behavior [18]. Large dynamo emf at the sawteeth was found to be dominated by nonlinear coupling, primarily due to the m = 1, n = 6, 7, and 8 modes through the m = 0, n = 1 tearing mode.

One last dynamo mechanism bears mention: the diamagnetic dynamo. This was originally measured in the edge of several RFP devices using probes [19]. The suggested mechanism for this dynamo term arose from electron drift effects; the electron velocity \vec{v}_e was expanded as a combination of the usual $\vec{E} \times \vec{B}$ drift and, additionally, the electron diamagnetic drift. Since $\vec{v}_e = \vec{v} - \frac{1}{n_e e}\vec{j}$, though, expansion of the electron velocity to include additional drift terms should not capture any physics beyond the MHD and Hall dynamos. Therefore, the diamagnetic dynamo is interpreted here to be a manifestation of MHD/Hall effects rather than an independent dynamo.

5.5 The kinetic dynamo

An alternative to the MHD and Hall dynamos, which has come to be known as the kinetic dynamo, was originally proposed by Ref. [20]. The 'standard' Ohm's law, $\vec{E} = \eta \vec{J}$, relies on a local balance between electron momentum gained from the electric field and momentum lost due to scattering. However, in stochastic plasmas, field aligned current is readily transported by magnetic perturbations with a radial component. The momentum gained through acceleration in an electric field at one flux surface is transported radially outward before loss due to scattering at a different flux surface. In this case, momentum balance is only achieved globally, requiring a non-local description of conductivity. The kinetic dynamo provides a picture of global conductivity by describing the emf generated by radial transport of parallel current. A consistent picture including Ampere's law demonstrated that this could not completely account for the RFP dynamo [21]. However, recent

measurements indicate that the kinetic dynamo may still play a role in the RFP dynamo process [22]. The dynamo contribution due to radial streaming identified by previous work is discussed below. While transport of field aligned current depends on fluctuations in $p_{e,\parallel}$, the general effect of anisotropy on the dynamo has not been dealt with in the literature. The discussion of previous work is supplemented by a description of the kinetic dynamo with contributions from both p_{\parallel} and p_{\perp} .

Radial streaming of field-aligned current

The drift kinetic equation has been used to determine electrostatic and electromagnetic fluctuation contributions to transport quantities [23]. The radial flux of a quantity R is given by an integral over the velocity distribution

$$\Gamma_{\rm R} = \int d\nu R(\nu) \left[\left(\tilde{\rm E} \times \langle {\rm B} \rangle / \langle {\rm B} \rangle^2 \right) \cdot \hat{\rm r} + \nu_{\parallel} \frac{\tilde{\rm B}_{\rm r}}{\langle {\rm B} \rangle} \right] \tilde{\rm f}$$
(5.13)

The total velocity distribution is given by $f = \langle f \rangle + \tilde{f}$, where $\langle f \rangle$ represents the mean distribution and \tilde{f} is the fluctuation component relevant to transport. Transport of parallel current is proportional to the transport of parallel electron momentum, $R = m_e v_{\parallel}$. Ignoring the $\tilde{E} \times \langle B \rangle$ term, this is determined by the second moment of fluctuations in the velocity distribution correlated with radial magnetic perturbations:

$$\Gamma_{\nu_{\parallel}} = m_e \int d\nu \tilde{f} \nu_{\parallel}^2 \frac{\tilde{B}_r}{\langle B \rangle}$$
(5.14)

Evaluation of this product yields $\Gamma_{\nu_{\parallel}} = \frac{1}{\langle B \rangle} \langle \tilde{p}_{e,\parallel} \tilde{B}_r \rangle$ [23]. Current transport itself is not sufficient to generate parallel emf, however. At a given flux surface, an imbalance between current transported from the interior and current transported to the exterior must be present to establish an electric field; in other words, the divergence of $\Gamma_{\nu_{\parallel}}$. Expressing this in notation consistent with Refs. [22, 24], the kinetic dynamo generated emf is given by:

$$\mathsf{E}_{\mathrm{kin}} = \frac{1}{e n_e B_0} \nabla \cdot \langle \tilde{\mathsf{p}}_{e,\parallel} \tilde{\mathsf{B}}_r \rangle \tag{5.15}$$

and the correlated product can be written as a voltage:

$$\Pi = \frac{1}{en_e B_0} \langle \tilde{p}_{e,\parallel} \tilde{B}_r \rangle$$
(5.16)

Inclusion of this effect in the generalized Ohm's law yields:

$$\eta \vec{J}_{\parallel} - \vec{E}_{\parallel} = \langle \tilde{v} \times \tilde{B} \rangle_{\parallel} - \frac{1}{en_{e}} \langle \tilde{j} \times \tilde{B} \rangle_{\parallel} + \frac{1}{en_{e}B_{0}} \nabla \cdot \langle \tilde{p}_{e,\parallel} \tilde{B}_{r} \rangle$$
(5.17)

The correlated product Π can be separated into contributions from density and temperature fluctuations: $\tilde{p}_e = n_e \tilde{T}_e + \tilde{n}_e T_e$. Contributions due to density fluctuations, $\Pi_n = \frac{T_{e,\parallel}}{e n_e B_0} \langle \tilde{n}_e \tilde{B}_r \rangle$, have been measured via interferometry/polarimetry during quiescent periods in reversed discharges [22]. The measured emf generated by density-fluctuation driven current transport was approximately 0.1 V/m, with the field changing sign in the mid-radius. Although this is smaller than the mean electric field ($E_{\parallel} \sim 1.5 \text{ V/m}$), and insufficient to balance the difference between parallel electric field and current, the contribution is non-trivial.

Anisotropy and the kinetic dynamo

To determine the effect of pressure anisotropy on the dynamo, the electron momentum balance equation provides a useful starting point:

$$m_e n_e \left(\frac{\partial \vec{v}_e}{\partial t} + \vec{v}_e \cdot \nabla \vec{v}_e \right) = -n_e e \left(\vec{E} + \vec{v}_e \times \vec{B} \right) - \nabla \cdot \overline{\vec{P}}_e - \vec{F}$$
(5.18)

Taking the parallel component of the spatial average allows a recovery of the generalized Ohm's law. As noted previously, the electron inertia on the left-hand side, $\frac{\partial \vec{v}_e}{\partial t}$, is small and ignored; the convective term $\vec{v}_e \cdot \nabla \vec{v}_e$ is also neglected. From the $\vec{v}_e \times \vec{B}$ term on the right-hand side, both the MHD and Hall dynamos are recovered, while the friction term, \vec{F} , is described by $\eta \vec{J}$. Typically, the pressure is treated as an isotropic, scalar pressure. In this case, the pressure contributions reduce to $\nabla \cdot \overline{\vec{P}}_e = \nabla p_e$. As noted in the previous section, the gradient of the scalar pressure is largely perpendicular to the equilibrium field and should not contribute significantly to the dynamo emf beyond Biermann battery-like effects.

With anisotropy, the pressure cannot be reduced to a scalar and the divergence of $\overline{\overline{P}}_e$ will retain additional contributions to the dynamo. Using a Chew-Goldberger-Low like pressure tensor, the pressure has the form [25]:

$$\overline{\overline{P}} = p_{\perp} \overline{\overline{I}} + (p_{\parallel} - p_{\perp}) \hat{b} \hat{b}$$
(5.19)

Here, p_{\parallel} and p_{\perp} represent the parallel and perpendicular components of the electron pressure. $\overline{\overline{I}}$ is the identity tensor, so $p_{\perp}\overline{\overline{I}}$ is diagonal, and \hat{b} is the unit vector in the direction of the magnetic field. The tensor $\hat{b}\hat{b}$ represents a traceless contribution which arises from the curvature of the magnetic fields. The divergence of $\overline{\overline{P}}$ is [25]:

$$\nabla \cdot \overline{\overline{P}} = \nabla p_{\perp} + \hat{b}\hat{b} \cdot \nabla (p_{\parallel} - p_{\perp}) + (p_{\parallel} - p_{\perp}) [\hat{b} (\nabla \cdot \hat{b}) + (\hat{b} \cdot \nabla) \hat{b}]$$
(5.20)

If each quantity X is represented as a mean value, $\langle X \rangle$, plus a spatial fluctuation, \tilde{X} , which averages to zero:

$$p_{\perp} = \langle p_{\perp} \rangle + \tilde{p}_{\perp}$$

$$p_{\parallel} = \langle p_{\parallel} \rangle + \tilde{p}_{\parallel}$$

$$\vec{B} = \vec{B}_{0} + \tilde{B}$$
(5.21)

The unit vector $\hat{\boldsymbol{b}}$ is then similarly broken into components

$$\hat{b} = \frac{\vec{B}_{0} + \tilde{B}}{\left|\vec{B}_{0} + \tilde{B}\right|}
= \frac{\vec{B}_{0} + \tilde{B}}{\sqrt{B_{0}^{2} + 2\vec{B}_{0} \cdot \tilde{B} + |\tilde{B}|^{2}}}
\approx \frac{\vec{B}_{0} + \tilde{B}}{B_{0}\sqrt{1 + 2\frac{|\tilde{B}_{\parallel}|}{B_{0}}}}
\approx \left(\frac{\vec{B}_{0}}{B_{0}} + \frac{\tilde{B}}{B_{0}}\right) \left(1 - \frac{|\tilde{B}_{\parallel}|}{B_{0}}\right)
\approx \left(1 - \frac{|\tilde{B}_{\parallel}|}{B_{0}}\right) \frac{\vec{B}_{0}}{B_{0}} + \frac{\tilde{B}}{B_{0}}
= \left(1 - \frac{|\tilde{B}_{\parallel}|}{B_{0}}\right) \hat{b}_{0} + \tilde{b}$$
(5.22)

and the tensor $\hat{b}\hat{b}$ as well as the terms representing the divergence and gradient of \hat{b} can be represented as:

$$\hat{b}\hat{b} = \left[\left(1 - \frac{|\tilde{B}_{\parallel}|}{B_{0}}\right)\hat{b}_{0} + \tilde{b}\right]\left[\left(1 - \frac{|\tilde{B}_{\parallel}|}{B_{0}}\right)\hat{b}_{0} + \tilde{b}\right] \\
\approx \left(1 - 2\frac{|\tilde{B}_{\parallel}|}{B_{0}}\right)\hat{b}_{0}\hat{b}_{0} + \hat{b}_{0}\tilde{b} + \tilde{b}\hat{b}_{0}$$
(5.23)

$$\nabla \cdot \hat{\mathbf{b}} = \nabla \left(\frac{1}{|\mathbf{B}|}\right) \cdot \vec{\mathbf{B}} + \frac{1}{|\mathbf{B}|} \nabla \cdot \vec{\mathbf{B}}
= -\left(\frac{\nabla |\mathbf{B}|}{|\mathbf{B}|^2}\right) \cdot \vec{\mathbf{B}}
\approx \frac{\nabla \left(\mathbf{B}_0 + |\tilde{\mathbf{B}}_{\parallel}|\right)}{\mathbf{B}_0^2 \left(1 + 2\frac{|\tilde{\mathbf{B}}_{\parallel}|}{\mathbf{B}_0}\right)} \cdot \left(\vec{\mathbf{B}}_0 + \tilde{\mathbf{B}}\right)
\approx \left(1 - 2\frac{|\tilde{\mathbf{B}}_{\parallel}|}{\mathbf{B}_0}\right) \frac{\nabla \mathbf{B}_0}{\mathbf{B}_0} \cdot \hat{\mathbf{b}}_0 + \frac{\nabla \mathbf{B}_0}{\mathbf{B}_0} \cdot \tilde{\mathbf{b}} + \frac{\nabla |\tilde{\mathbf{B}}_{\parallel}|}{\mathbf{B}_0} \cdot \hat{\mathbf{b}}_0$$
(5.24)

The $\nabla |\tilde{B}_{\parallel}| \cdot \hat{b}_0$ term of the divergence can be neglected, as it has $(m - nq)\tilde{B}$ dependence.

$$(\hat{\mathbf{b}} \cdot \nabla) \, \hat{\mathbf{b}} = \left[\left(1 - \frac{|\tilde{\mathbf{B}}_{\parallel}|}{B_0} \right) \hat{\mathbf{b}}_0 + \tilde{\mathbf{b}} \right] \cdot \nabla \left[\left(1 - \frac{|\tilde{\mathbf{B}}_{\parallel}|}{B_0} \right) \hat{\mathbf{b}}_0 + \tilde{\mathbf{b}} \right]$$

$$\approx \left(1 - \frac{|\tilde{\mathbf{B}}_{\parallel}|}{B_0} \right) \hat{\mathbf{b}}_0 \cdot \nabla \hat{\mathbf{b}}_0 + \hat{\mathbf{b}}_0 \cdot \nabla \tilde{\mathbf{b}} + \tilde{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}_0 - \hat{\mathbf{b}}_0 \cdot \nabla \left(\frac{|\tilde{\mathbf{B}}_{\parallel}|}{B_0} \hat{\mathbf{b}}_0 \right)$$

$$= \left(1 - 2 \frac{|\tilde{\mathbf{B}}_{\parallel}|}{B_0} \right) \hat{\mathbf{b}}_0 \cdot \nabla \hat{\mathbf{b}}_0 + \hat{\mathbf{b}}_0 \cdot \nabla \tilde{\mathbf{b}} + \tilde{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}_0 - \hat{\mathbf{b}}_0 \cdot \nabla \left(\frac{|\tilde{\mathbf{B}}_{\parallel}|}{B_0} \right) \hat{\mathbf{b}}_0$$

$$(5.25)$$

The $\hat{b}_0 \cdot \nabla \hat{b}_0$ term is perpendicular to \vec{B}_0 , so can be neglected in the calculation of parallel emf. The $\nabla \left(\frac{|\tilde{B}_{\parallel}|}{B_0}\right)$ term yields $\frac{1}{B_0^2} \left(B_0 \nabla |\tilde{B}_{\parallel}| - |\tilde{B}_{\parallel}| \nabla B_0\right)$. Again, the $\nabla |\tilde{B}_{\parallel}|$ term can be neglected.

Taking the spatial average of $\nabla \cdot \overline{\overline{P}}$ and dotting into \hat{b}_0 for the parallel component yields the parallel emf. Breaking this down term by term yields, for the gradient of the scalar pressure ∇p_{\perp} :

$$\left\langle \nabla \langle \mathbf{p} \rangle_{\perp} + \nabla \tilde{\mathbf{p}}_{\perp} \right\rangle \cdot \hat{\mathbf{b}}_{0} = 0$$
 (5.26)

where $\nabla p \cdot \hat{b}_0$ is assumed zero generally and the fluctuating term averages to zero.

The $\hat{b}\hat{b}\cdot\nabla\left(p_{\parallel}-p_{\perp}\right)$ term yields:

$$\left\langle \hat{b}\hat{b} \cdot \nabla \left(\mathbf{p}_{\parallel} - \mathbf{p}_{\perp} \right) \right\rangle \cdot \hat{b}_{0} = \left\langle \left\{ \left\{ \left(1 - 2\frac{|\tilde{B}_{\parallel}|}{B_{0}} \right) \hat{b}_{0}\hat{b}_{0} + \hat{b}_{0}\tilde{b} + \tilde{b}\hat{b}_{0} \right\} \nabla \left(\mathbf{p}_{\parallel} - \mathbf{p}_{\perp} \right) \right. \right\}$$

$$\left. \cdot \hat{b}_{0} \\ = \left\langle \tilde{b} \cdot \nabla \left(\tilde{p}_{\parallel} - \tilde{p}_{\perp} \right) \right\rangle$$

$$(5.27)$$

where, again, the gradients are perpendicular to \hat{b}_0 and terms with only a single fluctuating quantity average to zero. The remaining terms, $(p_{\parallel} - p_{\perp}) \hat{b} (\nabla \cdot \hat{b})$ and $(p_{\parallel} - p_{\perp}) (\hat{b} \cdot \nabla) \hat{b}$, are largely toroidicity induced but have some contributions which do not depend on this.

$$\left\langle \left(p_{\parallel} - p_{\perp} \right) \hat{b} \left(\nabla \cdot \hat{b} \right) \right\rangle \cdot \hat{b}_{0} = \left\langle \left(\left\langle p_{\parallel} \right\rangle + \tilde{p}_{\parallel} - \left\langle p_{\perp} \right\rangle - \tilde{p}_{\perp} \right) \left(\hat{b}_{0} + \tilde{b} - \frac{|\tilde{B}_{\parallel}|}{B_{0}} \hat{b}_{0} \right) \right. \\ \left. \cdot \left(\left\{ 1 - 2 \frac{|\tilde{B}_{\parallel}|}{B_{0}} \right\} \frac{\nabla B_{0}}{B_{0}} \cdot \hat{b}_{0} + \frac{\nabla B_{0}}{B_{0}} \cdot \tilde{b} \right) \right\rangle \cdot \hat{b}_{0}$$

$$(5.28)$$

Note that the $\langle (\cdots) \tilde{b} \rangle \cdot \hat{b}_0$ and $\langle (\cdots) \frac{|\tilde{B}_{\parallel}|}{B_0} \hat{b}_0 \rangle \cdot \hat{b}_0$ terms originating from \hat{b} will cancel after averaging, so only the \hat{b}_0 term is carried through.

$$\left\langle \left(p_{\parallel} - p_{\perp} \right) \hat{b} \left(\nabla \cdot \hat{b} \right) \right\rangle \cdot \hat{b}_{0} = \left(\left\langle p_{\parallel} \right\rangle - \left\langle p_{\perp} \right\rangle - 2 \left\langle \left(\tilde{p}_{\parallel} - \tilde{p}_{\perp} \right) \frac{|\tilde{B}_{\parallel}|}{B_{0}} \right\rangle \right) \frac{\nabla B_{0} \cdot \vec{B}_{0}}{B_{0}^{2}} + \left\langle \left(\tilde{p}_{\parallel} - \tilde{p}_{\perp} \right) \frac{\nabla B_{0} \cdot \tilde{B}}{B_{0}^{2}} \right\rangle$$

$$(5.29)$$

The $\nabla B_0 \cdot \vec{B}_0$ term requires toroidicity to contribute, but the $\nabla B_0 \cdot \tilde{B}$ term is present

even in a cylinder. Finally:

$$\begin{split} \left\langle \left(p_{\parallel} - p_{\perp} \right) \left(\hat{b} \cdot \nabla \right) \hat{b} \right\rangle \cdot \hat{b}_{0} &= \left\langle \left(\left\langle p_{\parallel} \right\rangle + \tilde{p}_{\parallel} - \left\langle p_{\perp} \right\rangle - \tilde{p}_{\perp} \right) \left(\hat{b}_{0} \cdot \nabla \tilde{b} + \tilde{b} \cdot \nabla \hat{b}_{0} \right. \\ &+ \frac{\left| \tilde{B}_{\parallel} \right|}{B_{0}^{2}} \hat{b}_{0} \cdot \nabla B_{0} \hat{b}_{0} \right) \right\rangle \cdot \hat{b}_{0} \\ &= \left\langle \left(\tilde{p}_{\parallel} - \tilde{p}_{\perp} \right) \left(\hat{b}_{0} \cdot \nabla \tilde{b} + \tilde{b} \cdot \nabla \hat{b}_{0} \right) \right\rangle \cdot \hat{b}_{0} \\ &+ \left\langle \left(\tilde{p}_{\parallel} - \tilde{p}_{\perp} \right) \left| \tilde{B}_{\parallel} \right| \right\rangle \frac{\nabla B_{0} \cdot \hat{b}_{0}}{B_{0}^{2}} \end{split}$$
(5.30)

The last term is toroidicity induced, while the first term can still contribute in a cylinder. The leading order contribution in a cylinder is $2\left(\tilde{p}_{\parallel}-\tilde{p}_{\perp}\right)\frac{\tilde{B}_{r}B_{0,\theta}^{2}}{rB_{0}^{3}}$, with a factor arising from both the $\hat{b}_{0} \cdot \nabla \tilde{b}$ and the $\tilde{b} \cdot \nabla \hat{b}_{0}$ terms.

Neglecting toroidicity, the net parallel kinetic emf is:

$$\frac{1}{n_{e}e}\left\langle \nabla \cdot \overline{\overline{P}} \right\rangle \cdot \hat{b}_{0} = \frac{1}{n_{e}e}\left\langle \tilde{b} \cdot \nabla \left(\tilde{p}_{\parallel} - \tilde{p}_{\perp} \right) \right\rangle + \frac{1}{n_{e}e}\left\langle \left(\tilde{p}_{\parallel} - \tilde{p}_{\perp} \right) \frac{\nabla B_{0} \cdot \tilde{B}}{B_{0}^{2}} \right\rangle + \frac{1}{n_{e}e}\left\langle 2 \left(\tilde{p}_{\parallel} - \tilde{p}_{\perp} \right) \frac{\tilde{B}_{r}B_{0,\theta}^{2}}{rB_{0}^{3}} \right\rangle$$
(5.31)

The second and third terms can be estimated based on the equilibrium magnetic field reconstructions from Chapter 4, where the value of $\frac{\nabla B_0}{B_0}$ in the core is roughly 0.5 m^{-1} and $\frac{B_{0,0}^2}{rB_0^2}$ is approximately 1.0 m⁻¹. This leads to an estimated contribution to the emf on the order of 0.01 V/m for the second term and 0.04 V/m for the third. This estimate uses only the correlated product Π_T shown in Figure 5.7, treating $n_e T_e$ as an upper bound for $p_{\parallel} - p_{\perp}$. While 0.01 V/m is an order of magnitude smaller than the value of $\nabla \cdot \langle \tilde{T}_e \tilde{B}_r \rangle$ estimated by linear fit, 0.04 V/m is nearly a third as large. Since all of these values are within error bars of zero, the order-of-magnitude estimate here should be interpreted with caution; the first term appears to be the dominant term, so the following discussion focuses solely on that dynamo contribution. The other terms may be non-negligible, however.

Note that, since the divergence of \vec{B} is zero, the magnetic perturbation can be drawn inside the del operator for a dynamo electric field given by:

$$\vec{\mathsf{E}}_{\text{kin}} = \frac{1}{en_e B_0} \nabla \cdot \langle \tilde{\mathsf{B}} \left(\tilde{\mathsf{p}}_{\parallel} - \tilde{\mathsf{p}}_{\perp} \right) \rangle \tag{5.32}$$

This is very similar to the term calculated due to radial streaming of parallel current, with a few exceptions. While the dynamo due to radial streaming was attributed exclusively to radial magnetic perturbations, \tilde{B}_r , here the full magnetic perturbation \tilde{B} may play a role. Since the gradient of the pressure fluctuations $\nabla (\tilde{p}_{\parallel} - \tilde{p}_{\perp})$ is expected to be dominated by the radial term, however, \tilde{B}_r should still be the primary contributing component. The crucial distinction arises from the difference $\tilde{p}_{\parallel} - \tilde{p}_{\perp}$. While the dynamo term arising from radial streaming of current nominally exists even with isotropic pressure, the results here indicate that no net current transport takes place under these conditions. A non-zero difference between \tilde{p}_{\parallel} and \tilde{p}_{\perp} must exist to generate a parallel emf. Therefore, any confirmation of kinetic dynamo effects requires a resolution of parallel and perpendicular pressure fluctuations. Like the MHD and Hall dynamos, the kinetic dynamo arises from fluctuations.

The measurements of electron temperature with Thomson scattering assume an isotropic distribution. If this assumption is true, then the thermal contributions to the kinetic dynamo are zero regardless of the ultimate magnitude of $\langle \tilde{T}_{e,\parallel} \tilde{B}_r \rangle$. Without an estimate of $\tilde{T}_{e,\parallel}$ and $\tilde{T}_{e,\perp}$, the results presented here do not provide a complete description of the dynamo. The emf estimated above is, at best, an upper bound on the total emf due to temperature fluctuations. Due to the variation in the wave vector of Thomson scattered light across the minor radius, the sampled dimension of the electron distribution changes. The resulting temperature is a radially varying combination of parallel and perpendicular temperatures; this effect may allow an estimate of anisotropy in future work.

5.6 Discussion

Measurements of the dynamo terms have been historically challenging, with work tending to focus on a single term or even particular components of a single term. It has become clear, however, that across much of the minor radius and for a variety of plasma conditions, multiple sources are responsible for the RFP dynamo. More complete measurement of all the RFP dynamo terms is needed. Given the constraints on measurement of the dynamo, however, the measurements presented here are a useful step forward in obtaining a more complete picture. Based on the observed behavior of temperature fluctuations correlated with \tilde{B}_r , a few topics for future study are suggested below.

While previous work has focused on the parallel pressure in terms of radial streaming of current, accounting for the effects of anisotropy shows that the difference between parallel and perpdendicular pressure is critical to the kinetic dynamo. There are also additional contributions to the kinetic dynamo beyond the $\nabla \cdot \langle \left(\tilde{p}_{e,\parallel} - \tilde{p}_{e,\perp} \right) \tilde{B} \rangle$ term discussed above. Pressure anisotropy has not yet been diagnosed by the major optical diagnostics (FIR, Thomson scattering) on MST. A more complete measurement of $\tilde{p}_{e,\parallel}$ and $\tilde{p}_{e,\perp}$ is necessary to understand the importance of the kinetic dynamo.

These measurements add to the evidence for a change in the nature of the RFP dynamo at the transition from reversed to non-reversed discharges. While the Hall dynamo is robust in reversed discharges and plays a significant role in sawtooth events, the removal of the q = 0 surface reduces non-linear mode coupling and the Hall contribution to current profile flattening. Conversely, while temperature fluctuations in reversed discharges are largely in phase with \tilde{B}_{θ} and the measured kinetic dynamo contributions are predominantly density fluctuation driven, the transition to F = 0 plasmas is marked by a significant increase in temperature fluctuations correlated with \tilde{B}_r . While the total emf generated by these fluctuations appears consistent with zero, this suggests that the nature of the kinetic dynamo may also change with the loss of reversal. Unlike the Hall dynamo, the measured dynamo is not expected to depend on non-linear coupling to the m = 0 modes,

so the reason for such a change is unclear–a change in stochasticity may impact particle streaming, for example. For ultra-low-q plasmas, with q_0 less than one and 0 < q(a) < 1, these changes to the dynamo may have significant ramifications. A dynamo may not be necessary to sustain a non-reversed configuration, as noted in the introduction. While the dynamo in MST has been studied extensively in reversed discharges, ultra-low-q plasmas may provide an interesting field for future dynamo studies.

Additionally, the measurements of temperature fluctuations in NBI heated, nonreversed discharges presented in Chapter 4 indicate that fast ion dynamics can affect the phase of temperature fluctuations correlated with magnetic perturbations. The amplitude of temperature fluctuations outside the remnant island structures is reduced, and the deviations of the fluctuation phase from b_{θ} are largely eliminated for radial locations where non-negligible fluctuation power is observed. While polarimetry measurements of increased on-axis current density suggest current redistribution leading to profile peaking, the measured changes to T_e fluctuation phase are the first indication that NBI might modify the RFP dynamo. The nature of these effects is not clear-the fast ions may be altering the pressure tensor, or the measured effect may be a back-reaction of the dynamo due to changes in the current profile. A large population of energetic ions may potentially impact all of the dynamo terms: the MHD dynamo through changes to the flow profile, the Hall dynamo through changes to the current profile, and the kinetic dynamo through pressure tensor terms. The impact of fast ions on the RFP dynamo may also provide a useful target for research.

While the currently available Spectron laser system for Thomson scattering offers 25 kHz repetition rates, an improved Fast Thomson system with repetition rates of 250 kHz is expected to be available soon. This system should dramatically improve fluctuation measurements. Feasible error bars for the n = 5-8 modes measured here would improve significantly with the increase in both time resolution and in data acquisition rates. Correlation of temperature fluctuations with modes with lower amplitude, for example the $n \ge 9$ tearing modes, should also be feasible.

5.7 References

- R. J. Bickerton. The amplification of a magnetic field by a high current discharge. *Proc. Phys. Soc.*, 72(4):618, 1958. doi: http://dx.doi.org/10.1088/0370-1328/ 72/4/316.
- [2] J. T. Chapman. *Spectroscopic measurement of the MHD dynamo in the MST reversedfield pinch*. PhD thesis, University of Wisconsin-Madison, 1998.
- [3] D. Craig. Personal communication, March 2014.
- [4] T. D. Tharp. *Measurements of Nonlinear Hall-driven Reconnection in the Reversed Field Pinch*. PhD thesis, University of Wisconsin-Madison, 2010.
- [5] J. S. Sarff. Personal communication, March 2014.
- [6] W. Shen, R. N. Dexter, and S. C. Prager. Current-density fluctuations and ambipolarity of transport. *Phys. Rev. Lett.*, 68(9):1319, 1992. doi: 10.1103/ PhysRevLett.68.1319.
- [7] D. Brotherton-Ratcliffe, C. G. Gimblett, and I. H. Hutchinson. Analysis of internal magnetic fluctuations in the HBTX1A reversed field pinch. *Plasma Phys. Control. Fusion*, 29(2):161, 1987. doi: http://dx.doi.org/10.1088/0741-3335/ 29/2/004.
- [8] J. R. King. *First order finite-Larmor-radius effects on magnetic tearing and relaxation in pinch configurations*. PhD thesis, University of Wisconsin-Madison, 2011.
- [9] D. A. Ennis. *Local measurements of tearing mode flows and the MHD dynamo in the MST reversed-field pinch*. PhD thesis, University of Wisconsin-Madison, 2008.
- [10] D. A. Ennis, D. Craig, S. Gangadhara, J. K. Anderson, D. J. Den Hartog, F. Ebrahimi, G. Fiksel, and S. C. Prager. Local measurements of tearing mode flows and the magnetohydrodynamic dynamo in the Madison Symmetric Torus reversed-field pinch. *Phys. Plasmas*, 17(8):082102, 2010. doi: http://dx.doi.org/10.1063/1.3458667.

- [11] A. Y. Aydemir and D. C. Barnes. Sustained self-reversal in the reversed-field pinch. *Phys. Rev. Lett.*, 52(11):930, 1984. doi: 10.1103/PhysRevLett.52.930.
- [12] D. D. Schnack, E. J. Caramana, and R. A. Nebel. Three-dimensional magnetohydrodynamic studies of the reversed-field pinch. *Phys. Fluids*, 28(1):321, 1985. doi: http://dx.doi.org/10.1063/1.865151.
- [13] H. Ji, A. F. Almagri, S. C. Prager, and J. S. Sarff. Time-resolved observation of discrete and continuous magnetohydrodynamic dynamo in the reversed-field pinch edge. *Phys. Rev. Lett.*, 73(5):668, 1994. doi: 10.1103/PhysRevLett.73.668.
- [14] P. W. Fontana, D. J. Den Hartog, G. Fiksel, and S. C. Prager. Spectroscopic observation of fluctuation-induced dynamo in the edge of the reversed-field pinch. *Phys. Rev. Lett.*, 85(3):566, 2000. doi: 10.1103/PhysRevLett.85.566.
- [15] D. J. Den Hartog, J. T. Chapman, D. Craig, G. Fiksel, P. W. Fontana, S. C. Prager, and J. S. Sarff. Measurement of core velocity fluctuations and the dynamo in a reversed-field pinch. *Phys. Plasmas*, 6(5):1813, 1999. doi: http://dx.doi.org/10.1063/1.873439.
- [16] W. X. Ding, D. L. Brower, D. Craig, B. H. Deng, G. Fiksel, V. Mirnov, S. C. Prager, J. S. Sarff, and V. Svidzinski. Measurement of the Hall dynamo effect during magnetic reconnection in a high-temperature plasma. *Phys. Rev. Lett.*, 93(4):045002, 2004. doi: 10.1103/PhysRevLett.93.045002.
- [17] W. X. Ding, D. L. Brower, B. H. Deng, A. F. Almagri, D. Craig, G. Fiksel, V. Mirnov, S. C. Prager, J. S. Sarff, and V. Svidzinski. The Hall dynamo effect and nonlinear mode coupling during sawtooth magnetic reconnection. *Phys. Plasmas*, 13(11):112306, 2006. doi: http://dx.doi.org/10.1063/1.2363353.
- [18] T. D. Tharp, A. F. Almagri, M. C. Miller, V. V. Mirnov, S. C. Prager, J. S. Sarff, and C. C. Kim. Measurements of impulsive reconnection driven by nonlinear Hall dynamics. *Phys. Plasmas*, 17(12):120701, 2010. doi: http://dx.doi.org/10. 1063/1.3521336.

- [19] H. Ji, Y. Yagi, K. Hattori, A. F. Almagri, S. C. Prager, Y. Hirano, J. S. Sarff, T. Shimada, Y. Maejima, and K. Hayase. Effect of collisionality and diamagnetism on the plasma dynamo. *Phys. Rev. Lett.*, 75(6):1086, 1995. doi: 10.1103/PhysRevLett.75.1086.
- [20] Abram R. Jacobson and Ronald W. Moses. Nonlocal dc electrical conductivity of a Lorentz plasma in a stochastic magnetic field. *Phys. Rev. A*, 29(6):3335, 1984. doi: 10.1103/PhysRevA.29.3335.
- [21] P. W. Terry and P. H. Diamond. A self-consistent theory of radial transport of field-aligned current by microturbulence. *Phys. Fluids B*, 2(6):1128, 1990. doi: http://dx.doi.org/10.1063/1.859248.
- [22] W. X. Ding, L. Lin, D. L. Brower, A. F. Almagri, B. E. Chapman, D. J. Den Hartog, J. Duff., and J. S. Sarff. Momentum and current transport in the MST reversed field pinch. *Bull. Am. Phys. Soc.*, 58(16):CP8.81, 2013.
- [23] S. C. Prager. Transport and fluctuations in reversed field pinches. *Plasma Phys. Control. Fusion*, 32(11):903, 1990. doi: http://dx.doi.org/10.1088/0741-3335/32/11/006.
- [24] W. X. Ding, L. Lin, D. L. Brower, A. F. Almagri, B. E. Chapman, G. Fiksel, D. J. Den Hartog, and J. S. Sarff. Kinetic stress and intrinsic flow in a toroidal plasma. *Phys. Rev. Lett.*, 110(6):065008, 2013. doi: 10.1103/PhysRevLett.110.065008.
- [25] N. A. Krall and A. W. Trivelpiece. *Principles of Plasma Physics*. McGraw-Hill, New York, NY, 1973. ISBN 0070353468.

Chapter 6 NBI and Instability

Since early neutral beam experiments on tokamaks in the 1980s, the propensity for energetic particles to drive instability has produced a rich field of research. Low-frequency MHD modes, like the m = n = 1 sawteeth in tokamaks, and Alfvénic modes have been destabilized by fast-ion populations. At sufficiently large fast-ion densities, entirely new modes appear with characteristic frequencies and growth rates determined by the particles themselves. Due to the large number of possible particle orbits and drifts, as well as the diversity of Alfvénic modes due to experimental geometries and plasma conditions, energetic particle studies have led to a diverse phenomenology that some authors have identified as a 'zoo'. Energetic particle induced instabilities are a fresh field on MST; as the only major RFP with neutral beam heating, MST provides a unique platform for energetic particle physics. So far, studies of beam-induced instability in the RFP have been primarily experimental in nature, with theoretical work awaited. Thomson scattering fluctuation measurements represent a useful diagnostic of internal structure/dynamics associated with energetic particle instabilities for validation of theoretical predictions.

Section 6.1 briefly reviews Alfvén eigenmodes and energetic particle modes that have been characterized in the RFP. Section 6.2 covers the results of attempted temperature fluctuation correlation measurements with the largest EPM and chirping modes. No significant structure has been observed to be correlated with these modes and upper bounds on fluctuation amplitude are provided. Future work with higher time resolution may provide clearer measurements. In Section 6.3, a previously unidentified, low-frequency burst instability in reversed discharges is characterized. This mode propagates at frequencies of only a few kHz in the plasma reference frame and appears to participate in EPM avalanches, leading to significant fast-ion transport. Additionally, anomalous increases in tearing mode velocity and amplitude are observed during these bursts along with possible bulk ion heating. Temperature fluctuations correlated with this mode for various plasma conditions are provided in Section 6.4. Observed structures are core-localized and similar in amplitude to tearing mode activity. This chapter concludes with a discussion of the identity of the low-frequency bursts.

6.1 Energetic particle modes in the RFP

Initial studies of Alfvén eigenmodes (AEs) and energetic particle modes (EPMs) in the RFP were performed in Ref. [1]. A thorough overview of the subject area, accessible even to 'uninitiated' graduate students, is provided by Ref. [2]. A more recent and more theoretically intensive overview is given in Ref. [3]. These sources heavily influence the following discussion.

Both compressional and transverse electromagnetic waves can propagate along a magnetic field. For transverse waves in a conducting fluid, known as shear Alfvén waves [4], the dispersion relation is:

$$\omega = k_{\parallel} \nu_A \tag{6.1}$$

where $v_A = \frac{B}{\sqrt{\mu_0 \rho_i}}$ is the Alfvén velocity, which depends on the equilibrium magnetic field B and the mass density ρ_i of the ions, and k_{\parallel} is the wave vector parallel to the magnetic field. In a uniform plasma, this wave is dispersionless. For axisymmetric cylindrical plasmas, however, magnetic shear leads to a radially non-uniform parallel wavevector which introduces dispersion [2]. The parallel wavevector is

given by:

$$k_{\parallel} = \frac{m - nq}{r} \frac{B_{\theta}}{|B|}$$
(6.2)

with B_{ϕ} , B_{θ} the axial (toroidal) and poloidal components of the equilibrium field, q the safety factor, and m and n the poloidal and toroidal wavenumbers. This is generally dependent on radius, so an Alfvén wave with finite radial extent will experience strong shear and rapidly disperse. The strong damping due to plasma non-uniformity makes excitation of these waves, referred to as the Alfvén continuum, difficult.

However, for toroidal plasmas, a field line winding helically around the torus will experience periodic variation in the amplitude of B due to inboard/outboard asymmetries. These asymmetries, which have functional dependence $B_0 \cos \theta$, couple poloidal harmonics [5]. For a mode with given n, the m and m + 1 modes couple. This can be seen in Figure 6.1. Two counter-propagating modes with different m numbers have a frequency crossing in cylindrical geometry. Accounting for coupling of poloidal harmonics in toroidal geometry leads to an avoided crossing, which creates a gap in the Alfvén continuum. This gap is important, as it allows marginally stable eigenmodes to exist which are not strongly damped like the continuum modes. These modes, known as toroidicity-induced Alfvén eigenmodes (TAEs), have finite radial extent but are localized to the location of the gap.

TAEs are well studied in tokamaks, but have only recently garnered attention in RFPs. TAEs have been reported in the edge of Extrap-T2R [6]. Alfvénic modes have also been observed in RFX-mod with a suggested identification as global Alfvén eigenmodes (GAEs), although TAE activity cannot be discounted [7]. Extensive studies of Alfvén eigenmodes for MST [1] have predicted a number of gap modes due to toroidicity effects. Due to the dynamic nature of typical MST equilibria, predicted TAE gaps vary significantly. However, no TAEs have been measured in MST to date. The reasons for this are discussed below.

Toroidicity is not the only cause of gaps in the Alfvén continuum. Gap eigenmodes can be divided into two general classes: 1) gaps due to an extremum in frequency of the continuum spectrum and 2) gaps due to frequency crossings



Figure 6.1: Cylindrical continua for counter-propagating poloidal harmonics (dashed lines) and toroidal continua (solid lines). In a torus, the modes couple and the frequency crossing is avoided. Figure from Ref. [1].

between counter propagating waves [2]. Extensive work has characterized the diverse zoo of eigenmodes: frequency crossings that result from various plasma shaping effects (ellipticity, non-circularity, and helicity, for example), continuum extrema due to reversed shear and finite beta effects, as well as gaps due to kinetic effects, compressional Alfvén waves, coupling to acoustic waves and many others [2, 3]. Studies in MST have focused primarily on TAEs and the beta-induced modes (BAEs).

The Alfvén eigenmodes are not as strongly damped as the continuum modes. An antenna or other source of free energy can excite these modes; frequently, a population of energetic particles provides sufficient energy to drive AEs unstable. Energy transfer between particles and waves requires [2]:

- Some component of particle velocity parallel to the wave electric field $(\vec{v} \cdot \tilde{E} \neq 0)$
- Particle orbit and wave phase matching
- A gradient in the spatial or velocity distribution

The first requirement, in a curved field, can be satisfied by particle drifts. The second requirement leads to the condition

$$\omega + (m+l)\omega_{\theta} - n\omega_{\phi} = 0 \tag{6.3}$$

where ω_{θ} and ω_{ϕ} are the poloidal and toroidal frequencies associated with the particle orbits and ω is the wave frequency. While l can be any integer value, for low energy particles in a circular cross section device, typically only the |l| = 1 terms are significant. Finally, spatial gradients in the fast-ion distribution typically drive instabilities. Distributions with df/dW < 0 are common (where *W* is the particle energy), leading to wave damping. However, the toroidal angular momentum is given by:

$$\mathsf{P}_{\phi} = \mathsf{m}\mathsf{R}\mathsf{v}_{\phi} - \mathsf{Z}\mathsf{e}\Psi \tag{6.4}$$

where Ψ is the poloidal flux. A particle distribution that peaks on axis has $df/d\Psi < 0$, but the gradient $df/dP_{\Phi} > 0$ and this spatial gradient supplies free energy to drive the wave.

Alfvén eigenmodes are normal modes of the plasma—energetic particles are not required for mode drive and their effects on mode characteristics are perturbative. The dispersion can be written more generally, however, using the energy principle technique for a plasma displacement ξ ; this leads to a description of additional modes [8, 9]:

$$i\Omega + \delta W_{\text{bulk}} + \delta W_{\text{hot}} = 0 \tag{6.5}$$

Here, δW_{bulk} is the ideal MHD potential energy and δW_{hot} is the kinetic contribution due to the energetic particles. Ω represents a generalized inertia term due to the thermal ions. In the appropriate limits, this dispersion relation recovers the wide variety of gap modes as well as the well-known fishbone mode [10]. When the fastion pressure becomes comparable to the bulk pressure a new class of instabilities appears known as energetic particle modes (EPMs). For these modes, the significant population of energetic particles represents more than just a perturbation; the mode frequency and growth rate are strongly determined by the characteristics of the energetic particle population, with the frequency usually corresponding to the orbital motion of the particles. Energetic particle modes may even be found in the Alfvén continuum owing to the strength of their drive.



Figure 6.2: Spectrograms of \dot{B}_{θ} signals during a non-reversed discharge (shot 1121014033). Modes with n = 5 (top left), n = 4 (top right), and n = 1 (bottom). Tearing mode activity is visible at ~10 kHz initially, accelerating to ~20 kHz after NBI turns on at 12.15 ms. EPM activity begins several milliseconds after beam turn on, at frequencies above 50 kHz.

Beam-driven instabilities have been studied most extensively in non-reversed discharges on MST [11, 12]. These plasmas are not as strongly dynamic as reversed discharges, offering more clarity in diagnosing observed mode activity. The dominant beam-driven instabilities in non-reversed discharges have mode numbers n = 1, 4, and 5, see Figure 6.2. The modes have burst-like time dependence, with typical

duration ~60 µs. While the n = 4 and 5 modes are near predicted TAE gaps, and the n = 4 frequency scaling does show some dependence on Alfvén velocity, the measured frequencies fall in the continuum. This is clearly observed from correlated FIR density fluctuation profiles in Figure 6.3. Density fluctuations indicate that the modes are core localized, with the observed frequencies clearly matching predicted continua. The n = 5 mode frequency also scales with the square root of the beam energy. The continuum frequency and scaling with beam energy, along with the large predicted fast-ion population, strongly support the identification of the n = 5 mode as an energetic particle mode. While the Alfvénic scaling of the n = 4 complicates identification, it has been speculated to be a form of EPM known as a resonant-TAE, among other possibilities. Observation of EPMs for both $v > v_A$ and $v < v_A$ implies that the spatial gradient, rather than the velocity gradient, provides the free energy [13].



Figure 6.3: FIR density fluctuation spectra for n = 4 and n = 5 modes, with predicted Alfvén continua overplotted. Each mode is clearly localized to the continuum, with extended radial structure. Figure from Ref. [13].

Profiles for \hat{B}_r correlated with the EPMs have also been measured with FIR, with a peak amplitude of approximately 2 G [12]. The fluctuations in \tilde{n}_e are estimated to be compressional in nature, rather than advective. Nonlinear coupling

is observed between the n = 1, 4, and 5 modes, contributing to significant transport/redistribution of energetic particles in the core. While the n = 5 kink-tearing mode is suppressed during neutral beam injection, bursts of coupled n = 1, 4, 5 EPMs also lead to a brief increase in amplitude of the kink-tearing mode. The rise in amplitude of the kink-tearing mode corresponds to the transport of fast ions, with the suppression factor returning to pre-burst levels on the same time scale as the recovery of the fast-ion population. Interestingly, the velocity of the n = 5 kink-tearing mode also increases during a burst event. This behavior is not currently well understood.

While only EPMs have been identified on MST so far, a wide variety of as yet unidentified mode activity has been observed. In non-reversed deuterium discharges with deuterium beam injection, chirping modes with n = 4 structure have been observed at lower current (200 kA). These modes are much longer in duration, roughly 0.5 ms, and chirp from approximately 90 kHz to 30 kHz, see Figure 6.4. No internal fluctuations correlated with these modes have been reported, but they are frequently triggered by m = 0 activity [14]. Behavior commonly associated with hole-clump formation in phase-space [15] is also frequently observed, as exemplified by the 'trilobyte' feature between 22.5 ms and 24.5 ms in Figure 6.4. The hole and clump in phase produce both an up-chirping and down-chirping branch; these are visible just above and below ~70 kHz.

6.2 \tilde{T}_e correlated with EPMs and chirping modes

Correlation of temperature fluctuations in non-reversed discharges with n = 5 EPMs and n = 4 chirping modes has been attempted. Due to the limited temporal resolution available with the Spectron laser system used for Thomson scattering, these measurements are challenging and no clear structure has been observed. Better resolution of correlated temperature fluctuations and structures awaits future work with the fast Thomson laser system under development. However, some upper limit on the fluctuation amplitude can be drawn for each case.



Figure 6.4: Spectrogram of \dot{B}_{θ} signals with n = 4 structure during a non-reversed discharge (shot 1130803045). Chirping activity is frequently triggered by m = 0 bursts (visible at ~20 kHz). Hole-clump behavior is also clearly visible.

n = 5 EPM correlated fluctuations

Using the same, 300 kA, $n_e \sim 1.0 \cdot 10^{13} \text{ cm}^{-3}$, F = 0 dataset used to diagnose current redistribution and q-profile modification in Chapter 4, temperature fluctuations are correlated with n = 5 EPMs observed via the toroidal array.

The EPM bursts themselves are roughly 60 μ s FWHM, so that even the besttimed TS bursts will only have 2 laser pulses during n = 5 activity. Due to the bursty nature of the modes, happening roughly every 0.5 ms, not every set of TS laser pulses will coincide with mode activity. This makes correlation measurements with the Spectron lasers inherently challenging. Additionally, for the plasma conditions described here, the EPM bursts are not as sharply defined (temporally) as at lower densities.

As described in Chapter 2, the unintegrated B_{θ} signals from the toroidal array are Fourier decomposed spatially. The n = 5 mode (or n = 4 mode for the next subsection) is represented as a sum of sine and cosine terms, which are individually Fourier transformed (temporally) to select the high-frequency beam-driven

Parameter	Min Value	Max Value	Mean Value
Peak \dot{B}_{θ} amplitude (G/s)	3.10^{5}	NA	NA
Frequency (kHz)	60	120	NA
Max $m = 0$ amplitude (G)	NA	5	NA
I _p (kA)	300	314	307.1
$n_e (10^{13} \text{cm}^{-3})$	0.9	1.1	1.0
T_e (eV)	190	290	236.1

Table 6.1: EPM Filter Parameters

instability of choice. The resulting spatial phase of the mode, as a function of time, is used in the temperature correlation analysis.

For the n = 5 mode, a bandpass sinc filter is used to select frequencies between 60 and 120 kHz. The peak burst amplitude from the \dot{B}_{θ} coils is required to be above $3 \cdot 10^5$ G/s (the absolute magnitude is not strictly important so the signals are not converted to B in Gauss). No filters on mode velocity (other than the bandpass filter) are imposed. The remaining filters are listed in Table 6.1 and 170 events satisfy these conditions. The correlated temperature fluctuations are shown in Figure 6.5.

No structure is apparent and the correlated fluctuations are all low-level, indicative of noise. Given the difficulties outlined above, the lack of apparent structure is not surprising. Any fluctuations correlated with this mode are estimated to be below 4 eV, or roughly 1.7% of the equilibrium temperature. For reference, the measured density fluctuations correlated with this mode have a peak amplitude of ~0.4% of the equilibrium density [12].

n = 4 chirping mode correlated fluctuations

Using an ensemble of shots in 200 kA, $n_e \sim 0.75 \cdot 10^{13} \text{ cm}^{-3}$, F = 0 discharges with full deuterium beam heating, temperature fluctuations are correlated with the n = 4 chirping modes discussed in the previous section. The same Fourier decomposition and filtering is used as before.

These modes are much longer in duration, lasting up to 0.5–1.0 ms. This eliminates the complications due to short burst widths encountered with EPMs. Unfor-



Figure 6.5: Electron temperature fluctuation amplitudes (top) and phases (bottom) correlated with n = 5 EPMs in non-reversed discharges. No structure is visible.

tunately, other complications are present. The modes are frequently triggered by m = 0 bursts, which lead to rapid heat transport from the core to the edge and reduce the observed temperature gradients. Tearing mode correlated fluctuations during periods of reduced gradients (near sawteeth in reversed discharges and in the core of improved confinement discharges) exhibit very little power. While other fluctua-

Parameter	Min Value	Max Value	Mean Value
Peak \dot{B}_{θ} amplitude (G/s)	$1.5 \cdot 10^5$	NA	NA
Frequency Low (kHz)	25	50	NA
Frequency Mid (kHz)	50	65	NA
Frequency High (kHz)	65	80	NA
Max $m = 0$ amplitude (G)	NA	10	NA
I _p (kA)	201	209	205.1
$n_e (10^{13} \text{cm}^{-3})$	0.65	0.80	0.71
T_e (eV)	130	170	141.7

Table 6.2: Chirp Filter Parameters

tion mechanisms may play a role here, the reduced temperature gradients do not bode well for fluctuation measurements. Furthermore, the chirping nature of the mode raises the possibility that any correlated temperature fluctuations may have time dependent amplitude and phase behavior. While some effort has been made to address this, the limited resolution of the Spectron system and the prohibitive demands of a dataset large enough to fully address time changing behavior leave this unresolved.

For the n = 4 mode, a bandpass sinc filter is used to select three different frequency bins with widths given in Table 6.2. For peaks above a suitable threshold in each bin, chirping behavior is distinguished from other energetic particle activity by requiring activity in each bin to be closely spaced in time and in the correct order for down-chirping. The extent of the bins also filters out modes that do not chirp all the way from 80 kHz to 25 kHz. The remaining filters are listed in Table 6.2—only 47 events meet the conditions listed. The correlated temperature fluctuations are shown in Figure 6.6.

Again, no structure is apparent in the fluctuations. The data above are for TS laser bursts occuring at any time point during the chirp. Due to insufficient statistics, binning the data into different time windows through the chirp is not feasible. Any fluctuations correlated with this mode are estimated to be below 8 eV, or roughly 5.6% of the equilibrium temperature.



Figure 6.6: Electron temperature fluctuation amplitudes (top) and phases (bottom) correlated with n = 4 chirping activity in non-reversed discharges. No structure is visible.

6.3 Ultra-low-frequency instabilities in reversed discharges

In reversed plasmas, the equilibrium profiles are significantly more dynamic than in the non-reversed discharges discussed previously. The periodic sawtooth events both accelerate ions [16] and transport energetic particles out of the core [17]. Estimates of fast-ion confinement times drop by approximately two orders of magnitude during reconnection, leading to rapid reduction in the fast-ion population. Following the sawtooth events, the fast-ion distribution continues to evolve as beam-deposited fast ions replenish the population. Measured current, safety factor, density, and temperature profiles all evolve rapidly around sawtooth events, as well. In short, reversed discharges pose a particularly challenging environment for understanding any instabilities driven by the fast-ion population.



Figure 6.7: Spectrogram of B_{θ} signals with n = 5 structure during a reversed discharge (shot 1130903079).

This can be seen clearly in the spectrogram of Figure 6.7. The edge B_{θ} signal for n = 5 activity is plotted for a single sawtooth period. The sawtooth events themselves are apparent in the vertical stripes of broadband fluctuations at either edge of the window; significant power cascades from the low-frequency MHD instabilities to very high frequencies during reconnection. Shortly after the sawtooth, the n = 5 kink-tearing mode is visible as a peak in the spectrum at low frequency (~12 kHz). During this period, little power is present at high frequencies due to insufficient

energetic particle drive. As the plasma rotation increases in the first milliseconds after the sawtooth, the peak frequency of the kink-tearing mode shifts upward (for this shot, a knee is visible at ~22 ms). Due to both natural safety factor evolution and beam-induced suppression, the kink-tearing mode decays during spin-up and the signal reduces briefly to background levels. During this time period, the energetic particle population also recovers sufficiently to drive EPMs at higher frequency (initially 70–110 kHz). Unlike the spectrograms for non-reversed plasmas, the bursts of n = 5 EPM activity are less clearly defined and exhibit significant chirping behavior. Over the course of the sawtooth period, the EPM frequency is observed to decrease. Not long after the kink-tearing mode dies, bursts of fluctuation power are visible near the plasma rotation frequency (f < 50 kHz). These bursts appear to be distinct from the kink-tearing mode and intimately connected to the chirping EPM behavior, which may be a trigger for the low-frequency burst or even the same mode.

A typical time series for the low-frequency n = 5 behavior, see Figure 6.8, shows kink-tearing activity early in the sawtooth cycle, followed by bursting events after the kink-tearing time of death. These events frequently have larger amplitude than the kink-tearing mode itself. The observed activity is qualitatively different, but distinguishing the bursts from kink-tearing activity via magnetic signals only is challenging based on the proximity to the plasma rotation frequency inferred from tearing mode phase velocities. While the flow profile decreases from the core to the edge, the tearing modes phase lock at the same apparent frequency on the toroidal array. The phase-locked tearing mode frequency is within a few kHz of the burst frequency, and given the width of the modes in frequency space, this difference is somewhat ambiguous.

Despite the dynamic equilibrium conditions in reversed discharges, important characteristics of these low-frequency bursts can be identified which distinguish them from the kink-tearing mode. These features are discussed in the following subsections. Measurements of plasma flow in the core via ion doppler spectroscopy, combined with measurements of phase velocity from the spatially filtered magnetic signals, resolve the ambiguity in mode frequency and demonstrate conclusively



Figure 6.8: Spectrogram (top) and time series (bottom) of integrated \tilde{B}_{θ} signals with n = 5 structure during a reversed discharge (shot 1130903133). Sawtooth events are marked by vertical red lines. The kink-tearing mode dies at approximately 21.5 ms and is followed by bursting events.

that the observed burst activity is propagating within the plasma reference frame. Furthermore, ANPA measurements during burst activity indicate that, unlike the kink-tearing mode, these bursts have a detrimental impact on fast-ion confinement. These results indicate that the observed low-frequency activity, like the higher frequency energetic particle modes, may be a beam-driven instability. As discussed above, the n = 5 kink-tearing mode velocity and amplitude has been observed to increase during EPM activity. Spontaneous momentum generation and increased amplitudes for tearing modes with $n \ge 6$ are also observed during these low-frequency bursts, and there may be some similarity between the two phenomena. Finally, possible ion heating of ~10 eV is observed during burst activity.

Unfortunately, due to technical difficulties, diagnostic coverage was not consistent and multiple datasets were accumulated at different plasma conditions for the various measurements presented here. The plasma current was 300 kA and density was $0.75 \cdot 10^{13}$ cm⁻³ for all conditions. However, the ion doppler spectroscopy measurements were taken in shallow reversal with F = -0.12 (249 shots) while the ANPA measurements were taken in deeper reversal with F = -0.2 (108 shots, from the set used in Chapter 4 Section 4.4).

For ensemble averages through burst activity, the low-frequency n = 5 burst signals from the magnetic coil array are low-pass filtered (f < 50 kHz) to eliminate contamination from the EPMs. Sawtooth periods are selected based on a clearly identifiable kink-tearing time-of-death. The t = 0 time reference (denoted t₀ below) is chosen to be the time at which the n = 5 amplitude at the beginning of a burst exceeds a threshold amplitude ($2.5 \cdot 10^5$ G/s for the \dot{B}_{θ} signal). The bursts are also filtered based on duration, peak amplitude, m = 0 amplitude, the time difference Δt between t₀ and the next sawtooth, and time delay between t₀ and the peak time. The m = 0 amplitude filter is applied to the time period [t₀ - 0.15 ms, t₀ + 0.15 ms]. The filter on Δt between t₀ and the next sawtooth ensures that the burst activity is distinguishable from rapid equilibrium changes around the sawtooth. The peak time-delay filter serves to eliminate bursts that are poorly defined temporally or that occur in quick succession too closely to be distinguished. These parameters are indicated in Table 6.3.

For the parameters given above, the ensemble averaged B_{θ} and B_{ϕ} amplitudes (from the integrated coil signals) for an n = 5 burst are shown in Figure 6.9. The
Parameter	Value
Threshold amplitude (G/s)	$2.5 \cdot 10^5$
$\operatorname{Min} \Delta t = t_{s a w} - t_0 (\mathrm{ms})$	1.0
Max $m = 0$ amplitude (G)	30
Min peak amplitude (G)	4
Min duration (ms)	0.3
$Max \Delta t = t_{peak} - t_0 (ms)$	0.2

Table 6.3: n = 5 Burst Filter Parameters

background fluctuation power present in the signals prior to a burst has been subtracted out. From Chapter 5, the anticipated scaling $\frac{na}{mR}$ for edge measurements, assuming m = 1, is applied to scale \tilde{B}_{θ} to \tilde{B}_{ϕ} . The scaled amplitude matches well with the measured \tilde{B}_{ϕ} time evolution, demonstrating that the burst activity has (m,n) = (1,5) structure.



Figure 6.9: Ensemble averaged integrated $n = 5 \tilde{B}_{\theta}$ and \tilde{B}_{ϕ} signals during a burst event. Scaling the poloidal waveform by na/mR, with m = 1, matches well with the measured toroidal waveform.

Mode propagation

Making the assumption that the tearing modes are stationary in the frame of the plasma enables straightforward diagnosis of the flow profile from edge magnetic signals. However, with the loss of the n = 5 kink-tearing mode, the core of the plasma inside the n = 6 rational surface becomes inaccesible with this technique. Alternative impurity ion toroidal flow (and temperature) measurements are possible with ion doppler spectroscopy [18]. The ion doppler spectroscopy diagnostic can operate passively, looking at electron impact emission for hydrogenic C⁺⁵ ions, or actively, in combination with a 50 keV diagnostic neutral beam using charge-exchange recombination of C⁺⁶ ions with hydrogen atoms [19, 20]. The active spectroscopy technique offers the advantage of localized temperature and velocity measurements at the intersection of the diagnostic neutral beam and the spectrometer viewing chord. The passive measurements, on the other hand, require line-integration; for toroidal velocity measurements the resulting viewing chord is quite long, significantly reducing resolution of local effects.

Both active and passive spectroscopy data are available for this ensemble. Due to hardware difficulties with the diagnostic neutral beam, however, the active data is only available for a limited number of shots (and is frequently interrupted by beam arcing). While the remaining good quality active data is insufficient for resolving flow profiles during enough burst events for an ensemble average, it does provide a useful check of the passive signals. The core electron temperature for these discharges is ~350 eV. At these temperatures, the C⁺⁵ ion population is highly core-localized. Despite the line-integration necessary for passive measurements, the localized nature of the impurity ion population makes the passive measurements comparable to the active measurements. Single-shot agreement is shown in Figure 6.10, where the active and passive velocity measurements show good agreement with each other as well as the n = 5 kink-tearing mode velocity prior to the time-of-death.

Although the active/passive IDS measurements are in good agreement with each other, small inaccuracies in the wavelength calibration for this dataset lead



Figure 6.10: Active (black) and passive (red) IDS velocity measurements on axis across multiple sawteeth for a single shot (shot 1140503103). Sawteeth are distinguishable by the sharp drop in velocity due to rapid momentum transport. The IDS measurements agree well with each other, as well as with the n = 5 kink tearing mode velocity (blue) while it is present.

to some uncertainty in the absolute velocity measurement. Ensemble averaged passive velocity (on-axis) and kink-tearing mode velocity time evolution around the kink-tearing time of death are shown in Figure 6.11. Early in the sawtooth cycle, impurity ion flow exceeds the kink-tearing mode velocity. As the plasma spins up and the kink-tearing mode approaches extinction, the mode velocity approaches the value of the impurity ion flow on-axis. The two measurements match just before the kink-tearing mode time-of-death. This suggests that the residual uncertainty in the absolute velocity calibration for passive spectroscopy is no more than a few kilometers per second.

Using the core flow measurements via spectroscopy and the inferred flow from the n = 6 tearing mode phase velocity, the core velocity profile is observed to flatten following the n = 5 kink-tearing mode death. See Figure 6.12 below for a single shot. This is in stark contrast to the centrally peaked profiles observed early in the sawtooth cycle, where a clear gradient is visible between the core ion



Figure 6.11: Ensemble averaged passive IDS (black) and n = 5 kink-tearing velocity (red) around the kink-tearing time-of-death. The kink-tearing mode approaches the core flow velocity as it nears death. Uncertainty in mode velocity increases near time-of-death due to low mode amplitude.

velocity, n = 5 kink-tearing, and n = 6 tearing mode phase velocities. Following the kink-tearing mode death, several low-frequency n = 5 bursts occur; the phase velocity of the filtered (f < 50 kHz) n = 5 signal is also shown for these bursts. While the spectrograms shown above were not sufficient to clearly distinguish the mode velocity relative to the plasma, the impurity ion flow and mode phase velocities unambiguously show that the bursting n = 5 activity is propagating in the plasma reference frame.

Ensemble averaged ion flow and phase velocity measurements through a lowfrequency burst are also shown in Figure 6.12. Prior to t = 0, when the mode amplitude is low, the n = 5 phase velocity measurements are unreliable and noisy. Near t = 0, with more power at higher frequencies associated with chirping behavior, the velocity ramps down to ~60 km/s before stabilizing. The bursting mode propagates at 13.2 ± 0.5 km/s above the equilibrium flow (IDS velocity on-axis) when the mode amplitude peaks. This corresponds to a toroidal rotation frequency of 7.0 ± 0.3 kHz in the plasma reference frame (f = vn/2 π R).



Figure 6.12: Impurity ion and mode velocities over a single sawtooth period (top, shot 1140502101) and ensemble averaged burst velocity in comparison to equilibrium flow (bottom).

The frequencies associated with particle motion are generally too high to correspond to the 7 kHz mode propagation frequency. The toroidal transit frequency ($f_{\phi} = v_{\parallel}/2\pi R$) is approximately 186 kHz for 25 keV ions with pitch = 0.8. The poloidal transit frequency ($f_{\theta} = v_{\parallel}/2q\pi R$) is even larger at approximately 1 MHz (assuming q ~ 0.18). Bounce frequencies associated with the trapped orbits have

not been systematically evaluated yet; while they are generally expected to be well above 7 kHz, a careful calculation is required to confirm this. Candidate particle and fluid drift frequencies also do not correspond well with the measured frequency. For drift waves and the fluid branch of the fishbone, among other instabilities, modes are frequently observed at the bulk-ion-diamagnetic frequency. The ion diamagnetic drift is:

$$v_{\mathrm{D},i} = -\frac{\nabla \mathrm{P}_i \times \vec{\mathrm{B}}}{\mathrm{ZenB^2}} \tag{6.6}$$

The core density profiles are generally flat in MST, so the pressure gradient corresponds predominantly to the temperature gradient. Based on the difference in impurity-ion temperature measured on-axis and 5 cm off-axis via IDS, the core gradient is on the order of 70 eV/m. This is generally consistent with measured electron temperature gradients, which are on the order of a few hundred eV/m near the core. With core magnetic field strength of B = 0.3 T, the drift velocity is approximately 0.23 km/s; at a minor radius of r = 5 cm, the ion diamagnetic frequency would be ~0.7 kHz. This is too low by an order of magnitude. The combined drift due to curvature and inhomogeneity in B is:

$$\nu_{\rm B} = \left(\varepsilon_{\perp} + 2\varepsilon_{\parallel}\right) \frac{\vec{B} \times \nabla B}{ZeB^3} \tag{6.7}$$

where ϵ_{\perp} and ϵ_{\parallel} represent the particle kinetic energies perpendicular and parallel to the magnetic field, respectively. Approximating \vec{B} with a Bessel function model having $B_0 = 0.3$ T yields a drift of approximately 63 km/s near the core. The resulting frequency ($f = \vec{k} \cdot \vec{v}_B/2\pi$), again at a minor radius of r = 5 cm, is ~200 kHz. As with the frequencies of particle motion, this is too high. Precession frequencies for both trapped and circulating particles may also be important. Analytical expressions for precession of trapped and circulating particles have been derived for tokamak geometry; similar expressions for the RFP need to be evaluated.

In the core of MST, both the ion diamagnetic drift and the curvature drift are predominantly in the poloidal rather than toroidal direction. A poloidal drift could produce a perceived toroidal phase velocity on the coil array. This issue is not addressed here; more work is required to distinguish such an effect.

Momentum generation

While the flow profiles shown in Figure 6.12 clearly indicate propagation of the n = 5 burst mode, the n = 6 mode is also observed to increase in velocity during a burst. Ensemble averages of both the velocities and amplitudes for n = 6 through 10 tearing modes are shown in Figure 6.13. The amplitudes are from the integrated \tilde{B}_{θ} signals, while the velocities are calculated from the unintegrated \dot{B}_{θ} signals for improved resolution. The increase in velocity observed with the n = 6 tearing mode is observed for higher n modes as well. Furthermore, the amplitudes of modes n \geq 6 increase during a burst event; for higher toroidal mode number, the amplitude clearly dips prior to increasing.

The jump in velocity/amplitude does not occur at the same time for each mode, however. Modes with higher n are observed to spin-up/ramp-up slightly later than modes with lower n. This is made clearer in Figure 6.14, where the perturbation to the equilibrium velocities/amplitudes is plotted over time. The equilibrium velocities and amplitudes are obtained by averaging the values during the time window from t = -0.1 ms to t = 0 ms. These equilibrium quantities are then subtracted from the measured time series to obtain the perturbation due to the burst activity.

The velocity/amplitude increase is observed shortly after t_0 for the n = 6 tearing mode, and a 'pulse' of increased flow/amplitude propagates radially outward on a time scale of hundreds of μ s. Additionally, the amplitudes for $n \ge 7$ appear to dip early in the burst before rising. The n = 7 itself may also be decreasing in amplitude before the burst, but a line with slope zero can be fit to the entire 0.5 ms prior to the burst without significant deviation from the error bars. Momentum transport studies during edge biasing in MST found that momentum injected at the edge took ~1.5 ms to reach the core and estimated the kinematic viscosity to be 55 m²/s based on the slowing down time [21]. The wavefront propagation speed (as estimated from the time for the 'pulse' to propagate from the n = 6 to n = 7 rational surfaces) is similar in magnitude, but the slowing down time following a pulse is less clear.



Figure 6.13: Time evolution of n = 6-10 tearing mode velocities (top) and amplitudes (bottom) through a burst event. The n = 5 burst amplitude is also shown for reference.

Due to the irregular timing between repeated bursts, the associated momentum generation at each subsequent burst obscures any slowing down that may occur after the initial burst.

While the ensemble averaged impurity ion flow (Figure 6.12) does not jump in the same manner as the tearing modes, both the single-shot time series and the



Figure 6.14: Time evolution of perturbations to n = 6-f10 tearing mode velocities (top) and amplitudes (bottom) through a burst event.

ensemble averaged impurity flow increase across the burst cycle. The momentum in the core appears to ramp upward in time with subsequent n = 5 bursts. The magnitude and radial extent of the flow generated during a burst may depend on burst amplitude. The time required for the 'pulse' to propagate to the magnetic axis may also be longer than the 0.5 ms window used for burst ensembling.

As with the increase in n = 5 kink-tearing mode velocity during an EPM burst in non-reversed plasmas, the origin of the momentum generated during these low-frequency n = 5 bursts is not clear. These bursts appear to be associated with fast-ion transport, discussed in the next section. In tokamaks, fishbone-induced nonambipolar transport leads to changing radial electric fields which oppose toroidal plasma flow [22]. Expected radial fields caused by non-ambipolar transport in MST would oppose toroidal flow as well, but most of the core-generated flow would be poloidal. Alternatively, the n = 5 bursting mode may be coupling with the tearing modes through m = 0 activity to transfer momentum.

Possible ion heating

Passive spectroscopy also provides a measurement of the core impurity ion temperature. The core temperatures are observed to be in the 200–250 eV range. Ensemble averages of core ion temperatures at two locations are shown in Figure 6.15. Prior to the burst, core temperatures appear to have little time dependence, although a modest increase in temperature may take place early in the window. Immediately after a burst, at both locations, apparent ion heating of approximately 10 eV occurs over a 200–300 μ s period. The increase in temperature is sustained through the remainder of the burst. While the eye may be drawn to the apparent trend, a linear fit between the first and last time points also fits the data reasonably well, so no conclusion can be drawn from this. Better time resolution and an ensemble with improved statistics could clarify whether or not the apparent break point at t₀ is real or an artifact. The mechanism for ion heating is not clear, but both magnetic reconnection and Alfvén waves have been observed to heat ions [23, 24, 25].

Fast-ion transport/redistribution

The advanced neutral particle analyzer (ANPA) diagnostic on MST [26, 27] is sensitive to core-localized fast ions with high-pitch. Hydrogen and deuterium ions can be measured simultaneously, and multiple channels allow measurement of the energy distribution with 3–4 keV resolution across a range of 35 keV. Ensemble



Figure 6.15: Core impurity ion temperature on-axis (black) and 5 cm off-axis (red) during an n = 5 burst. Approximately 10 eV of heating is observed at both locations.

averaged ANPA signals near the beam energy show that non-linearly coupled EPM bursts (n = 1, 4, 5) in non-reversed plasmas lead to redistribution of the fast-ion population, with a drop of ~14% in signal during burst activity [12].

Single shot time series for several energy channels are shown in Figure 6.16 across an entire sawtooth period. Following the sawtooth, the fast-ion population recovers from energization/loss. While the kink-tearing mode is still active, the signals near the beam energy roughly equilibrate. High-frequency EPM activity begins just before 24 ms, shortly after the kink-tearing death. Low-frequency chirping begins almost immediately after the first EPM burst, but the first low-frequency n = 5 burst of appreciable amplitude does not occur until approximately 25 ms. Following this low-frequency burst the ANPA channels closest to the beam energy show reduction in signal strength. The reduction in ANPA signals persists and becomes more severe through subsequent bursts, eventually affecting all channels shown. Later, during a pause between bursts, the signal strength recovers across multiple channels. While either particle redistribution or pitch angle scattering could contribute to the observed drop in signal, redistribution appears to be the



Figure 6.16: Spectrogram of n = 5 activity (top) and time series (bottom, upper panel) with ANPA signals (bottom, lower panel) for several channels near the beam energy across a full sawtooth cycle (shot 1130903139). Sawteeth are indicated by vertical red lines. Signals at all energies begin to drop when n = 5 low-frequency bursting activity begins. The energetic particle population recovers briefly during a pause in bursting activity.

more likely culprit.

The ensemble averaged ANPA signal at 24 keV through a burst is shown in Figure 6.17. Signals from each burst are averaged over 24 samples (24 μ s) to reduce noise. Since multiple bursts in a row can significantly change the overall signal level, each burst is normalized to the pre-burst signal amplitude during the window [-0.5 ms, -0.2 ms]. The ANPA signal is also normalized to the D_{α} signal. The amplitude of the 24 keV signal drops by ~30% through a burst. While previously observed EPM-driven transport leads to a drop on time scales of ~60 μ s, during low-frequency n = 5 activity reduction is observed over much longer time scales of ~200 μ s. Additionally, while the ANPA signal near the beam energy begins to recover immediately after an initial, EPM-induced transport event, in this case the ANPA signal does not recover while the n = 5 amplitude remains significant.



Figure 6.17: Ensemble average of the ANPA signal at 24 keV (closest to the beam energy) during an n = 5 burst.

Ensemble averaged amplitudes for n = 1, 4, 5, and 6 EPMs are shown during a burst in Figure 6.18. The frequency filters used to isolate each EPM are listed in Table 6.4, and exclude both the low-frequency n = 5 bursts and tearing mode activity. The behavior of the energetic particle modes observed here exhibits many



Figure 6.18: Ensemble averages of n = 1, 4, 5, and 6 energetic particle mode amplitudes from the \dot{B} coils during a low-frequency n = 5 burst.

Toroidal mode number	Lower cutoff (kHz)	Upper cutoff (kHz)
1	50	100
4	140	220
5	60	110
6	90	160

Table 6.4: EPM Frequency Filters

similarities to previously reported behavior during bursts of non-linearly coupled EPMs. The rise in amplitude of the n = 5 EPM flattens the fast-ion distribution locally, but steepens the gradient outside of this region, driving the n = 1 and n = 4 energetic particle modes unstable. 'Avalanches' of this type are predicted by theory [28], observed on many devices, for example NSTX [29], and lead to enhanced transport. Following the initial avalanche, however, the behavior becomes somewhat more complicated as the low-frequency n = 5 mode is driven unstable. This is then followed by another avalanche involving the n = 1, 5, and 6 energetic particle modes.

Although the drop in ANPA signal could be due to pitch angle scattering [30],

the correlation of low-frequency n = 5 bursts with EPM activity implicates particle transport. Furthermore, the behavior of the n = 5 bursts can be distinguished from n = 5 kink-tearing activity in non-reversed plasmas, where EPM bursts lead to a brief increase in mode amplitude and velocity. In this case, the n = 5 mode clearly participates in EPM avalanches, acting as both a trigger and a successor. This strongly indicates that the n = 5 bursts are tied to gradients in the fast-ion density profile. Finally, after the EPM avalanche have finished, the n = 5 burst maintains elevated levels of particle transport that prevent the fast-ion population from fully recovering.

6.4 $ilde{\mathsf{T}}_e$ correlated with ultra-low-frequency instabilities

While the low-frequency n = 5 bursts are sufficiently long for the entire pulsetrain of a TS laser burst to fall inside, their bursty nature still makes correlating temperature fluctuations a challenge. Across all 249 shots of the F = -0.12 ensemble, 393 bursts meet the requirements of Table 6.3. Of these, only 258 coincide with a TS laser burst. As with the EPMs discussed in the previous sections, the rapidly varying behavior of the mode over time also exceeds the ability of the Spectron system to adequately resolve. However, some idea of the internal structure of the mode is attainable.

Mode structures observed in F = -0.12, -0.2, and -0.3 plasmas are discussed below. While the mode frequency and particle transport results in the previous section applied a 4 G minimum peak amplitude filter, that requirement is relaxed slightly here. Instead, the average mode amplitude during a TS laser burst is required to be above 2 G. Since TS laser bursts frequently do not overlap with the peak in amplitude and the 200 µs window is wide enough that any average across the peak will be noticeably reduced, the relaxation of this filter is not expected to change the results significantly. The $\Delta t = t_{saw} - t_0$ filter is also relaxed from 1.0 ms to 0.7 ms. While the previous results required a 0.5 ms window for ensemble averaging,

Parameter	Value
Threshold amplitude (G/s)	$2.5 \cdot 10^5$
$\operatorname{Min} \Delta t = t_{s a w} - t_0 (\mathrm{ms})$	0.7
Max $m = 0$ amplitude (G)	25
Min peak amplitude (G)	2
Min duration (ms)	0.3
$Max \Delta t = t_{peak} - t_0 (ms)$	0.2

Table 6.5: n = 5 Burst Filter Parameters for \tilde{T}_e Correlation

Start	End	Number of Events
0.0 ms	0.35 ms	94
0.35 ms	0.70 ms	92
0.70 ms	3.20 ms	117

here the TS burst is only 0.2 ms wide, so the burst t_0 can occur slightly closer to a sawtooth while maintaing the same minimum clearance. Finally, the m = 0 amplitude filter is made more stringent, with a maximum amplitude of 25 G over a wider window from t_0 - 0.25 ms to t_0 + 0.5 ms. The other filter parameters are listed in Table 6.5.

F = -0.12 plasmas

The dataset for F = -0.12 plasmas is sufficiently large that correlations with n = 5 burst events can be binned into multiple time windows. The windows chosen are given in Table 6.6. Two windows of equal length, 0.35 ms, are selected at the beginning of the burst. For most events, these two windows cover the full duration of the burst. The last window is significantly wider at 2.5 ms; many bursts can last a few ms and obtaining reasonable time resolution for such long durations is not feasible with the present dataset. The correlated temperature fluctuation amplitudes and phases are shown in Figure 6.19.

The initial temperature structure is core-peaked and localized within z/a = 0.2. Similar structure is seen in the later time windows, but at reduced amplitude; for



Figure 6.19: Correlated electron temperature fluctuation amplitudes (top) and phases (bottom) in F = -0.12 plasmas. Fluctuations are binned into time windows from [0.0, 0.35] (top panel), [0.35, 0.70] (middle panel), and [0.70, 3.20] (bottom panel).

these time windows only a few of the core-most points show non-zero fluctuation amplitude. Some structure in the mid-radius may be possible, but the error bars obscure this. The fluctuations are also observed to be in-phase with \tilde{B}_{θ} at the wall. These results demonstrate the rapidly changing nature of the correlated temperature fluctuations. Like the n = 5 kink-tearing mode structures, the fluctuations are corelocalized, but they lack the broad, off-axis peak characteristic of the kink-tearing mode. Narrowing the filter parameters for mode amplitude and velocity has no appreciable effect on the observed structure.

F = -0.2 plasmas

At deeper reversal, the duration of n = 5 kink-tearing activity following a sawtooth is decreased. The low-frequency n = 5 bursts are still observed in these plasmas, beginning after the death of the kink-tearing mode and the resumption of EPM bursts. The dataset for plasmas with F = -0.2 is less extensive (108 shots), so no time windowing is feasible. Correlated temperature fluctuations are shown in Figure 6.20. Similarly to the results for F = -0.12 plasmas, the observed structures are core-peaked and largely in-phase with \tilde{B}_{θ} at the wall. The fluctuation amplitudes in these plasma conditions are more highly core-localized, with little fluctuation power observed outside z/a = 0.1. The peak amplitude has similar magnitude to the peak observed for shallower reversal.

F = -0.3 plasmas

This dataset is even less extensive than the F = -0.2 dataset (95 shots). While n = 5 bursts are still observed in these plasmas, they occur less frequently, with shorter duration and lower peak amplitudes. No significant structure is observed in the temperature fluctuations, shown in Figure 6.21.



Figure 6.20: Correlated electron temperature fluctuation amplitudes (top) and phases (bottom) in F = -0.2 plasmas.

Comparison

The results at F = -0.12, the best diagnosed dataset, indicate strongly time-dependent structure. While higher time resolution is required to adequately resolve this behavior, the general structures show consistent behavior between datasets. Where significant fluctuation power is observed, the structures are core-localized, with



Figure 6.21: Correlated electron temperature fluctuation amplitudes (top) and phases (bottom) in F = -0.3 plasmas.

amplitude ~10 eV. As reversal deepens, the peak in fluctuation amplitude appears to shift further core-ward. At the deepest reversal, the fluctuations shift entirely out of view. Since the electron dynamics, particularly the temperature fluctuations, are generally well-correlated with magnetic perturbations, these results suggest a core-localized electromagnetic perturbation. The n = 5 rational surface is expected to be

completely removed from the plasma during these burst events, so the magnetic perturbations should be purely sinusoidal in nature with no island structure.

The improved time resolution attainable with the Fast Thomson laser system should provide a better picture of how the structure evolves through a burst cycle, as well as providing better overall statistics. Even with the low resolution in the measurements presented here, correlating temperature fluctuations with a beam-driven instability is a significant step for Thomson scattering. Correlated fluctuations so far have focused on the kink-tearing and tearing modes, which are stationary in the plasma reference frame. Although a mode with similar amplitude to the n = 6 tearing mode and propagating at only a few kHz is a relatively easy target, this demonstrates the versatility of the Thomson scattering system. Higher frequency instabilities provide a promising target for the Fast Thomson system.

6.5 Discussion

While no clear temperature fluctuation structures are observed to be correlated with known EPMs and chirping modes, a new instability has been characterized in reversed discharges. This instability has very low frequency, effectively 'hiding' it near the rotation frequency with the tearing modes. Several important characteristics distinguish this mode from the kink-tearing and purely tearing modes, though.

- Association with chirping behavior, followed by propagation only a few kHz above the plasma rotation
- Fast-ion transport and participation in EPM 'avalanches'
- Anomalous momentum generation and increased tearing mode amplitude

Possible bulk-ion heating of 10 eV is observed. The correlated temperature structures also indicate a core-localized mode with an amplitude of roughly 10 eV. The structure is strongly dependent on reversal parameter, moving core-ward with deeper F. The observed characteristics allow for informed speculation about the identity of this instability.

Fishbone

Drawing on Eqn. 6.5, where mode behavior depends on both the ideal MHD potential associated with the perturbation, δW_{bulk} , and the kinetic contribution of the energetic particles, δW_{hot} , a few general inferences can be made about the n = 5 driving terms. During a typical sawtooth cycle in non-beam heated discharges, δW_{bulk} is positive early in the cycle, contributing to instability, but negative later in the cycle. While δW_{hot} is consistent with stabilizing behavior in non-reversed plasmas, the bursting behavior in standard plasmas suggests that it is capable of providing sufficiently strong drive to overcome ideal MHD stability late in the sawtooth cycle of a reversed discharge. The implied dependence on both q and β_{hot} bears strong similarities with fishbone behavior in tokamak plasmas.

The fishbone mode was the first energetic particle instability observed in tokamaks [31]. It was initially observed with perpendicular neutral beam injection, where it was most virulent, but also later observed with parallel injection. Fishbones commonly down-chirp by several kHz at the beginning of a burst, and propagate at low frequency (typically near the ion diamagnetic frequency). They are most notorious for fast-ion transport, with a single burst capable of reducing neutron emission rates by up to 40%. These characteristics are in close agreement with the most prominent features of the instability measured here.

Furthermore, stability studies of non-resonant kinks (NRK) in NSTX have identified an important transition between the NRK and fishbone at appropriate values of the safety factor and fast-ion β [32]. In the tokamak, the (1,1) kink may still be unstable with q_{min} above but sufficiently close to one. The mode is most strongly unstable for values of q_{min} close to one. At these values, fast ions have a stabilizing effect on the NRK, with mode stability improving for increasing values of β_{hot} . Mode stability also improves with increasing q_{min} , with intermediate values of q_{min} and β_{hot} capable of completely stabilizing the NRK. However, at these intermediate values of q_{min} , increasing β_{hot} too far beyond the NRK stability threshold leads to the crossing of another threshold, driving the fishbone unstable.



Figure 6.22: Speculative stability diagram for n = 5 perturbations. Decreasing q_0 stabilizes a kink-like mode, while increasing β_{hot} can stabilize and destabilize different branches of the kink mode. The dashed line indicates probable evolution of plasma parameters over a sawtooth cycle.

Combining the physical picture developed for the NRK-fishbone transition in tokamaks with the known features of the n = 5 kink-tearing mode in MST leads to a similar picture. This is illustrated in Figure 6.22. For values of q_0 sufficiently close to the 1/5 resonance, only a non-resonant kink may be unstable, with fast ions entirely stabilizing. At deeper reversal, q_0 is reduced and the NRK is completely stabilized by both the fast ions and the evolution of the safety factor over the course of a sawtooth cycle. At sufficiently deep reversal, however, the time-evolving safety factor strays across the stability boundary of the fishbone. So far, the bursty n = 5 mode has not been observed in plasmas with reversal shallower than F = -0.1,

providing an indicator of where this boundary may lie. The bursting n = 5 mode also becomes weaker and less frequent at very deep reversal (F \leq -0.3), so there may be a lower bound on q_0 for fishbone instability.

Alternatives

Estimates of the beta-induced Alfvén eigenmode (BAE) for MST indicate that an n = 5 gap is possible in the core [1, 33]. This gap is predicted to be near the location of the observed temperature fluctuation structures, but extends up to 24 kHz in frequency. Some unknown effect may be responsible for reducing the width of the gap in frequency. A BAE could produce similar transport in participation with 'avalanches', and the frequency chirping may be explained by the transport induced by EPMs prior to the burst. Additionally, Alfvén waves have been implicated in ion heating, so a BAE may concievably lead to the observed bulk ion heating.

Finally, bulk ion heating is also observed during reconnection events on MST. Some as yet unidentified reconnection event may be responsible for the observed ion heating. Reconnection with m = 0 activity causes significant fast-ion transport and mode chirping, but also transports a significant fraction of the momentum out of the plasma. Reconciling this picture with the anomalous momentum generation may be more challenging. Given the wide range of activity observed during these bursts, there is plenty of room for speculation. Future studies would benefit from measurements of mode frequency scaling (beam energy, Alfvén velocity, other plasma parameters), parallel/perpendicular ion heating comparisons, and poloidal flow.

6.6 References

- [1] J. J. Koliner. *Neutral Beam Excitation of Alfvén Continua in the Madison Symmetric Torus Reversed Field Pinch.* PhD thesis, University of Wisconsin-Madison, 2013.
- [2] W. W. Heidbrink. Basic physics of Alfvén instabilities driven by energetic

particles in toroidally confined plasmas. *Phys. Plasmas*, 15(5):055501, 2008. doi: http://dx.doi.org/10.1063/1.2838239.

- [3] Philipp Lauber. Super-thermal particles in hot plasmas—kinetic models, numerical solution strategies, and comparison to tokamak experiments. *Phys. Rep.*, 533(2):33, 2013. doi: http://dx.doi.org/10.1016/j.physrep.2013.07.001.
- [4] H. Alfvén. Existence of electromagnetic-hydrodynamic waves. *Nature*, 150: 405, 1942. doi: 10.1038/150405d0.
- [5] C.Z Cheng, Liu Chen, and M.S Chance. High-n ideal and resistive shear Alfvén waves in tokamaks. *Ann. Phys.*, 161(1):21, 1985. doi: http://dx.doi.org/10. 1016/0003-4916(85)90335-5.
- [6] G. Regnoli, H. Bergsåker, E. Tennfors, F. Zonca, E. Martines, G. Serianni, M. Spolaore, N. Vianello, M. Cecconello, V. Antoni, R. Cavazzana, and J.-A. Malmberg. Observations of toroidicity-induced Alfvén eigenmodes in a reversed field pinch plasma. *Phys. Plasmas*, 12(4):042502, 2005. doi: http: //dx.doi.org/10.1063/1.1861896.
- [7] S. Spagnolo, M. Zuin, F. Auriemma, R. Cavazzana, E. Martines, M. Spolaore, and N. Vianello. Alfvén eigenmodes in the RFX-mod reversed-field pinch plasma. *Nucl. Fusion*, 51(8):083038, 2011. doi: http://dx.doi.org/10.1088/ 0029-5515/51/8/083038.
- [8] Liu Chen. Theory of magnetohydrodynamic instabilities excited by energetic particles in tokamaks. *Phys. Plasmas*, 1(5):1519, 1994. doi: http://dx.doi.org/ 10.1063/1.870702.
- [9] Fulvio Zonca and Liu Chen. Resonant and non-resonant particle dynamics in Alfvén mode excitations. *Plasma Phys. Control. Fusion*, 48(5):537, 2006. doi: http://dx.doi.org/10.1088/0741-3335/48/5/004.

- [10] Liu Chen, R. B. White, and M. N. Rosenbluth. Excitation of internal kink modes by trapped energetic beam ions. *Phys. Rev. Lett.*, 52(13):1122, 1984. doi: 10.1103/PhysRevLett.52.1122.
- [11] J. J. Koliner, C. B. Forest, J. S. Sarff, J. K. Anderson, D. Liu, M. D. Nornberg, J. Waksman, L. Lin, D. L. Brower, W. X. Ding, and D. A. Spong. Fast-particledriven Alfvénic modes in a reversed field pinch. *Phys. Rev. Lett.*, 109(11):115003, 2012. doi: 10.1103/PhysRevLett.109.115003.
- [12] L. Lin, W. X. Ding, D. L. Brower, J. J. Koliner, S. Eilerman, J. A. Reusch, J. K. Anderson, M. D. Nornberg, J. S. Sarff, J. Waksman, and D. Liu. Measurement of energetic-particle-driven core magnetic fluctuations and induced fast-ion transport. *Phys. Plasmas*, 20(3):030701, 2013. doi: http://dx.doi.org/10.1063/ 1.4798397.
- [13] L. Lin. Energetic-particle-driven instabilities and induced fast ion transport in a reversed-field pinch. *Bull. Am. Phys. Soc.*, 58(16):NI2.1, 2013.
- [14] J. K. Anderson, A. F. Almagri, D. J. Den Hartog, S. Eilerman, C. B. Forest, J. J. Koliner, V. V. Mirnov, L. A. Morton, M. D. Nornberg, E. Parke, J. A. Reusch, J. S. Sarff, J. Waksman, V. Belykh, V. I. Davydenko, A. A. Ivanov, S. V. Polosatkin, Y. A. Tsidulko, L. Lin, D. Liu, G. Fiksel, H. Sakakita, D. A. Spong, and J. Titus. Fast ion confinement and stability in a neutral beam injected reversed field pinch. *Phys. Plasmas*, 20(5):056102, 2013. doi: 10.1063/1.4801749.
- [15] H. L. Berk, B. N. Breizman, and N. V. Petviashvili. Spontaneous hole-clump pair creation in weakly unstable plasmas. *Phys. Lett. A*, 234(3):213, 1997. doi: http://dx.doi.org/10.1016/S0375-9601(97)00523-9.
- [16] S. Eilerman. Ion Runaway During Magnetic Reconnection in the Reversed-Field Pinch. PhD thesis, University of Wisconsin-Madison, 2014.
- [17] G. Fiksel, B. Hudson, D. J. Den Hartog, R. M. Magee, R. O'Connell, S. C. Prager,A. D. Beklemishev, V. I. Davydenko, A. A. Ivanov, and Yu. A. Tsidulko. Obser-

vation of weak impact of a stochastic magnetic field on fast-ion confinement. *Phys. Rev. Lett.*, 95(12):125001, 2005. doi: 10.1103/PhysRevLett.95.125001.

- [18] R. M. Magee, D. J. Den Hartog, G. Fiksel, S. T. A. Kumar, and D. Craig. Toroidal charge exchange recombination spectroscopy measurements on MST. *Rev. Sci. Instrum.*, 81(10):10D716, 2010. doi: http://dx.doi.org/10.1063/1.3478665.
- [19] S. Gangadhara, D. Craig, D. A. Ennis, and D. J. Den Hartog. Modeling fast charge exchange recombination spectroscopy measurements from the Madison Symmetric Torus. *Rev. Sci. Instrum.*, 77(10):10F109, 2006. doi: http://dx.doi. org/10.1063/1.2219412.
- [20] D. J. Den Hartog, D. Craig, D. A. Ennis, G. Fiksel, S. Gangadhara, D. J. Holly, J. C. Reardon, V. I. Davydenko, A. A. Ivanov, A. A. Lizunov, M. G. O'Mullane, and H. P. Summers. Advances in neutral-beam-based diagnostics on the Madison Symmetric Torus reversed-field pinch. *Rev. Sci. Instrum.*, 77(10):10F122, 2006. doi: http://dx.doi.org/10.1063/1.2217920.
- [21] A. F. Almagri, J. T. Chapman, C. S. Chiang, D. Craig, D. J. Den Hartog, C. C. Hegna, and S. C. Prager. Momentum transport and flow damping in the reversed-field pinch plasma. *Phys. Plasmas*, 5(11):3982, 1998. doi: http://dx. doi.org/10.1063/1.873118.
- [22] W. W. Heidbrink, M. E. Austin, R. K. Fisher, M. García-Muñoz, G. Matsunaga, G. R. McKee, R. A. Moyer, C. M. Muscatello, M. Okabayashi, D. C. Pace, K. Shinohara, W. M. Solomon, E. J. Strait, M. A. Van Zeeland, and Y. B. Zhu. Characterization of off-axis fishbones. *Plasma Phys. Control. Fusion*, 53(8): 085028, 2011. doi: http://dx.doi.org/10.1088/0741-3335/53/8/085028.
- [23] R. M. Magee, D. J. Den Hartog, S. T. A. Kumar, A. F. Almagri, B. E. Chapman, G. Fiksel, V. V. Mirnov, E. D. Mezonlin, and J. B. Titus. Anisotropic ion heating and tail generation during tearing mode magnetic reconnection in a high-temperature plasma. *Phys. Rev. Lett.*, 107(6):065005, 2011. doi: 10.1103/PhysRevLett.107.065005.

- [24] Varun Tangri, P. W. Terry, and Gennady Fiksel. Anomalous impurity ion heating from Alfvén cascade in the reversed field pinch. *Phys. Plasmas*, 15(11): 112501, 2008. doi: http://dx.doi.org/10.1063/1.2998829.
- [25] D. A. Gates, N. N. Gorelenkov, and R. B. White. Ion heating by fast-particleinduced Alfvén turbulence. *Phys. Rev. Lett.*, 87(20):205003, 2001. doi: 10.1103/ PhysRevLett.87.205003.
- [26] S. Eilerman, J. K. Anderson, J. A. Reusch, D. Liu, G. Fiksel, S. Polosatkin, and V. Belykh. Time-resolved ion energy distribution measurements using an advanced neutral particle analyzer on the MST reversed-field pinch. *Rev. Sci. Instrum.*, 83(10):10D302, 2012. doi: http://dx.doi.org/10.1063/1.4728312.
- [27] J. A. Reusch, J. K. Anderson, V. Belykh, S. Eilerman, D. Liu, G. Fiksel, and S. Polosatkin. Calibration of an advanced neutral particle analyzer for the Madison Symmetric Torus reversed-field pinch. *Rev. Sci. Instrum.*, 83(10): 10D704, 2012. doi: http://dx.doi.org/10.1063/1.4729493.
- [28] H. L. Berk, B. N. Breizman, J. Fitzpatrick, M. S. Pekker, H. V. Wong, and K. L. Wong. Nonlinear response of driven systems in weak turbulence theory. *Phys. Plasmas*, 3(5):1827, 1996. doi: http://dx.doi.org/10.1063/1.871978.
- [29] E.D. Fredrickson, N.N. Gorelenkov, R.E. Bell, J.E. Menard, A.L. Roquemore, S. Kubota, N.A. Crocker, and W. Peebles. Fast ion loss in a 'sea-of-TAE'. *Nucl. Fusion*, 46(10):S926, 2006. doi: http://dx.doi.org/10.1088/0029-5515/46/10/ S09.
- [30] C. F. Kennel and F. Engelmann. Velocity space diffusion from weak plasma turbulence in a magnetic field. *Phys. Fluids*, 9(12):2377, 1966. doi: http://dx. doi.org/10.1063/1.1761629.
- [31] K. McGuire, R. Goldston, M. Bell, M. Bitter, K. Bol, K. Brau, D. Buchenauer, T. Crowley, S. Davis, F. Dylla, H. Eubank, H. Fishman, R. Fonck, B. Grek, R. Grimm, R. Hawryluk, H. Hsuan, R. Hulse, R. Izzo, R. Kaita, S. Kaye,

H. Kugel, D. Johnson, J. Manickam, D. Manos, D. Mansfield, E. Mazzucato, R. McCann, D. McCune, D. Monticello, R. Motley, D. Mueller, K. Oasa, M. Okabayashi, K. Owens, W. Park, M. Reusch, N. Sauthoff, G. Schmidt, S. Sesnic, J. Strachan, C. Surko, R. Slusher, H. Takahashi, F. Tenney, P. Thomas, H. Towner, J. Valley, and R. White. Study of high-beta magnetohydrodynamic modes and fast-ion losses in PDX. *Phys. Rev. Lett.*, 50(12):891, 1983. doi: 10.1103/PhysRevLett.50.891.

- [32] Feng Wang, G. Y. Fu, J. A. Breslau, and J. Y. Liu. Linear stability and nonlinear dynamics of the fishbone mode in spherical tokamaks. *Phys. Plasmas*, 20(10): 102506, 2013. doi: http://dx.doi.org/10.1063/1.4824739.
- [33] J. J. Koliner. Personal communication, March 2014.

Chapter 7 Conclusion

An innovative Bayesian analysis technique has been expanded to provide a full description of correlated fluctuations. Using a temperature fluctuation model of the form given in Eqn. 2.35, where:

$$T_{e} = T_{e,0} + \tilde{T}_{e,n} \cos\left(\zeta_{n} + \delta\right)$$

information about both the amplitude and phase of correlated fluctuations can be extracted using a sine/cosine expansion similar to pseudospectral techniques:

$$T = \langle \tilde{T}_{e} | \exp\{i\zeta_{n}\} \rangle$$

$$T^{\dagger} = \langle \tilde{T}_{e} | \exp\{i(\zeta_{n} - \pi/2)\} \rangle$$

Application of this technique to temperature measurements from an advanced Thomson scattering diagnostic with high time resolution yields a powerful tool for diagnosing a wide range of plasma phenomena. Thomson scattering has traditionally been used only for equilibrium temperature and density profile measurements; the original development of the Bayesian analysis technique used throughout this thesis successfully demonstrated the ability to measure internal magnetic structures with Thomson scattering fluctuation measurements as well as dynamic changes to the magnetic equilibrium [1]. Constraint of the magnetic equilibrium with Thomson scattering fluctuation measurements is demonstrated with a reasonable degree of diagnostic maturity in this thesis. Fluctuation-induced contributions to current transport have been measured with Thomson scattering for the first time. Finally, measurement of internal magnetic structures for modes propagating in the plasma reference frame has been demonstrated for the first time. The major physics conclusions achieved with this technique are summarized below, followed by a brief outline of suggestions for future work.

7.1 Summary of results

Equilibrium profile constraint

In Chapter 4, rational surface locations were measured with 3-6 mm accuracy from electron temperature fluctuation structures. Modification of the axisymmetric equilibrium reconstruction routine MSTFit has made possible constraint of the equilibrium safety factor with these measurements. The errors in Thomson scattering rational surface estimates are sufficiently small that equilibrium current and safety factor profiles can be inferred with errors of only a few percent in the core.

Inferred current profiles and safety factors for non-reversed discharges with neutral beam heating indicate redistribution of current which reduces q_0 and magnetic shear. This observation supports one possible mechanism for observed suppression of core MHD activity during NBI: current profile modification which eliminates the n = 5 rational surface. In reversed plasmas, on the other hand, any changes to the current and q profiles are too small to be accurately observed with Thomson scattering. For all cases, the n = 5 kink-tearing mode is found to be, at best, marginally resonant. A non-resonant mode is expected to be highly sensitive to changes in the value of q_0 ; the equilibrium changes observed in non-reversed plasmas may significantly impact the stability of the n = 5 kink-tearing mode and a rigorous theoretical examination is required to fully understand this effect. However, the observation of n = 5 suppression in varying plasma conditions without consistent current profile modification casts doubt on this mechanism. Multiple effects due to energetic particles may contribute to the observed behavior.

Current transport

In Chapter 5, deviation of the correlated temperature fluctuation phase from the phase of \tilde{B}_{θ} at the wall was observed, with fluctuation power at some radial locations completely out of phase with \tilde{B}_{θ} . The correlated fluctuation product with radial perturbations $\langle \tilde{T}_e \tilde{B}_r \rangle$, which is associated with current transport, has been measured for the first time. While the net current transport is consistent with zero, contributions due to individual modes are non-zero. The n = 6 mode is the dominant driver of transport and tends to flatten the current profile in the core. Fluctuations due to the other modes drive opposing current transport, however, and the sum over all modes from n = 5 to n = 8 largely cancels. Previous work has established $\langle \tilde{p}_{e,\parallel} \tilde{B}_r \rangle$ as a contributor to the RFP dynamo. Analysis of pressure anisotropy effects on electron momentum balance indicate that, in addition to $\tilde{p}_{e,\parallel}$, current transport due to $\tilde{p}_{e,\perp}$ is also critical to the kinetic dynamo. The kinetic dynamo is shown to depend on the difference $\tilde{p}_{e,\parallel} - \tilde{p}_{e,\perp}$ and cannot contribute to the RFP dynamo without anisotropy. Since the measurements presented here assume an isotropic temperature, the estimate of current transport is at best an upper bound.

Beam-driven instabilities

In Chapter 6, a new beam-driven instability was characterized in reversed discharges. This bursting mode is associated with frequency chirping, followed by propagation at only a few kHz above the plasma rotation frequency (7 kHz for the plasma and beam conditions used here). It participates in EPM avalanches, contributing to significant fast ion transport. Spontaneous momentum generation for higher n tearing modes occurs during a burst. The tearing mode amplitudes also change, with an overall increase although modes with n > 6 dip in amplitude before eventually rising. Possible impurity ion heating of 10 eV has also been observed. Electron temperature fluctuations correlated with this mode indicate a core-peaked structure with amplitude of 10-15 eV. This structure shifts inward with deepening reversal, indicating a sensitive dependence on the safety factor.

While no theoretical predictions are available to compare with the experimental results presented here, the observations afford a variety of speculative mode identifications. The strongest possibility is a pressure driven kink, with temporally evolving safety factor and fast ion beta over a sawtooth cycle bringing the discharge out of a non-resonant-kink unstable region of parameter space into a fishbone-unstable region. Alternatives include an Alfvén eigenmode (possibly a beta-induced eigenmode), or some type of reconnection event.

7.2 Suggestions for future work

The anticipated availability of a Fast Thomson scattering laser system with time resolution up to 250 kHz opens many possibilities for expanding the work presented here. The increase in number of sample points in a laser burst should improve statistics for faster convergence of correlated fluctuations—allowing both improved temporal resolution of dynamic behavior as well as correlations with lower amplitude modes (i.e. - tearing modes with $n \ge 9$ and the EPMs). Even without Fast Thomson scattering, though, several interesting topics are accessible with current diagnostic capabilities.

n = 6 behavior during NBI

In non-reversed discharges, the observed changes in the equilibrium current and safety factor profiles indicate that the n = 6 tearing mode lies in a region of reduced shear and enhanced current gradient. The increase in mode amplitude at the wall reported here appears consistent with this result, but the reduction in island width is not. Additionally, previously reported results in slightly different conditions suggest mode suppression. Multiple mechanisms may be in play, affecting both n = 5 and n = 6 activity. The behavior of the n = 6 mode is much better understood, exhibiting typical tearing parity, and Thomson scattering measurements can resolve

the full island width. Measurements of mode amplitude and island width across a variety of plasma and beam conditions may provide a better opportunity to diagnose mode behavior during neutral beam injection and compare to theoretical predictions.

3D equilibrium reconstruction with TS rational surfaces

Given the successful application of Thomson scattering rational surface measurements to axisymmetric MSTFit equilibria, the next step is diagnosis of three-dimensional equilibria. Significant progress has already been made adapting the VMEC/V3FIT codes to the RFP [2]. These compute fully three-dimensional equilibrium solutions to the general radial force balance equation: $\vec{J} \times \vec{B} = \nabla p$. Applications of these equilibrium solvers have focused primarily on Single-Helical Axis (SHAx) states in the RFP. Given the kink-like nature of the n = 5 mode, however, any MST discharge with n = 5 MHD activity may be inherently three-dimensional and not accurately described by axisymmetric equilibria. SHAx states generally lock, making Thomson scattering tearing mode island measurements significantly more challenging. Application of VMEC/V3FIT equilibria to fully diagnosed multi-helicity discharges with n = 5 activity may provide a better opportunity to identify helical perturbations of the magnetic axis.

Dynamo dependence on reversal parameter

Previous measurements have identified the q = 0 surface as critical for the Hall dynamo, with a removal of this surface from the plasma reducing non-linear mode coupling and the Hall contribution to current profile flattening. While non-linear coupling to m = 0 modes is not expected to play a role in the kinetic dynamo, non-reversed discharges exhibit temperature fluctuation driven contributions to the kinetic dynamo that have not been observed in reversed discharges. This may be compensation for reduced Hall dynamo emf. Dynamo activity in non-reversed discharges has not generally been studied in MST as extensively as in reversed discharges. The full behavior of the kinetic dynamo, including density fluctuation

driven terms, as well as the behavior of the MHD and Hall dynamos in non-reversed discharges may provide a valuable topic for future dynamo studies. For ultra-low-q plasmas, with q_0 less than one and 0 < q(a) < 1, the observed changes to the dynamo may have significant ramifications.

NBI and dynamo

Fast ions are observed to eliminate temperature fluctuation contributions to the kinetic dynamo. The amplitude of temperature fluctuations outside the remnant island structures is reduced, and the deviations of the fluctuation phase from \tilde{b}_{θ} are largely eliminated for radial locations where non-negligible fluctuation power is observed. Clear evidence of current profile modification has been obtained previously with polarimetry, and confirmed here. The measurements reported here are the first indication that NBI may modify the RFP dynamo, however. A large population of energetic ions may potentially impact all of the dynamo terms: the MHD dynamo through changes to the flow profile, the Hall dynamo through changes to the current profile, and the kinetic dynamo through pressure tensor terms. The impact of fast ions on the dynamo may also provide a unique opportunity not available on other devices.

Identification of low-frequency beam-driven instability

Many characteristics of the low-frequency, n = 5 bursting mode have been detailed here. The measurements still leave considerable leeway for speculation about mode identity, however. In addition to theoretical examination, several experimental observations are still required. Most critically, mode frequency scalings with beam energy, Alfvén velocity, plasma current and other parameters should provide a clearer distinction between energetic particle mode and Alfvén eignenmode predictions. Additionally, CHERS or IDS measurements of parallel and perpendicular impurity ion temperatures can clarify whether or not heating is isotropic. Poloidal flow measurements may also be necessary to fully understand the dynamic behavior during these bursts.

7.3 Final remarks

Looking back at the suggestions for future work listed in Ref. [1], it is encouraging to note that, in addition to other studies, the work presented here finally completes all of the recommendations (with the exception of Thomson scattering density measurements, which are being addressed in a systematic manner). Significant progress has been made during the four intervening years, with much more to look forward to in the near future. I can only hope that, in four more years, the same can be said of the recommendations made here.

7.4 References

- H. D. Stephens. *Electron Temperature Structures Associated With Magnetic Tearing Modes in the Madison Symmetric Torus*. PhD thesis, University of Wisconsin-Madison, 2010.
- [2] J. J. Koliner. *Neutral Beam Excitation of Alfvén Continua in the Madison Symmetric Torus Reversed Field Pinch*. PhD thesis, University of Wisconsin-Madison, 2013.
DISCARD THIS PAGE

Colophon

This thesis was typset in the Gyre Pagella font using pdfTeX and B_BT_EX. A modified version of the thesis class created by William Benton was used. This class was based on the memoir class and Eric Benedict's wi-thesis class, has also been modified by Steven Baumgart, and is available for download on the Memorial Library website. All plots, except where noted, were made in Python using the matplotlib package (created by John Hunter).