NONLINEAR MAGNETOHYDRODYNAMICS OF AC HELICITY INJECTION

by

Fatima Ebrahimi

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Physics)

at the

UNIVERSITY OF WISCONSIN-MADISON

2003

NONLINEAR MAGNETOHYDRODYNAMICS OF AC HELICITY INJECTION

Fatima Ebrahimi

Under the supervision of Professor Stewart C. Prager At the University of Wisconsin–Madison

AC magnetic helicity injection is a technique to sustain current in plasmas in which the current distribution relaxes by internal processes. The dissipation of magnetic helicity is balanced by magnetic helicity injected by oscillating the surface poloidal and toroidal loop voltages. The technique is considered for steady-state current sustainment in the reversed field pinch (RFP). The resulting current profile, and the accompanying magnetic fluctuations in these configurations are determined by 3-D MHD dynamics. We have completed a comprehensive 3-D MHD computational study of Oscillating Field Current Drive (OFCD), a form of AC helicity injection, in the RFP. Our results are compared with both 1-D computations and quasilinear analytical solution. The one-dimensional model provides a benchmark for comparison to the full 3-D plasma response. In a classical 1-D plasma, the oscillating voltages produce a steady current in the plasma, driven by the dynamo-like effect associated with the oscillating axisymmetric velocity and magnetic fields. This current is localized to the plasma edge region. With full 3-D dynamics, tearing fluctuations relax the plasma current toward the core, by the tearing mode dynamo, yielding a steady plasma current over the entire cross-section. The tearing fluctuations are comparable in magnitude to those that occur in standard RFP plasmas, although a global mode resonant at the edge occurs.

We have also studied current profile control by OFCD, as a separate application. We find that OFCD at appropriate frequency flattens the current density profile such that magnetic fluctuations are reduced. The current modification by OFCD is better understood when the effect of poloidal and toroidal oscillating electric fields are studied separately. We find that in OFCD, through the combination of poloidal and toroidal oscillating fields, a more favorable parallel electric field results which causes the reduction of magnetic fluctuations for most part of the cycle.

We have performed MHD simulations of a standard RFP at high Lundquist number up to $S = 5 \times 10^5$. Since in OFCD plasmas the axisymmetric oscillations decrease with S, using high Lundquist numbers is crucial for determining the viability of OFCD. It is also important for a more realistic picture of the MHD dynamics in the standard RFP. High-S computation elucidates the dynamics of sawtooth oscillations and the associated m=0 fluctuations. The effect of m=0 nonlinear mode coupling on the sawtooth oscillations is investigated by eliminating m=0 modes in the MHD computations. The sawtooth oscillations are not observed without m=0 modes. The m=0 mode is driven by the m=1 mode (the trigger for the sawtooth), leading to energy transfer from the m=1 mode to the m=0mode and a rapid decay of the m=1 amplitude (the sawtooth crash).

As the RFP moves toward improved confinement conditions and high-beta plasmas, finite pressure effects become important. We have performed a linear MHD stability analysis for the pressure-driven instabilities in conditions exceeding the Suydam limit. We found that the transition from the resistive to ideal pressure-driven modes occurs only at high beta values, several times the Suydam limit.

Acknowledgments

First and foremost I would like to thank my adviser Prof. Stewart Prager for his guidance and support throughout the research presented in this thesis. I also thank my committee members, Profs. Paul Terry, Cary Forest, Chris Hegna and, in particular, Carl Sovinec for his careful review of my dissertation and valuable comments on both computations and physics.

I thank John Wright for helping me get up to speed with the DEBS code and Dalton Schnack for making his expertise available. I would also like to thank the OFCD experimental staff: John Sarff, Karsten McCollam and Art Blair. John for the fruitful discussions about OFCD techniques and relaxed state modeling in the high-S limit and Karsten for the physics discussions about OFCD in the MST experiment. I thank Ben Hudson for providing me with the field line code. I also want to thank all of the MST group and the people at CPTC. I learned a great deal of plasma physics at the MST and CPTC group talks and seminars.

I dedicate this PhD thesis to my parents, Mohammadghasem Ebrahimi and Robabe Momeni, from whom I learned the value of hard work and persistence. I wish to thank my husband, Johan Carlsson, for supporting me over the past four years. Without him I couldn't have made it. I also want to acknowledge all the teachers, from high school to grad school, who gave me the inspiration and encouragement to pursue physics. Finally, thank you to all of my family and friends for understanding and supporting me.

Contents

1	Introduction			1
	1.1	1 AC magnetic helicity injection		3
	1.2	2 Overview of this Thesis		7
2	Hig	ligh Lundquist number MHD simulations of star	ndard RFP	13
	2.1	1 Introduction		13
	2.2	2 The DEBS code		15
	2.3	3 Equilibrium models		17
	2.4	4 Linear computation		19
		2.4.1 m=1 core tearing modes and m=0 modes $\ .$		22
		2.4.2 Resistive edge-resonant modes		24
	2.5	5 Dependence on Lundquist number		24
		2.5.1 Radial profiles		28
		2.5.2 Temporal nonlinear evolution		30
	2.6	6 Sawtooth oscillations and m=0 modes $\ldots \ldots$		34
		2.6.1 Calculations of linear magnetic energy for m	$=0 \mod \dots \dots$	39
		2.6.2 The dynamics in the absence of m=0 modes		41
	2.7	7 Summary		44
3	One	ne-dimensional classical response to the oscillati	ing fields	60
	3.1	1 Introduction		60
 3.2 One-dimensional computations		2 One-dimensional computations		60
			61	
	3.4	4 Parameter dependences		67
	3.5	5 Summary		70
4 Three-dimensional computation of AC helicity injection		ection	75	
	4.1	1 Introduction		75

	4.2	$S = 10^5$				
		4.2.1 The axisymmetric quantities	77			
		4.2.2 The cycle-averaged quantities	82			
		4.2.3 Temporal behavior during a cycle	84			
	4.3	$S = 5 \times 10^5$	87			
	4.4	The excitation of edge-resonant modes – linear and quasi-linear computations	93			
	4.5	Summary	102			
5	Cur	rent profile control by AC helicity injection	L06			
	5.1	Introduction	106			
	5.2	Time-averaged quantities	108			
	5.3	Time-dependence	111			
		5.3.1 Oscillating poloidal current drive (OPCD)	111			
		5.3.2 Oscillating toroidal current drive (OTCD)	113			
		5.3.3 The combination of the oscillating fields – OFCD	120			
	5.4	The frequency dependence	130			
	5.5	Summary	136			
6	Conclusions and future work					
	6.1	Conclusions	141			
	6.2	Future work	144			
$\mathbf{A}_{]}$	Appendix A					
$\mathbf{A}_{]}$	Appendix B					

1 Introduction

Magnetic helicity is a measure of the degree of structural complexity of the magnetic field lines in both laboratory and astrophysical plasmas. In the plasma within a magnetic flux surface, magnetic helicity characterizes the field line topology, and represents the linkage of lines of force with one another. It can be shown that the magnetic helicity is a measure of the knottedness of the magnetic field lines. Magnetic helicity decays on the resistive diffusion time. However, if helicity is created and injected into a plasma configuration, the additional linkage of the magnetic fluxes can sustain the configuration indefinitely against the resistive decay. Injection of magnetic helicity into the plasma is closely related to current drive. Thus, in a magnetically confined laboratory plasma, in order to drive current, magnetic helicity must be injected. Both conventional inductive ohmic current drive and non-inductive current drive techniques can be used to inject helicity into plasmas.

Inductive ohmic current drive technique is not steady-state. For steady-state reactor scenarios, inductive ohmic current drive alone is not sufficient. Various techniques such as DC and AC helicity injection can be used for steady-state current drive. Both DC and AC helicity injection techniques can sustain current in plasmas in which the current distribution relaxes by internal processes. In AC helicity injection, the magnetic helicity dissipation is balanced by magnetic helicity injection by oscillating the surface poloidal and toroidal loop voltages. Internal relaxation processes are expected to enable current penetration to the core. The technique is considered for current sustainment in the reversed field pinch (RFP), and similar helicity injection schemes are being studied for the spherical tokamak and spheromak. The resulting current profile, and the accompanying magnetic fluctuations are determined by nonlinear 3-D MHD dynamics. In this dissertation, we present a comprehensive 3-D MHD computational study of helicity injection in the RFP. Our objective for this research is to investigate the MHD dynamics of AC helicity injection for steady-state current drive and current profile control.

The reversed field pinch is a toroidal magnetized plasma characterized by relatively small magnetic field where both magnetic field components are comparable $(B_T \sim B_p)$. The direction of the toroidal magnetic field in the plasma edge is opposite to that in the core, making this configuration highly sheared. The safety factor $q = \frac{rB_T}{RB_p}$, is less than unity in the RFP, which cause nonlinear interaction of a large number of tearing resonant modes with poloidal mode number m=1 and relatively small radial spacing. The mode overlap can cause the magnetic field lines to wander chaotically, leading to rapid energy loss. In the standard inductive RFP, parallel electric field is small near the edge, and large in the core and has a steep gradient. As a result, the current profile is linearly unstable for current driven long wavelength tearing modes. The tearing mode amplitudes become large enough to relax the current from the core toward the edge through a fluctuation induced dynamo process. Therefore, current drive techniques which depend upon plasma relaxation process are more effective in this configuration.

The DC and AC helicity injection techniques rely upon magnetic fluctuations to relax the current density profile. In a plasma fully sustained by AC helicity injection, fluctuations are generated by the edge-driven current, that then generates current in the plasma core via the fluctuation induced MHD dynamo. Thus, the asymmetric tearing fluctuations transport the current from the plasma edge to the core region, opposite to standard ohmically sustained RFP.

In recent years, RFPs have advanced toward improved confinement conditions. Profile control, steady-state sustainment and single helicity state are the features currently being investigated both experimentally and theoretically. In conventional RFPs the energy confinement is limited by the resistive current-driven magnetic fluctuations which lead to enhanced transport. To improve confinement, parallel current profile control is required. Several profile control techniques have been developed to suppress magnetic fluctuations and transport. A surface poloidal electric field in addition to toroidal loop voltage programming have been experimentally applied to modify parallel current profile. [1, 2] The long wave-length core tearing modes have been substantially reduced through inductive current profile control and a dynamo free RFP configuration has been obtained. Tokamaklike energy confinement conditions have been achieved in the MST RFP experiment at low



Figure 1.1: Two interlinked flux tubes

toroidal magnetic field. [3] Non-inductive current drive techniques such as RF have potential for localized profile control and auxiliary heating, which have not been used in RFP to date. RF current drive techniques are also being tested experimentally. [4]

In addition to profile control, steady-state current sustainment is important and desirable for reactor type operations. One promising candidate for steady-state current sustainment is AC helicity injection which is computationally investigated in this thesis.

The inductive current profile techniques, non-inductive current drive and auxiliary heating, quasi-single helicity states, and steady-state current sustainment by AC helicity injection are the new features of RFP configuration being explored both experimentally and theoretically. The present research aims to provide understanding of AC helicity injection dynamics using MHD computation.

1.1 AC magnetic helicity injection

Magnetic helicity is a measure of the knottedness of the magnetic field lines, and is defined as $K_l = \int \mathbf{A} \cdot \mathbf{B} d\mathbf{v}_l$, where \mathbf{A} is the magnetic vector potential and the integral extends over the volume of a flux tube whose closed, bounding surface is s_l (flux tube is the volume swept out by all the field lines passing through a given closed curve l). [5] Helicity, a topological concept, represents the linkage of the magnetic field lines. Consider two flux tubes that follow two closed space curves l_1 and l_2 , with magnetic fluxes Φ_1 and Φ_2 , and volumes \mathbf{v}_1 and \mathbf{v}_2 . The flux tubes link each other once, as shown in Fig. 1.1. For the first flux tube, we can use $Bd\mathbf{v}_1 = B \cdot \hat{n}ds_1dl_1 = \Phi_1dl_1$, and helicity then becomes, $K_1 = \int \mathbf{A} \cdot \mathbf{B}d\mathbf{v}_l =$ $\Phi_1 \int \mathbf{A} \cdot dl_1 = \Phi_1\Phi_2$; similarly for the second flux tube, we obtain $K_2 = \Phi_1\Phi_2$. Thus, K_1 and K_2 measure the linkage of the two flux tubes. If the tubes are not interlinked, the line integrals would vanish, and if they link N times, we would get $K_1 = K_2 = - + N\Phi_1\Phi_2$, where the sign shows the right or left handed of the relative orientation. It can be shown that for an ideal MHD plasma, the integrals K_l are invariant for each flux surface.

However, in a plasma with small resistivity, under some conditions, total magnetic helicity over the plasma volume is approximately conserved and a specified class of solutions called Taylor states is obtained. After an initial unstable phase, a slightly resistive turbulent plasma inside a conducting boundary spontaneously relaxes to the minimum magnetic energy state subject to the constraint of conservation of total magnetic helicity (Taylor 1974). [6] In this particular model, it can be shown analytically that the magnetic helicity is closely related to the plasma current. Magnetic helicity, K, is defined as $K = \int \mathbf{A} \cdot \mathbf{B} d\mathbf{v}$, where the integral extends over the plasma volume. The relaxed Taylor state is obtained from the following equation,

$$\nabla \times \mathbf{B} = \lambda \mathbf{B},\tag{1.1}$$

where, $\lambda = J_{\parallel}/B$ is a constant. The cylindrical symmetric solution to Eq. 1.1 is the wellknown Bessel function solution; $B_z = B_0 J_0(\lambda r), B_\theta = B_0 J_1(\lambda r)$ It can be shown that the final relaxed state only depends on $\lambda \propto K/\phi_z^2$; thus the final state is completely determined by the two invariants magnetic helicity K and toroidal flux ϕ_z . We note that helicity closely relates to plasma current density through this ratio $\lambda \propto K/\phi_z^2$ obtained from the Taylor theorem. During the relaxation, the magnetic energy decays while the total magnetic helicity remains constant. This is because magnetic energy and helicity have different decay rates. The decay rates are $\dot{W} \sim -\eta \int \mathbf{J}^2 d\mathbf{v}$ and, $\dot{K} \sim -2\eta \int \mathbf{J} \cdot \mathbf{B} d\mathbf{v}$. The Fourier transformation of B (B_k) and J (kB_k) gives $\dot{W} \sim -\eta \sum k^2 B_k^2$ and, $\dot{K} \sim -2\eta \sum k B_k^2$ indicate that the high k small scale fluctuations tends to dissipate the magnetic energy faster than the helicity. The experimental measurement of helicity during relaxation has been examined in Ref. [7].

Under a gauge transformation $A \to A + \nabla \chi$, the change in the helicity K is $K \to K + \int \chi B \cdot ds$, which for the boundary conditions with $B_n \neq 0$, gauge invariance may be violated. To maintain gauge invariance for a toroidal plasma, helicity is redefined as,

$$K = \int \mathbf{A} \cdot \mathbf{B} d\mathbf{v} - \phi_p \phi_z, \qquad (1.2)$$

where, $\phi_z = \int A \cdot dl_{\theta}$ and $\phi_p = \int A \cdot dl_z$, and the line integrals are along the azimuthal and axial paths. The second term represents the linkage of toroidal flux within the plasma (ϕ_z) with poloidal flux (ϕ_p) that passes through the center of the torus. The second term is subtracted from the volume integral to maintain gauge invariance. [8, 9, 10] From Eq. 1.2, the rate of change of helicity for a resistive MHD plasma is

$$\frac{\partial K}{\partial t} = 2\phi_z v_z - 2\int \Phi \mathbf{B} \cdot d\mathbf{s} - 2\int \mathbf{E} \cdot \mathbf{B} d\mathbf{v}$$
(1.3)

where Φ is the electrostatic potential on the plasma surface and v_z is the toroidal loop voltage. Any technique to sustain the plasma current must also maintain helicity constant in time. In the usual toroidal induction, as in a tokamak, helicity dissipation is balanced by the DC toroidal loop voltage present in the first term on the right hand side. In DC electrostatic helicity injection helicity is maintained by the second term, which represents the intersection of a field line with a surface held at a constant electric potential.

In AC helicity injection the helicity is provided by oscillating fields in the first term. In steady-state,

$$\overline{\hat{\phi}_z \hat{v}_z} = \overline{\eta \int \mathbf{J} \cdot \mathbf{B} d\mathbf{v}}$$
(1.4)

where the over-bar denotes a time average over a cycle of the oscillating fields, $\hat{\phi}_z$ and \hat{v}_z (the "hat" denotes an oscillating quantity). The oscillation in the poloidal flux is provided by an oscillating surface toroidal loop voltage. Hence, if toroidal and poloidal surface voltages

are oscillated, with a 90 degree phase difference, then helicity is injected steadily, even in the absence of a DC loop voltage. This technique was suggested by Bevir and Gray [8] to sustain the current in an RFP. It has also been referred to as $F - \Theta$ pumping or oscillating field current drive (OFCD). In this thesis, we will use the acronym OFCD.

Both DC and AC helicity injection have been examined experimentally. Spheromaks have been formed by electrostatic helicity injections. [11] Electrostatic helicity injection has also been studied experimentally in spherical tokamaks [12, 13]. In both electrostatic and AC helicity injection, the core current penetration relies on relaxation process and is more effective in configurations close to relaxed Taylor states. However, electrostatic helicity injection has also been used for edge current drive and non-inductive startup current drive in spherical tokamaks. [14] OFCD has been examined in in the ZT40-M RFP and is being tested in the MST experiment. [15] The technique was shown to demonstrate a small amount of current (about 5% of the total) in the ZT40-M RFP [16], with a phase dependence in agreement with theory. However, plasma-wall interactions generated by the oscillating plasma position precluded tests with larger voltages.

Considerations of helicity balance provide little information on the dynamics of the current drive. A somewhat more complete view is obtained through examination of the effect of the applied voltages on the fields within the plasma, using the mean-field parallel (to the cycle-averaged mean magnetic field) Ohm's law,

$$\overline{\mathbf{E}_{\parallel}} + \overline{(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}} + \overline{\langle \widetilde{\mathbf{V}} \times \widetilde{\mathbf{B}} \rangle_{\parallel}} = \overline{\eta \mathbf{J}_{\parallel}}$$
(1.5)

where \mathbf{V}_{00} and \mathbf{B}_{00} are the oscillating velocity and magnetic fields with poloidal and toroidal mode numbers $\mathbf{m} = \mathbf{n} = 0$, $\mathbf{\tilde{V}}$ and $\mathbf{\tilde{B}}$ are the fields with \mathbf{m} , $\mathbf{n} \neq 0$, <> denotes an average over a magnetic surface, ()_{||} = () $\cdot \mathbf{\bar{B}}/B$, and $\mathbf{\bar{B}}$ is the cycle-averaged mean (0,0) magnetic field. The first term $\mathbf{\overline{E}}_{||}$ is the ohmic toroidal electric field which is zero for the full current sustainment by OFCD in the absence of a DC loop voltage. We see that there are two dynamo-like current drive terms on the left hand side, one arising from the one-dimensional oscillating fields that occur at the OFCD frequency (the second term) and one that arises from non-axisymmetric plasma fluctuations and instabilities (the third term). In the absence of fluctuations (neglecting the third term) a current is driven by the symmetric oscillating fields. The oscillating radial velocity combines with the oscillating magnetic field to produce a DC current. This current is confined to within a classical resistive skin depth near the plasma surface, and decays to zero at the plasma center. It is a classical effect, although one that is absent in a plasma without flow. Considering that $\mathbf{V}_{00} = \mathbf{E}_{00} \times \mathbf{B}/B^2$, the first two terms can also be combined and written as $\overline{(\mathbf{E}_{00} \cdot \mathbf{B}_{00})}/B$. Hence, we can consider the first two terms on the LHS of Eq. 1.5 as a time-averaged parallel component of electric field which has both (AC) oscillating and DC components. The fluctuation induced dynamo term (the third term on the LHS) transports the OFCD-driven edge current into the plasma core. For the partial current sustainment by OFCD, the cycle-averaged parallel current density is sustained by all the three term on the LHS. However, the two dynamo terms from the axisymmetric oscillations and the asymmetric fluctuations can steadily sustain the plasma current through AC helicity injection in the absence of an ohmic DC loop voltage i.e. full sustainment by OFCD.

1.2 Overview of this Thesis

The objective for the research presented in this thesis is to understand the MHD dynamics of AC helicity injection for steady-state current drive and current profile control in RFP. We have investigated the full nonlinear dynamics of OFCD, using 3-D nonlinear MHD computation. We have employed 3-D nonlinear MHD computations, the DEBS code, to study the dynamics of AC helicity injection for both steady-state current sustainment and for controlling the current profile.

MHD computations at high Lundquist number provide more regular and pronounced oscillations similar to the experimental observations of sawtooth oscillations. Chapter 2 provides 3-D MHD simulations of a standard RFP at high Lundquist number up to $S = 5 \times 10^5$. The goals are to examine the effects of high Lundquist numbers and to provide a benchmark for OFCD plasmas. The dynamics of sawtooth oscillations and the associated m=0

magnetic fluctuations can also be studied using high-S computations. The code description is presented in this chapter. The linear computations of both core tearing modes and edge-resonant modes are also investigated. Simplified linear computations show the localized radial tearing mode structure around the resonant surface as S is increased (shown up to $S = 10^{6}$). Because edge-resonant modes, resonant outside the reversal surface, can be excited in 3-D full current sustainment by OFCD, these modes are particularly discussed, and the linear S-scaling of these modes is presented. The dependence of the radial profiles and the magnetic fluctuations on S are also examined. The effect of m=0 nonlinear mode coupling on the sawtooth oscillations is investigated by eliminating m=0 modes in the MHD computations. It is shown that the sawtooth oscillations are not observed without m=0 modes and the transfer of energy from m=1 modes to m=0 modes through the dynamo relaxation (the sawtooth crash phase) does not occur.

The classical OFCD effect, which occurs in the absence of fluctuations, is calculated in chapter 3, both through 1-D computation and analytic quasilinear calculation. This calculation provides a benchmark to which the additive effect of the fluctuations can be compared. In a 1-D classical plasma, OFCD generates a steady-state current confined to within a resistive skin depth of the plasma surface. The current is generated by the cycleaveraged dynamo-like $(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}$ effect from the axisymmetric velocity and magnetic field oscillations. We also find that, at large amplitude of the oscillating voltages, transient fields are generated that persist for about a resistive diffusion time.

We employ 3-D, resistive MHD computation to study the nonlinear dynamics of OFCD. This permits us to address two key questions: what is the effectiveness of OFCD as a current drive technique and what is its effect on plasma fluctuations? The full 3-D results of full current sustainment by OFCD are presented in chapter 4 for Lundquist numbers of 10^5 and 5×10^5 . Investigation of the cycle-averaged quantities reveals that the plasma current (and helicity) can indeed be sustained by OFCD. Examination of the surface-averaged quantities throughout a cycle indicates that the plasma current oscillates substantially, although the magnitude of the oscillation decreases with Lundquist number. Plasma fluctuations increase

significantly with OFCD; however the increase is concentrated mainly in a global mode that is nearly ideal (resonant at the extreme plasma edge). The core-resonant tearing modes are not increased significantly.

In recent years, improved reversed field pinch (RFP) confinement conditions have been achieved through inductive current profile control using surface electric fields. In chapter 5, we investigate AC helicity injection as an alternative technique for partial current drive and current profile control. We present 3-D MHD simulations of AC helicity injection demonstrating both partial current sustainment and significant shaping of ohmic current profile. It is shown that tearing fluctuations are reduced with the modification of the current profile. The detailed MHD dynamics including both the cycle-averaged quantities and the temporal variations of axisymmetric fields and asymmetric fluctuations during a cycle are studied. The current modification by OFCD is better understood when the effect of poloidal and toroidal oscillating electric fields are studied separately. The detailed dynamics of oscillating poloidal electric field (OPCD) in which only poloidal electric field is oscillated and oscillating toroidal electric field (OTCD) in which only toroidal electric field is oscillated are also studied. The optimal driving frequency range for effective current relaxation with low modulation amplitudes is also discussed.

The current-driven tearing modes are typically the dominant instabilities in the RFP core region. However, as present RFP experiments can operate at high beta using auxiliary heating techniques and current profile control in the improved confinement regimes, pressure-driven instabilities are expected to increase and the stability limit become important. In Appendix B, the linear MHD stability of local and global resistive pressure-driven instabilities is examined computationally in a cylinder. We find two results. First, the high-k localized interchange is resistive (in growth rate and radial structure) at beta values up to several times the Suydam limit, transitioning to an ideal mode at extremely high beta. Only at very high beta values is the mode ideal in its radial structure and its growth rate (which becomes independent of S). No sudden changes in growth rate occur at the Suydam limit. Second, we find that global pressure-driven modes (of tearing spatial parity)

are equally unstable and have a similar transition from resistive to ideal as beta increases. Since the localized modes are more subject to stabilization mechanisms beyond MHD (such as finite Larmor radius stabilization), the global modes will likely be more influential in RFPs at high beta.

References

- J. S. Sarff, S. A. Hokin, H. Ji, S. C. Prager, and C. R. Sovinec, Phys. Rev. Lett. 72, (1994).
- [2] B. E. Chapman, A. F. Almagri, J. K. Anderson *et al.*, Phys. of Plasmas, 9, 2061 (2002); B. E. Chapman, J. K. Anderson, T. M. Biewer, *et al.*, Phys. Rev. Lett, 87, 205001-1 (2001)
- [3] J. S. Sarff *et al.*, Nuclear Fusion 2002.
- [4] C. B. Forest *et al.*, Phys. Plasmas **7**, 1352 (2000).
- [5] H. K. Moffatt, Magnetic Field Generation in Electrically Conducting Fluids, 1978
- [6] J. B. Taylor, Phys. Rev. Lett. **33**, 1139 (1974).
- [7] H. Ji, S. C. Prager and J. S. Sarff, Phys. Rev. Lett. 74, 2945 (1995)
- [8] M. K. Bevir and J. W. Gray, in Proceedings of the Reversed-Field Pinch Theory Workshop, edited by H. R. Lewis and R. A. Gerwin (Los Alamos Scientific Laboratory, Los Alamos, NM, 1981), Vol. III, p. A-3.
- [9] T. H. Jensen and M. S. Chu, Phys. Fluids 27, 2881 (1984).
- [10] M. K. Bevir, C. G. Gimblett and G. Miller, Phys. Fluids 28, 1826 (1985).
- [11] T. R. Jarboe, I. Henins, A. R. Sherwood, C. W. Barnes, and H. W. Hoida, Phys. Rev. Lett. 51, 39 (1983).
- [12] M. Ono, G. J. Greene, D. Darrow, C. Forest, H. Park, and T. H. Stix, Phys. Rev. Lett. 59, 2165 (1987).
- [13] T. R. Jarboe, M. A. Bohnet, A. T. Mattick, B. A. Nelson, and D. J. Orvis, Phys. Plasmas 5, 1807 (1998).
- [14] S. Kaye, M. Ono, Y. K. M. Peng et al., Fusion Technol. 36, 16 (1999).

- [15] K. Mccollam et al. and A. Blair et al., APS poster presentation, 2002
- [16] K. F. Schoenberg, J. C. Ingraham, C. P. Munson et al., Phys. Fluids **31**, 2285 (1988).
- [17] B. A. Nelson, T. R. Jarboe, D. J. Orvis, L. A. McCullough, J. Xie, C. Zhang, and L. Zhou, Phys. Rev. Lett. 72, 3666 (1994).
- [18] L. Marrelli, P. Martin, G. Spizzo et al., Phys. of Plasmas, 9, 2868 (2002)

2 High Lundquist number MHD simulations of standard RFP

2.1 Introduction

In the past two decades, numerical simulations within the framework of the resistive MHD model have successfully demonstrated the RFP dynamo effect and the characteristics of the magnetic fluctuations. Most of the past MHD simulations have been performed at Lundquist numbers, S, about two order of magnitude lower than the values of the operating experiments, limited by computer speed and memory. A more realistic picture of the RFP dynamics requires computations at parameters closer to experimental values. The extended MHD models with two-fluid and kinetic closures need to be explored numerically for an even more detailed picture of experimental observations. MHD computations at high Lundquist number provides more regular and pronounced field reversal oscillations similar to the experimental observations of sawtooth oscillations. In this section, the result of nonlinear MHD computations at more realistic Lundquist number, close to the experiment are presented. The goals are to examine the effects of high Lundquist numbers and to provide a benchmark with which to compare plasmas with OFCD.

The Lundquist number scaling of fluctuations in conventional RFP has been explored both experimentally and numerically. Experimental scaling of standard RFP fluctuations with Lundquist number up to 10^6 indicated a weak dependence on S (Stoneking 1998). [1] A computational study by Cappello and Biskamp obtained a magnetic fluctuation scaling of $S^{-0.22}$ in the range $3 \times 10^3 \leq S \leq 10^5$. [2] Sovinec studied the Lundquist number scaling of the magnetic fluctuation level using 3-D MHD computations without plasma pressure. [3] A weak scaling of $S^{-0.18}$ for the total magnetic fluctuation level (rms of the total volume averaged magnetic field including all poloidal and toroidal modes) for S from 2.5×10^3 to 4×10^4 was obtained. A more recent numerical study of the confinement scaling with finite pressure effects indicates that the magnetic fluctuation level remains high at Lundquist number up to $S = 7 \times 10^5$ (Scheffel and Schnack 2000). [4] These simulations were performed at low aspect ratio R/a=1.25 and while the scaling laws were presented, the detailed dynamics and the radial profiles at high S were not shown. It is worth mentioning that all the past experimental and numerical S scaling have been obtained for the standard RFP. The S scaling for RFP plasmas with improved confinement conditions using current profile control which might result a strong S dependence, remains for future investigations.

Here, we present the temporal evolution and radial profiles of both axisymmetric and asymmetric quantities at Lundquist number up to $S = 5 \times 10^5$ and aspect ratio R/a=2.88 at zero pressure. Because of the need for high temporal and spatial resolutions for high S computations, these computations are numerically challenging and require both a large amount of CPU time and memory. The results presented here agree with the previous study by Sovinec at lower S ($S \le 4 \times 10^4$), but here high S computations show more regular temporal behavior of magnetic fluctuations and reversal parameter similar to the sawtooth crashes observed experimentally.

Some of the experimental observations such as sawtooth oscillations and m=0 bursts are not fully understood and require both analytical and computational studies. The observation of sawtooth oscillations at high S computations presented here enable studying and understanding of the sawteeth dynamics. Here, we study the behavior of sawtooth oscillations regarding with m=0 fluctuations. We find that m=0 modes have significant effect on the sawtooth oscillations.

The nonlinear 3-D resistive MHD code, DEBS, is described in Sec. 2.2. In Sec. 2.3, the equilibrium models used both for linear stability analysis and the nonlinear simulations throughout this thesis are reviewed. The linear radial structure of the tearing modes obtained from the linear computations are shown in Sec. 2.4. The high-S nonlinear MHD computations are discussed in Secs. 2.5 and 2.6. The magnetic fluctuation dependence on Lundquist number, including the detailed radial profile variations with S and the temporal evolution, are presented in Sec. 2.5. The detailed dynamics during sawtooth oscillations, such as m=0 fluctuations, at high-S are discussed in Sec. 2.6. The sawtooth oscillations associated with the plasma relaxation and dynamo activity are illustrated. The linear and total magnetic energy for m=0 modes are calculated in Sec. 2.6.1 and the energy drive for

the growth of m=0 mode is shown for standard plasma. To understand the dynamics of sawtooth oscillations, the m=0 modes are artificially removed from the computations. The dynamics in the absence of m=0 modes and nonlinear m=0 mode coupling are examined in Sec. 2.6.2.

2.2 The DEBS code

Resistive MHD instabilities are important in the analysis of the nonlinear dynamics of fusion plasmas; examples include the reconnection dynamics in tokamaks through the growth and saturation of resistive modes and the tearing dynamo relaxation in RFPs and spheromaks. However, these instabilities evolve on times scales that are long compared to ideal time scales (e.g., fast compressional and shear Alfvén). Therefore, the simulation of phenomena governed by these low frequency and long wavelength dynamics is difficult and requires algorithms that eliminate the rapid ideal time scales. The explicit algorithms are restricted by stability limits associated with wave propagation (small time steps) and are not suitable for studying the nonlinear evolution of resistive MHD modes. Incompressible models have been used to remove fast compressional (Aydemir and Barnes, 1984), [5] and larger time steps are possible in these models. However, some important physics may be eliminated in these models, for example the incompressible codes, Aydemir et.al. showed that compressibility is an important feature of RFP physics and the symmetric radial pinch flow $V_r(r)$ can be important in the RFP dynamo effect and field reversal sustainment. [6]

Implicit schemes which allow time step larger than the compressional time scale, are more complicated to implement and require the solution of large block matrix equations (Aydemir and Barnes, 1985). [7] Using implicit algorithms on nonlinear equations leads to a nonlinear system, and direct solvers are not applicable, an iterative solver has to be used. Recently, fully implicit, nonlinear time differencing of the resistive MHD equations have been explored using a 2-D reduced viscous-resistive MHD model, supporting shear Alfvén and sound waves(Chacon et al. 2002). [8] Newton-Raphson iterative algorithm and Krylov iterative techniques has been used for the implicit time integration and the required algebraic matrix inversions. A physics-based preconditioning has also been employed for the efficiency of the Krylov method. The implicit algorithm allows time steps much larger than the explicit stability limit. In 2-D reduced MHD, the magnetic field component in the ignorable direction B_z is much larger than the magnitude of the poloidal magnetic field B_p ($B_z \sim$ constant and the poloidal velocity is incompressible). Thus, the reduced MHD model is limited to configurations like tokamaks and is not applicable for RFPs in which magnitudes of the poloidal and toroidal magnetic fields are comparable. The 3-D version of the fully implicit scheme is under development (Chacon et al.)

The semi-implicit algorithm used in DEBS [9] and NIMROD [10] for long time simulations, allow time steps larger than the explicit stability limit by eliminating both fast compressional and shear Alfvén waves time restrictions. Fully implicit treatment of the nonlinear convolution terms, $(V \times B)_{m,n}$ and $(J \times B)_{m,n}$, result in a coupling of all Fourier coefficients and requires the inversion of large matrices. In the semi-implicit method, a linear MHD term (semi-implicit operator) is added to the original momentum equation to relax the stability limit. In this method, only the dissipation terms are treated implicitly. Since in the semi-implicit schemes, only part of the equations is integrated implicitly at a given time step, this method requires less work than a full implicit integration. The combination of leapfrog and predictor-corrector methods are used for time discretization of the wave-like terms and advective terms, respectively.

We have employed the 3-D resistive MHD code, DEBS, to study the nonlinear dynamics in the RFP both for standard and OFCD plasmas. The DEBS code solves the compressible nonlinear resistive MHD equations in periodic cylindrical geometry. The code also evolves the energy equation and can be used for finite pressure studies. The pressure equation is not included in most of the computations, except in Appendix B where we study the linear pressure-driven instabilities using the adiabatic pressure equation. The set of the resistive MHD equations evolved in the code are,

$$\begin{aligned} \frac{\partial \mathbf{A}}{\partial t} &= S\mathbf{V} \times \mathbf{B} - \eta \mathbf{J} \\ \rho \frac{\partial \mathbf{V}}{\partial t} &= -S\rho \mathbf{V} \cdot \nabla \mathbf{V} + S\mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{V} - S \frac{\beta_0}{2} \nabla P \\ \frac{\partial P}{\partial t} &= -S\nabla \cdot (PV) - S(\gamma - 1)P\nabla \cdot \mathbf{V} + 2 \frac{(\gamma - 1)}{\beta_0} \eta J^2 - \nabla \cdot \mathbf{q} \end{aligned}$$
(2.1)
$$\mathbf{B} &= \nabla \times \mathbf{A} \\ \mathbf{J} &= \nabla \times \mathbf{B} \end{aligned}$$

where time and radius are normalized to the resistive diffusion time $\tau_R = 4\pi a^2/c^2\eta_0$ and the minor radius a, respectively, velocity to the Alfvén velocity V_A , and magnetic field **B** to the magnetic field on axis B_0 . $S = \frac{\tau_R}{\tau_A}$ is the Lundquist number (where $\tau_A = a/V_A$), and ν is the viscosity coefficient, which measures the ratio of characteristic viscosity to resistivity (the magnetic Prandtl number). $\beta_0 = \frac{8\pi P_0}{B_0^2}$ is the initial β on axis. Table 1 summarizes the normalizations used in the code. The mass density ρ is assumed to be uniform in space and time. The resistivity profile has been chosen to resemble the experimental profiles (increasing near the plasma edge), $\eta = (1 + 9(r/a)^{20})^2$. The vector potential is advanced directly and magnetic field and current are then calculated. The time advance is a combination of the the Leapfrog and semi-implicit methods. The code uses a finite difference method for radial discretization and pseudospectral method for periodic azimuthal and axial coordinates. Both fast compressional ($\tau = \frac{a}{V_A}$) and slow shear Alfvén modes ($\tau = \frac{R}{V_A}$) can be resolved by this code. However, the semi-implicit method allows large time steps and elimination of the Alfvén modes.

2.3 Equilibrium models

The equilibrium force balance equation $(J \times B = \nabla p)$ and Ampere's law can be combined to obtain the dimensionless equilibrium equation

$$\nabla \times \mathbf{B} = \lambda(r)\mathbf{B} + \beta_0 \frac{\mathbf{B} \times \nabla p(r)}{2B^2}$$
(2.2)

Normalized quantity	Units
B, magnetic field	B_0 (gauss) initial toroidal field (r=0)
r, length	a (cm) minor radius
t, time	$\tau_R = 4\pi a^2/c^2 \eta_0$ (s) (resistive diffusion time)
E, electric field	$E_0 = \mathrm{a}B_0/\mathrm{c}\tau_R \; (\mathrm{[V]/cm})$
V, voltage	$V_0 = E_0 \mathbf{a} ([\mathbf{V}])$
ρ , mass density	$ \rho_0 (g.cm^{-3}) $ (initial density on axis)
P, pressure	$P_0 \ (\mathrm{erg.} cm^{-3})$
	(initial pressure on axis)
T, temperature	$T_0 = P_0/m_i n_0 \mathbf{k}$ (kev) (initial temperature on axis)
V, velocity	$V_{A0} = B_0 / \sqrt{4\pi\rho_0} \; (\mathrm{cm.}S^{-1}) \; (\mathrm{Alfvén \; velocity})$
$\mathrm{S} = \tau_R / \tau_A$	$\tau_R V_{A0}/a$ (Lundquist number)
$P_m = \nu/\eta$	$\nu_0 \tau_R / a^2$ (Prandtl number)
β_0	$\beta_0 = \frac{8\pi P_0}{B_0^2}$ (initial beta)

Table 1: The normalization of the fields and quantities used in the DEBS code.

where $\lambda(r) = \mathbf{J} \cdot \mathbf{B}/B^2$. This equation is written in terms of parallel and perpendicular components ($\nabla \times B = J_{\parallel} + J_{\perp}$). Equation (2.2) yields equilibrium magnetic field profiles close to experimental equilibrium profiles by allowing the λ profile to vary with radius and including a finite pressure gradient. In the limit of small β , ∇p can be neglected in the equilibrium force balance equation and current flows parallel to the magnetic field line $(J = \lambda B)$ or,

$$\nabla \times \mathbf{B} = \lambda(r)\mathbf{B} \tag{2.3}$$

Equation (2.3) presents the force free model. The θ and z components of Eq. 2.2 are

$$\frac{dB_{\theta}}{dr} = \lambda B_z - \frac{B_{\theta}p'}{B^2} - B_{\theta}/r$$

$$\frac{dB_z}{dr} = -\lambda B_{\theta} - \frac{B_z p'}{B^2}$$
(2.4)

In the alpha equilibrium model, the parallel current profile and pressure profile are given as $\lambda(r) = J \cdot B/B^2 = 2\theta_0(1 - r^{\alpha})$ and $p(r) = p_0(1 - p_1r^{\delta})$ respectively, where $\alpha, \theta_0, \delta, p_0$ and p_1 are free constants. Other equilibrium quantities can be computed from Eq. 2.4. The typical equilibrium profiles from this model are shown in Fig. 2.1. In this model the free parameters can be chosen to obtain equilibrium profiles very close to the RFP profiles with toroidal field reversal. Further, by varying these free parameters, stable and unstable equilibrium profiles are found for resistive current-driven and pressure-driven instability analysis. The pressure term could also be ignored for the current-driven instabilities. The $(\alpha - \Theta_0)$ stability diagram obtained in the past (Antoni et al. 1986) [11] makes this model convenient to use for stability analysis based on Δ' theory. Throughout this thesis, we use the alpha model for linear stability analysis when needed.

Another equilibrium model is the paramagnetic equilibrium model commonly used as an initial equilibrium for nonlinear RFP simulations. In steady state, there is a uniform electric field in the z direction $(E_{\theta} \sim 0)$ and using parallel Ohm's law $E_{\parallel} = \eta J_{\parallel}$, λ is obtained

$$\lambda(r) = E_0 B_z / (\eta B^2) \tag{2.5}$$

Equations (2.5) and (2.4) can be solved to obtain the paramagnetic equilibrium fields. The equilibrium magnetic field profiles obtained from the paramagnetic model are shown in Fig. 2.2. Although, this model gives rise to equilibrium profiles close to RFP profiles, it does not produce toroidal field reversal.

The modified Bessel function model (MBFM) can also be used as an equilibrium model. In this model, $\lambda(r)$ profile is constant to a break radius, r_b , $(\lambda(r) = \lambda_0 \text{ for } r \leq r_b)$ and falls linearly to zero $(\lambda(r) = \lambda_0(1-r)/(1-r_b)$ for $r > r_b)$. The λ profile from MBFM is used in Eq. 2.4 to yield the equilibrium profiles.

2.4 Linear computation

The linear stability of current-driven and pressure-driven instabilities is numerically studied using the DEBS code in the linear regime. The ideal and resistive pressure-driven instabilities will be discussed in Appendix B. Here, the radial profile features of the linear tearing instability are presented. For almost every nonlinear 3-D computation of standard or OFCD



Figure 2.1: Equilibrium magnetic field and pressure profiles (B_Z, B_θ, p) from the alpha model.

plasmas presented in this thesis, we have performed the linear stability analysis using single mode computation. We have also studied the quasilinear effects by allowing the equilibrium quantities to evolve. The linear and quasilinear studies are important to understand the 3-D nonlinear behavior in the presence of all the tearing modes.

The free energy from the plasma current (or pressure) gradient can give rise to MHD instabilities. The ideal MHD theory provides a thorough description of the plasma equilibrium and stability in the limit of zero resistivity. In ideal MHD, the magnetic field lines are frozen to the fluid and the solutions may become singular at the resonance surface $(K \cdot B = 0$, where K is the wave vector). The ideal stability limit can be determined using the



Figure 2.2: Paramagnetic equilibrium model (a) B_z and B_θ magnetic fields, (b) q(r) and $\lambda(r)$ profiles.

energy principle (based on the loss or gain of potential energy of the plasma). [27] The ideal current-driven instability (kink modes) and pressure-driven instabilities (interchange modes) grow on a fast time scale (the Alfvenic time scale) and can cause plasma disruption. The inclusion of a small resistivity (or dissipation) in the plasma introduces another class of the instabilities called resistive instabilities. The resistive instabilities grow on a time scale much slower than the Alfvén time (τ_A) and much faster than the resistive diffusion time (τ_R). The small resistivity allows the magnetic field lines to break and reconnect. The singularity at the resonant surface is removed in resistive MHD. The Faraday equation and Ohm's law can be combined to give the following equation for the reconnected component of magnetic field (in the r direction in cylindrical geometry) $\gamma \tilde{B}_r = i \frac{B_{\theta}}{r} (m - nq) \tilde{V}_r + \eta \nabla^2 (r \tilde{B}_r)/r$, where at the resonant surface q=m/n (F = K · B = 0). In the ideal limit ($\eta = 0$) $\tilde{V}_r = -i\gamma \tilde{B}_r/F$. At the resonance surface, F=0, so that for V_r to be well-behaved, B_r must vanish and reconnection can not occur. Resistivity clearly becomes important around the resonant surface. With the addition of resistivity at the resonance surface (F=0), reconnection can occur.

Linear resistive MHD stability has been studied both numerically and analytically. Furth et al. [12] classified the resistive MHD instabilities into the tearing modes cause by the current gradient, gravitational interchange modes (g-modes) cause by the pressure gradient in the bad curvature region and the rippling modes caused by the resistivity gradient. The growth rate of resistive modes can be calculated by matching the solutions in the outer regions (ideal regions, $\eta = 0$) to that in the inner resistive layer. Equating $\Delta' = (B'_r|_{r_s^+} - B'_r|_{r_s^-})/B_r|_{r_s}$, the jump in logarithmic derivative of B_r across the resistive layer, with Δ'_{in} gives the growth rate for tearing modes, $\gamma_{\text{tearing}} \propto S^{-3/5} \Delta'^{4/5}$. The mode is unstable if $\Delta' > 0$. Δ' is a measure of the magnetic energy to be gained by the perturbed magnetic field at the resonant surface. The S-scaling of the growth rates of resistive MHD modes using linear analytical calculations are given as, $\gamma_{\text{tearing}} \propto S^{-3/5}$, $\gamma_{\text{g-mode}} \propto S^{-1/3}$ and $\gamma_{\text{rippling}} \propto S^{-3/5}$. The linear numerical calculations of growth rates of resistive MHD instabilities (by several authors) yield the same asymptotic S-scaling.

2.4.1 m=1 core tearing modes and m=0 modes

Here, we examine the radial mode structure of the tearing modes resonant in the core region using linear computations, to compare later with the nonlinear radial structure. In these linear computations, the alpha equilibrium model (see Sec. 2.3) has been used with $\alpha = 3$ and $\lambda_0 = 3.2$. Equilibrium profiles that are unstable for tearing modes have been chosen according to the linear Δ' theory in RFPs ($\Delta' > 0$ for instability) reported by Antoni *et al.* [11] The equilibrium profiles are fixed during the evolution of a single mode. Here, we denote the core tearing modes, resonant inside the reversal surface, with negative axial mode number ($k_z = n/R < 0$) and the edge-resonant modes, resonant outside the reversal surface, with positive axial mode number ($k_z = n/R > 0$). The m=0 modes are resonant at the reversal surface. We find that the radial mode structure around the resonant surface become more localized as the Lundquist number increases.

Figure 2.3(a) shows the linear radial profile of the reconnected magnetic field, B_r , and the radial flow velocity, V_r , at $S = 10^4$ for the m=1, $k_z = -1.8$ tearing mode in cylindrical geometry. The resonant location is shown with a vertical line. As is seen, the mode has tearing parity. The growth rate for this mode is $\gamma \tau_A = 0.0165$ and $\Delta' = (B'_r)_{r_e^+} - C_r^+$ $B'_r|_{r_s})/B_r|_{r_s}$ calculated from the unstable eigenfunction (B_r) is positive $(\Delta' = +0.82)$, where r_s is the resonant surface radius. The magnitude of B_r is nonzero at the resonant surface indicating a reconnecting resistive mode. The eigenfunctions for $S = 10^6$ are shown in Fig. 2.3(b). It is seen that the radial velocity is more localized at higher S. This is because at high S the plasma gets close to the ideal regime (with singular solutions at the resonant surface), and the radial structure of the modes become more localized around the resonant surface. The resistive layer width equation, $\delta = \left[\frac{\rho\eta\gamma r^2}{B_a^2 n^2 q'^2}\right]^{1/4}$, obtained from the linear theory also indicates the reduction of δ with inverse resistivity. For the tearing modes $(\gamma \propto S^{-3/5})$, the resistive layer width S-scaling is obtained as $\delta \propto S^{-2/5}$. Thus, the resistive layer becomes narrow at higher S, as seen from the numerical eigenfunctions. The growth rate at $S = 10^6$ is $\gamma \tau_A = 0.74 \times 10^{-3}$ as expected from linear S-scaling for tearing modes $(\gamma \propto S^{-3/5})$. The magnetic Prandtl number, P, used in the linear computations is of the order of unity. However, here for comparison with the nonlinear eigenfunctions, P = 10 is used for the linear tearing mode at $S = 10^6$ (Fig. 2.3(b)).

The single mode dynamo terms $\langle \tilde{V} \times \tilde{B} \rangle_{\parallel}$ are shown in Fig. 2.4. More local features around the resonant surface is seen at $S = 10^6$. Using linear computations, we have confirmed that the linear radial mode structure becomes more localized as S increases. Experimental measurement of the dynamo term show a global total dynamo effect which may arise from the superposition of the single mode dynamo terms (Fontana *et al.* 2000). [13] Nonlinear dynamo mode structure is discussed further later.

Tearing modes resonant at the reversal surface are the m=0 modes and contribute to the nonlinear fluctuation induced dynamo in the edge region (will be shown in Sec. 2.5). The tearing m=0 modes are linearly stable in a plasma surrounded by a perfectly conducting wall. As we will discuss later, m=0 is nonlinearly driven in the nonlinear simulations.

Here, we use a resistive wall with a constant resistive time scale τ_{wall} placed at r = a and the perfectly conducting wall placed at r > a as the boundary condition to drive m=0 mode linearly unstable. Figure 2.5(a) shows the eigenfunctions for m=0, $k_z = 0.3$ mode at $S = 10^4$. The radial flow velocity has odd parity and is confined near the edge region. The linear m=0 dynamo term for this mode is also shown in Fig.2.5(b).

2.4.2 Resistive edge-resonant modes

Tearing modes resonant outside the reversal surface, edge-resonant modes, can also become unstable in RFPs. However, the amplitudes of short wavelength edge-resonant modes are generally small in the edge region of standard plasmas. With the application of surface inductive electric fields, the oscillations of the axisymmetric field can be large and longer wavelength edge-resonant modes may be excited. As will be shown in chapter 4, the current sustainment of plasma current by OFCD causes the fluctuation amplitudes to increase mainly because of the excitation of the long wavelength edge-resonant modes (m=1, n=+2) in low-S plasmas.

Here, we examine resistive edge resonant modes under extremely unstable equilibrium conditions. The equilibrium profiles obtained from the alpha model with $\alpha = 4$. and $\theta_0 = 2$ are shown in Fig. 2.6. The eigenfunctions for m=1, n=+6 are also shown in Fig. 2.6. The magnitude of B_r is nonzero at the resonant surface and the flow velocity is localized around the resonant surface. These modes have also the same S-scaling as the core tearing modes. We have also performed the S-scaling for the resistive edge-resonant modes and obtained $\gamma \propto S^{-3/5}$ asymptotic scaling. The result is shown in Fig. 4.25.

2.5 Dependence on Lundquist number

In the previous section, the linear MHD computations for tearing modes were examined. Here, the nonlinear high S MHD computations are presented. The radial profile variations with S, magnetic fluctuations dependence on S, and the temporal behavior of axisymmetric and asymmetric quantities are investigated. The computations are started with a specified



Figure 2.3: The linear tearing mode structure obtained from the linear computations for (a) $S = 10^4$, (b) $S = 10^6$ (m=1, $k_z = -1.8$).



Figure 2.4: The single tearing mode dynamo term $\langle \tilde{V} \times \tilde{B} \rangle_{\parallel}$ obtained from the linear computations for (a) $S = 10^4$, (b) $S = 10^6$ [m=1, $k_z = -1.8$ (n=6)]. The vertical line denotes the location of the resonant surface.



Figure 2.5: The linear mode structure for m=0, $k_z = 0.3$ (n=1) mode obtained from the linear computations at $S = 10^4$. (a) the eigenfunctions (b) the linear dynamo term.

time-independent axial electric field at the wall, $E_z(r = 1)$. The boundary condition on $E_z(r = 1)$ can be a fixed value or such that the pinch parameter Θ is kept constant. The paramagnetic equilibrium is used. In these simulations, the nonlinear resistive MHD equations are evolved with nonzero asymmetric fluctuations which affect the axisymmetric profiles. The parallel electric field is small near the edge and has a steep gradient. This parallel electric field results in a current profile which is linearly unstable against current-driven resistive MHD instabilities. The resulting tearing fluctuations grow and through nonlinear mode coupling a quasi stationary-state forms. The tearing fluctuations distribute the plasma current through the dynamo process. The net volume average dynamo effect is almost zero.

We have performed computations for the two aspect ratios of 1.6 and 2.88. For low aspect ratio (less than 2), the radial spacing of m=1 resonances is more sparse and fewer Fourier modes make contributions to the dynamo process. For high aspect ratio, there will be more



Figure 2.6: (a) The unstable edge-resonant equilibrium profiles chosen from the alpha model. (b) The linear eigenfunctions for m=1, $k_z = +1.8$ (n=+6) resistive edge-resonant mode obtained from the linear computations at $S = 10^5$.

unstable m=1 modes (resonant in the core region) which are more closely spaced. The toroidal mode numbers of the dominant m=1 modes are found near $n \sim 2R/a$; thus higher spectral resolution is needed at high aspect ratio. For the aspect ratio 1.6, the axial mode resolution $-42 \leq n \leq 42$ is found to be sufficient, and for aspect ratio 2.88, $-84 \leq n \leq 84$ has been used. The poloidal mode resolution $0 \leq m \leq 5$ has been employed for all the cases. A large number of radial mesh points is needed to resolve the small-scale fluctuations at high S. The largest number of radial grid points used is 260. Some of the computations require expensive diagnostics and need to be run for a large fraction of diffusion time. These computations are performed at lower aspect ratio 1.6 with lower resolution to save CPU time and memory. In numerical simulations some form of dissipation is required to avoid energy cascade into small scale fluctuations (short wave length modes). An artificial viscosity is therefore used for numerical stability. The minimum magnetic Prandtl number

 $P = \nu/\eta = 10$ is used for high $S = 5 \times 10^5$ computations. For some cases, larger P (50-100) have been used for smoother eigenfunctions. With the dissipation coefficients used here, sawtooth oscillations are observed at high S and the results are largely independent of the magnitude of viscosity used in these simulations. The radial profiles at $S = 10^5$ and $S = 5 \times 10^5$ are presented in the following section. The results in Sec. 2.5.1 are for aspect ratio R/a = 2.88. The temporal evolution and the modal magnetic energies are shown in Sec. 2.5.2. Some of the results presented in Sec. 2.5.2 are for aspect ratio R/a = 1.6.

2.5.1 Radial profiles

The computations at $S = 10^5$ and $S = 5 \times 10^5$ are performed for aspect ratio R/a = 2.88with the constant boundary electric field $E_z(r = 1) = 5$ which results a pinch parameter $\Theta \sim 1.67$. The three terms in parallel Ohm's law are shown in Fig. 2.7 for $S = 5 \times 10^5$. As is seen, the fluctuation induced dynamo term $S < \tilde{V} \times \tilde{B} >_{\parallel}$ suppresses current in the core region and drives current near the edge. The λ profile and parallel current density profile J_{\parallel} are shown in Fig. 2.8. It is seen that the current on axis is reduced at higher S and increased near the edge region. The dynamo terms for these two cases are also shown in Fig. 2.9. The dynamo activity is higher both in the core and in the edge at $S = 5 \times 10^5$ which explains the current density profiles in Fig. 2.8. Although the magnetic fluctuations are reduced at higher S, the dynamo effect (the contribution of all the modes) is larger at higher S and is transporting more current.

The contributions of the m=0, m=1 and m=2 modes in the dynamo term are shown in Fig. 2.10. As is seen the suppression of the current in the core is mostly due to the m=1 dynamo and the m=0 dynamo drives current near the edge. The contribution of all other modes, including the m=2 dynamo, is rather small. These results agree with the prior results at lower S [Ho 1990 and Sovinec 1995]. [14, 3] Both m=0 and m=1 dynamos increase at higher S. The increase in current density profile (Fig. 2.8) near the edge r/a =0.8-1 at $S = 5 \times 10^5$ is therefore caused by the increased m=0 dynamo activity. Although the dominant m=1 mode amplitudes are reduced at higher S, the m=1 magnetic spectrum become broader at higher S as shown in Fig. 2.11(a). Because of enhanced mode coupling, the amplitudes of high-n modes increase with S. Similarly, the m=0 magnetic spectrum is broader at higher S (see Fig. 2.11(b)). The magnetic spectrum broadening at higher S will lead to larger total dynamo term at higher S. [15]

The single mode dynamo of the dominant mode is reduced with S. Using a simple argument from Ohm's law, $S < \tilde{V} \times \tilde{B} >_{\parallel} = \eta J$, and assuming current of the order of unity, the single mode dynamo product scales as $< \tilde{V} \times \tilde{B} >_{\parallel} \approx S^{-1}$. When the relative phase between \tilde{V} and \tilde{B} is ignored, individual velocity and magnetic fluctuations scales as $\tilde{B} = \tilde{V} \approx S^{-1/2}$. The experimental measurement of the single mode dynamo products m=1, n=-7 and m=1, n=-9 yields S-scaling of $< \tilde{V}_{\theta}\tilde{B} > \approx S^{-0.64}$ and $< \tilde{V}_{\phi}\tilde{B} > \approx S^{-0.88}$ which is much stronger than the individual empirical scaling of the fluctuation amplitudes \tilde{B} and \tilde{V} , indicating the role of phase effects. [16]

The nonlinear computations also show the reduction of the single mode dynamo with S. The dynamo term for the dominant m=1 modes (1,-6) and (1,-7) at $S = 10^4$ and $S = 10^5$ are shown in Fig. 2.12. As is seen the (1,-6) mode dynamo is reduced on axis at higher S and the (1,-7) mode dynamo is also smaller at $S = 10^5$. According to linear theory, the radial mode structure for higher-n modes is more localized which is also seen for the nonlinear single mode dynamo (1,-7) in Fig. 2.12. The nonlinear dynamo structure can be compared with the linear mode dynamo shown in Sec. 2.4. The nonlinear (1,-6) dynamo term shown in Fig. 2.12 is broader than the linear (1,-6) dynamo in Fig. 2.4. We also note that at higher S the nonlinear single mode dynamo becomes more localized. As shown in Sec. 2.4 using linear computations, at high S as plasma get close to ideal regime, the mode structure becomes very localized around the resonant surface. As seen in Fig. 2.12, the nonlinear mode structure also show slightly higher localization at higher S. We can then conclude that at high S plasma, the total global dynamo effect results from the superposition of the localized high-n single mode dynamo. Further investigation of the nonlinear mode dynamos requires computations at higher S.

The experimental measurements of fluctuation-induced dynamo by Fontana et al. 2000 [13]

indicate that m=0 fluctuations are responsible for the dynamo at the edge, which is consistent with the results obtained here. Further, the velocity fluctuation measurements show that the radial flow velocity has odd parity around the reversal surface consistent with the linear MHD theory discussed in Sec. 2.4 (Figs. 2.3, 2.4, 2.5), and as is seen in Fig. 2.10 the nonlinear m=0 dynamo term changes sign near the reversal surface.

The time-averaged q profile shown in Fig. 2.13 indicates that at higher S the reversal becomes stronger. Deeper field reversal at higher S is also seen from the reversal parameter F (will be shown in the next section) indicating stronger nonlinear dynamo activity at higher S.

2.5.2 Temporal nonlinear evolution

The temporal behavior of the non-axisymmetric fluctuations and the toroidal field reversal are discussed here. The radial average modal magnetic energies of the dominant m=1 modes and m=0, n=1 are shown in Fig. 2.14. The sawtooth oscillations in both m=1 modal magnetic energy and field reversal are more pronounced at higher $S = 5 \times 10^5$, and resemble the experimental measurements (see Fig. 2.19(a)). The m=0 mode starts to grow as the amplitudes of dominant m=1 modes become large. The energy drive of the m=0 is discussed further in the next section.

As mentioned before, the detailed empirical and numerical calculations of S-scaling of magnetic fluctuations obtain weaker scaling than $\tilde{B} \propto S^{-1/2}$ (from the simple Ohm's law). As expected, the core modal magnetic energies are reduced at higher S as shown in Fig. 2.14. The total magnetic fluctuation S-scaling for the few points obtained here is between $\tilde{B}/B \approx S^{-0.18}$ and $\tilde{B}/B \approx S^{-0.2}$ that have been obtained in the past calculations of S-scalings. Figure 2.15 illustrates the magnetic spectrum for m=1 and m=2 modes with 84 toroidal mode numbers for $S = 5 \times 10^5$.

The oscillations of reversal parameter F around its time-averaged negative value, are shown in Fig. 2.16 for three different S computations which have fixed pinch parameter $\Theta = 1.8$ for aspect ratio R/a=1.6. The low aspect ratio simulations can be performed with a


Figure 2.7: The time averages of the three terms in $E_{\parallel} + S < \tilde{V} \times \tilde{B} >_{\parallel} = \eta J_{\parallel}$ at $S = 5 \times 10^5$, R/a = 2.88.

smaller number of Fourier modes. As is seen in Figs. 2.14 and 2.16, the field reversal becomes deeper at higher Lundquist number and the oscillations become more regular. The reversal is also deeper at higher current ($\Theta = 1.8$) as seen in Fig. 2.16. The period of the sawtooth oscillations have been roughly calculated and the S-scaling of $\tau_F/\tau_R \approx S^{-0.4}$ is obtained which has a resistive-MHD hybrid character. The result obtained here is consistent with that found for quasi-periodic oscillations in Ref. [2]. The sawtooth period is not governed by a pure resistive diffusion time scale and doesn't scale linearly with S, the scaling which was reported in Ref. [17]. The experimental scaling of the sawtooth period in MST also shows scaling as $\approx S^{-1/2}$ (Stoneking 1998) which is governed by the resistive-MHD hybrid time ($\tau_{saw} \approx \sqrt{\tau_R \tau_A}$). [1] We should note that the collapse or crash time is much faster than the resistive MHD hybrid time and might be governed by the time scales beyond MHD time scales.



Figure 2.8: Time-averaged $\lambda(r)$ and J_{\parallel} profiles $(R/a=2.88,\Theta=1.67).$



Figure 2.9: Time-averaged dynamo term, $S < \tilde{V} \times \tilde{B} >_{\parallel}$ profile $(R/a = 2.88, \Theta = 1.67)$.



Figure 2.10: The m=0, m=1, m=2 dynamo terms, $S < \tilde{V} \times \tilde{B} >_{\parallel}$. (a) $S = 10^5$ (b) $S = 5 \times 10^5$. <> denotes surface averaged and sum over all toroidal mode numbers, n (time-averaged).



Figure 2.11: Toroidal mode number spectrum for m=1 and m=0 magnetic energy for $S = 10^4$ and $S = 10^5$ (time-averaged over ten data points).



Figure 2.12: The nonlinear time-averaged single mode dynamo term for the dominant modes (1,-6),(1,-7).

2.6 Sawtooth oscillations and m=0 modes

Most of the experimental observations related to large-scale magnetic fluctuations of RFP plasmas have been successfully explained through resistive MHD computations. Observations such as the relaxation process and the fluctuation-induced dynamo effect in RFPs have had strong computational support and have been computationally demonstrated over the last two decades. However, some of the important features of the experiments such as sawtooth oscillations and the source of m=0 bursts in RFPs have not yet been fully understood. Further theoretical models and computations are required to explain these observations. Here we study the physics of sawtooth oscillations and m=0 modes, using high Lundquist number MHD computations. Regular sawtooth oscillations which can only be obtained in high S computations are discussed in the following section. The linear and



Figure 2.13: Time-averaged q profile.

total magnetic energy drive for m=0 modes are calculated in Sec. 2.6.1. The effect of m=0 and m=1 mode coupling on sawtooth oscillations is investigated by eliminating the m=0 modes in the MHD computation. The dynamics in the absence of m=0 modes is presented in Sec. 2.6.2.

Sawtooth oscillations of core temperature and magnetic field occur in both tokamaks and RFPs. The measurement of sawtooth oscillations in MST were performed by Prager *et al.* 1990. [18] Sawtooth crashes are interpreted as a sudden reconnection event due to resistive MHD activities. The first theoretical model to explain the sawtooth crashes in tokamaks was proposed by Kadomtsev (1975). In this model the sawtooth oscillation was explained through the nonlinear evolution of the resistive m=1 kink mode. The nonlinear evolution of the resistive kink mode is characterized by the nonlinear time scale $\tau \sim S^{1/2}$ which can be obtained from the equation for the time evolution of a magnetic island. However, this model could not explain some of the features of the sawtooth disruption including the fast



Figure 2.14: The magnetic modal energy, $W_{m,n} = 1/2 \int \tilde{B}_{r(m,n)}^2 d^3r$, for modes (1,-6) = --, (1,-7) = --, (1,-5) = --, (0,1) = --, and field reversal F vs.time, for $S = 10^5$ and $S = 5 \times 10^5$ [$\overline{F} = -0.118$ for $S = 10^5$ and $\overline{F} = -0.14$ for $S = 5 \times 10^5$].



Figure 2.15: The magnetic energy spectrum for m=1 and m=2 (R/a=2.88).

reconnection time scale during the crash which is much smaller than the time predicated above. Other mechanisms such as two fluid effects and collisionless kinetic effects have been proposed to shorten the long time scales associated with resistive reconnection. The inclusion of the Hall term, electron inertia and electron pressure in the generalized Ohm's law $[E + V \times B = \eta J + J \times B + \frac{dJ}{dt} + \nabla P_e]$ allows shorter reconnection times. [19] Therefore, the collisionless and two fluid effects provide time scales not too far from the observed collapse times.

In the RFPs, however, the broad spectrum of Fourier modes coupling nonlinearly affect the dynamics of the sawtooth crashes. The sawtooth oscillations in RFPs are associated with the plasma relaxation and turbulent dynamo activity. Experimental observations show that plasma relaxation (i.e. the minimization of the ratio W/K, where W is the magnetic energy) occurs during the sawtooth crash phase [20] in RFP. The relaxation event was explained through three nonlinear processes by Ho and Craddock 1991. [21] First, free energy provided



Figure 2.16: The field reversal parameter, F, at three different S. The sawtooth oscillations are more regular at higher S. The time averaged F and τ_F are also shown. For all three cases, $\Theta = 1.8$ and R/a=1.6.

by the current gradient leads to the linear instability and transfer of energy to the m=1 modes. In this phase, the profiles have been driven away from the relaxed state as a result of resistive diffusion. Second, the transfer of energy from the low-n core m=1 modes to the higher-n modes resonant near the reversal surface through the nonlinear coupling with m=0 modes. And finally the transfer of energy from the m=1 modes resonant near the reversal surface to the mean toroidal field through the dynamo effect and the field reversal is sustained. During the last two dynamo phases, the nonlinear dynamo has the major role of rearranging the current distribution and transporting the current from the core to the

edge region. The dynamo relaxation and the sawtooth crashes occur in the last two phases.

The three nonlinear phases can be interpreted using the m=0 and m=1 modal energies, F, W/K and q(0) oscillations during the sawtooth crash shown in Figs. 2.17 and 2.18. During the first phase (marked in Figs. 2.17 and 2.18 by t_1), the current density on axis gradually peaks and m=1 mode amplitudes become large enough to cause nonlinear growth of the m=0 mode and reduction of q on axis. The dynamo relaxation occurs during the crash time (between t_1 and t_2), the energy is transferred from dominant m=1 modes to m=0 and m=1 with higher n, the ratio of W/K is minimized, toroidal flux is generated and field reversal is maintained (time t_2). The current density is flattened in the core through the m=1 dynamo relaxation and q on axis is increased. The reduction of m=1 mode activities after the relaxation cause the m=0 to decay. The experimental decay of m=0 mode after relaxation is faster than the decay observed in Fig. 2.18.

We compare the characteristic of the magnetic fluctuations and the sawtooth oscillations of an MST shot with the MHD computation at $S = 5 \times 10^5$ and aspect ratio R/a=2.88. Figure 2.19(a) shows the magnetic fluctuation amplitudes for the dominant core mode m=1, n=-6 [$b_{n=6}$ in Fig. 2.19(a)] along with the m=0,n=1 modal amplitude ($b_{n=1}$) obtained experimentally from an MST shot. The magnetic modal energies obtained from MHD computations at $S = 5 \times 10^5$ for the core mode m=1,n=-6 and m=0,n=1 are also shown in Fig. 2.19(b) for comparison. As is seen the periodic sawtooth oscillations of the fluctuations from the code are similar to the experimental measurement. The core mode amplitudes reveal a linear growth and a rapid damping in both experiment and the code. However, the m=0 bursts observed experimentally (shown in Fig. 2.19(a), bottom graph) damp faster than the computational m=0 mode.

2.6.1 Calculations of linear magnetic energy for m=0 modes

The question of whether m=0 modes are driven linearly or nonlinearly in standard plasma is investigated using the calculation of m=0 linear and nonlinear magnetic energies in time. As is seen in Figs. 2.17, the rapid growth of m=0 mode amplitudes follows the growth of m=1 dominant modes; suggesting that m=0 modes are driven by nonlinear mode coupling. The fact that m=0 modes are driven nonlinearly is known from previous low S MHD computations and linear stability analysis shows that m=0 is linearly stable with a close fitting conducting wall boundary condition. To further illustrate this, we calculate the linear contribution to the volume average radial magnetic energy drive for the m=0, n=1 and m=1, n=-4 modes for $S = 5 \times 10^5$. The sawtooth oscillations of F and the energy of these modes are shown in Fig. 2.20. The energy terms can be calculated from the following equation,

$$\frac{\partial B_{1r}^2}{\partial t} = S[(\mathbf{B}_0 \cdot \nabla)\mathbf{V}_1 - (\mathbf{V}_1 \cdot \nabla)\mathbf{B}_0]_r \cdot B_{1r}^* + C.C + N.L.$$
(2.6)

where, subscript '0' and '1' denote equilibrium and linear perturbed quantities, respectively, and N.L. denotes the nonlinear terms. The dissipative terms on the RHS are small over most of the plasma and have been ignored. The linear contribution of the linear energy terms on the RHS shown in Fig. 2.21 is negative during the m=0, n=1 mode, indicating that m=0 mode is nonlinearly driven. The linear contribution during the m=0 decay is also negative and is different from the total energy contribution indicating that the decay is partially nonlinear (see Fig. 2.22). The linear energy contribution for m=1 and n=-4 mode has also been calculated in Fig. 2.23 which shows the linear contribution is positive during the slow growth of this mode ; thus the m=1 mode is linearly driven as expected from the linear stability analysis. The same result is obtained for the other dominant core modes [i.e. (1,-3)]. The core mode energy decay is mostly due to the nonlinear energy contribution. The linear energy terms for the m=0 mode have been measured and calculated experimentally in MST for standard RFP plasma, [22] and is consistent with the numerical results obtained here. However, the energy drive for m=0 modes in plasmas with the auxiliary current drive is not known and needs to be examined both experimentally and computationally.

2.6.2 The dynamics in the absence of m=0 modes

To understand the dynamics of sawtooth oscillations, we have performed computation without m=0 modes. In this case m=0 fluctuations are artificially suppressed at every time step. The nonlinear coupling between m=1 core modes and m=0 modes are removed in the absence of the m=0 modes. As is seen in Fig. 2.24, m=0 modes are set to zero in the case shown in Figs. 2.17 – 2.20 at time $t/\tau_R = 0.16$. As a result the field reversal F begins to weaken and the sawtooth oscillations are not observed. The field reversal becomes smaller but the plasma remains reversed and saturates to almost a fixed value of field reversal and magnetic fluctuations (Fig. 2.24(b)), a steady non-oscillatory state. The sawtooth behavior of magnetic fluctuations is also not seen in the absence of m=0 modes, which prove the important role of m=0 modes in the regular behavior of symmetric and asymmetric fields observed experimentally. Similar behaviors have also been studied by Ho and Craddock for lower-S computation ($S = 3 \times 10^3$). However, they did not observe the sawtooth oscillations at low S computation. Therefore, we find that at high-S computation shown here (with more regular sawtooth behavior), m=0 modes determine the dynamics of observed sawtooth oscillations.

Since m=0 modes are responsible for driving the edge dynamo, the dynamo radial profile and the current profile would change in the absence of the m=0 dynamo. Figure 2.25 illustrates the change of $\lambda(r)$ and parallel current density profiles when m=0 modes are eliminated. Because of the absence of m=0 modes and edge dynamo, the peak in the current density profile near the edge region is not observed. The current density gradient is larger in the plasma core which might cause other m=1 core modes with higher n numbers to grow. The safety factor on axis q(0) is lower without the m=0 modes as seen in Fig. 2.26. This allows stronger nonlinear mode coupling because of the closer (less sparse) resonant surfaces and cause growth of higher-n core modes. However, the q profile near the edge region becomes flatter in the absence of m=0 mode coupling. As expected without m=0 modes the contribution of the dynamo term near the edge region is not significant (see Fig. 2.26(b)) and the dynamo term does not drive current near the edge (outside the reversal surface) which can explain the zero current density near the edge in Fig. 2.25. The dynamo is reduced everywhere in the absence of the m=0 fluctuations resulting in weaker toroidal field reversal and higher current on axis. The radial profile of the dynamo term for the dominant core modes has been shown in Fig. 2.27. Similar to the symmetric quantities, the mode dynamo profile also becomes stationary as the plasma settles into a steady state. This behavior is not observed for the standard case in the presence of m=0 modes and the sawtooth oscillations of the magnetic fluctuations. In addition to the n=-3 and n=-4 core modes for R/a=1.6, n=-7 develops the largest amplitude and dynamo term (see Fig. 2.27).

Figure 2.28 shows the m=1 and m=2 magnetic energy spectrums at time t_1 with m=0 modes (standard case) and time t_3 when the plasma saturates to a steady non-oscillatory state (t_1 and t_3 are marked in Fig. 2.24). Because of the removal of m=0 modes and their nonlinear coupling with other modes, the small scale fluctuations (high n fluctuations, n > 10 and n < -30) have been reduced. The core mode (1,-7) has the largest amplitude for the steady nonoscillatory state without m=0 modes. The addition of the (1,-7) core mode to the standard dominant core modes (1,-3) and (1,-4) (for R/a=1.6) cause the total magnetic fluctuation level to increase (Fig. 2.24). It is expected that by eliminating m=0 modes (experimentally operating with F > 0), the nonlinear coupling between m=0 and m=1 is removed and lower fluctuation amplitudes may result. However, here we see that the elimination of m=0 nonlinear mode coupling between stronger resulting in excitation of other core modes. The m=2 spectrum also shown in Fig. 2.28(b) similarly saturates to a state with reduction of high-n fluctuations and increase in the core m=2 modes [with the dominant m=2, n=-10 mode generated from the nonlinear coupling of (1,-3) and (1,-7)].

The magnetic fluctuation and the spectrum does not change as the plasma settle into the steady non-oscillatory state. However, before this saturation there is a transition from the quasi-oscillatory state to a non-oscillatory state. The spectrum during this transition (at time t_2) is seen in Fig. 2.29. The spectrum is similar to a quasi single helicity state and has a narrow structure in the absence of m=0 modes. When the m=1 modes reach larger amplitudes, the magnetic fluctuations settle into a broader spectrum shown in Fig. 2.28 without m=0 modes. We conclude that the elimination of m=0 modes removes the sawtooth oscillations but does not reduce the total magnetic fluctuation level in spite of the absence of m=0 mode nonlinear coupling. Although the magnetic spectrum during the transition from the oscillatory to the non-oscillatory state becomes narrow and resembles a quasi single helicity state, the stochasticity of the magnetic field lines does not improve when the plasma reaches the final steady state. Figure 2.30 illustrates the magnetic field lines intersections with a fixed RZ plane (toroidal plane) at time t_1 (standard RFP case), t_2 (during the transition to the narrow spectrum) and t_3 (final non-oscillatory steady state). The stochasticity of the magnetic field lines is proportional to the magnetic fluctuation amplitude. The stochasticity parameter is described by [Rechester and Rosenbluth], [25]

$$s = \frac{1}{2} (W_{mn} + W_{m'n'}) / |r_{mn} - r_{m'n'}|$$
(2.7)

where W_{mn} is the width of the separatix of an island near the resonant surface and is given by

$$W_{mn} = 4\sqrt{\frac{rq}{q'}\frac{\tilde{B}_r(r)}{B_\theta}\frac{1}{m}}$$
(2.8)

m,n and m',n' are the mode numbers for two neighboring resonant surfaces. The magnetic field lines become highly stochastic (s >> 1) when the resonant surfaces are closely spaced (dense), such as in RFPs, and when magnetic fluctuation amplitudes are high resulting in large magnetic islands ($W \sim \sqrt{B_r}$). Because of a safety factor less than unity, the resonant surfaces are closely packed causing high magnetic stochasticity in most of an RFP. At high Lundquist number, the magnetic fluctuation amplitudes and associated stochasticity decrease, as seen in Fig. 2.30(a). For $S = 5 \times 10^5$ the field lines in the plasma core are more ordered out to about radius 0.2. At lower S stochasticity develops over the whole plasma region. When the magnetic spectrum becomes narrow, island overlapping and the subsequent stochasticity decreases. As is seen in Fig. 2.30(b), the core region is less stochastic and the n=-3 and n=-4 islands structure are more distinct. The magnetic surfaces are also more ordered near the edge region. As the m=1 nonlinear coupling increases



Figure 2.17: The magnetic modal energy, $W_{m,n} = 1/2 \int \tilde{B}_{r(m,n)}^2 d^3r$, vs time for (1,-3), (1,-4), (1,-5) and (0,1) modes ($S = 5 \times 10^5$ and R/a=1.66).

(evidenced by the growth of (1,-7) island) and the plasma reaches a non-oscillatory steady state, the stochasticity increases (Fig. 2.30(c)).

2.7 Summary

We have investigated MHD computations of standard RFP at high Lundquist number. A more realistic picture of RFP dynamics close to the experimental observations is studied using high S MHD computations. One of these observations is sawtooth oscillations which have not been fully understood. These oscillations are observed in high S computations. The goal is to understand the dynamics of sawtooth oscillations and the associated m=0 magnetic fluctuations.

We have shown the radial profiles at high S and the profile variations with S. The results agree with the earlier computations at lower S. The linear and nonlinear single



Figure 2.18: The regular oscillations of m=0 modal energy, F, W/K and q(0).

mode dynamo are also shown. The radial structure becomes more localized around the resonant surface at higher S. However, the nonlinear single mode structure is broader than the linear one. At high S, the total dynamo term is global similar to the experiment, which arises from the superposition of the single mode dynamo terms. It is shown that because of the enhanced nonlinear mode coupling at high S, the magnetic spectrum broaden. Strong nonlinear dynamo activity at high S results in deeper toroidal field reversal. Total magnetic fluctuations S-scaling similar to the previous computational S-scaling is obtained. We have also examined the dependence of the period of the sawtooth oscillations on S. It is shown that the scaling is governed by the resistive MHD hybrid time.

We have also investigated the dynamics of the sawtooth oscillations and m=0 modes



Figure 2.19: (a) The measurement of magnetic field and magnetic modal amplitudes from a MST shot (from D. Craig) (b) Magnetic modal energies for the core mode m=1,n=-6 and m=0 mode obtained from MHD computation.

using high S computations. The m=0 modes which limit the confinement in the standard RFP experiment require better understanding. We have studied the relaxation process during a sawtooth crash using the temporal behavior of m=1 and m=0 fluctuations along with the ratio W/K, F, and q_0 . The crash occurs after a resistive diffusion phase when plasma is driven away from the relaxed state and m=1 modes have reached large amplitudes. The rapid growth of m=0 mode amplitudes follows the growth of m=1 dominant modes, suggesting that m=0 modes are driven by nonlinear m=1 mode coupling. The dynamo relaxation occurs during the crash time. The m=1 and m=0 dynamos transport the current from the core to the edge. To further investigate the growth of m=0 mode, we have calculated the linear energy term for m=0 mode. We show that the m=0 mode is driven nonlinearly in a standard plasma, and is consistent with the experimental measurement of the linear energy



Figure 2.20: The field reversal F sawtooth oscillations and modal energy for m=0, n=1 and m=1, n=-4 during a sawtooth oscillation.



Figure 2.21: (a) The radial component of volume averaged magnetic energy for m=0,n=1 during the sudden growth of the mode and the linear contribution (linear part of RHS of Eq. 2.6) (b) The separate linear energy terms.

term.

To understand the dynamics of sawtooth oscillations, we have performed computation without m=0 modes. The effect of m=0 and m=1 nonlinear mode coupling on the sawtooth oscillations is investigated by eliminating m=0 modes in the MHD computations. The dynamo relaxation process discussed above is studied when m=0 modes have been removed. The sawtooth oscillations are not observed without m=0 modes. This proves the important role of m=0 modes in the sawteeth dynamics. In the absence of the m=0 nonlinear mode



Figure 2.22: The radial component of volume averaged magnetic energy for m=0,n=1 during the decay.

coupling, the plasma transitions to a non-oscillatory steady state; however, the total magnetic fluctuation level does not reduce. The plasma settles into a steady state and a weak reversal is maintained. The transfer of energy from high-n m=1 modes to m=0 modes (i.e. the m=0 dynamo relaxation phase) can not occur; thus the sawtooth crash is not observed in the absence of m=0 modes. The m=0 modes are necessary for sawtooth oscillations to occur, but they do not trigger the sawteeth.



Figure 2.23: The radial component of volume averaged magnetic energy for m=1,n=-4 mode.



Figure 2.24: The m=0 fluctuations are removed at $t/\tau_R = 0.16$. (a) Field reversal parameter F. (b) Total magnetic fluctuations. $(S = 5 \times 10^5)$



Figure 2.25: $\lambda(r)$ and parallel current density $J_{\parallel}(r)$ profiles for standard RFP (case shown in Figs. 2.17 – 2.20) and in the absence of m=0 modes.



Figure 2.26: Time-averaged (a) q profile (b) $<\tilde{V}\times\tilde{B}>_{\parallel}$ dynamo term.



Figure 2.27: The dynamo $\langle \tilde{V} \times \tilde{B} \rangle_{\parallel}$ for separate dominant core modes at time t₃ in Fig. 2.24. (case without m=0 modes)



Figure 2.28: The magnetic energy spectrum for the standard case with m=0 modes (at t_1) and for the case without m=0 modes (at t_3). (a) m=1 spectrum (b) m=2 spectrum.



Figure 2.29: The m=1 magnetic spectrum at time t_2 marked in Fig. 2.24 during the transition from the standard sawtooth oscillatory behavior to a steady nonoscillatory state when m=0 modes are removed.



Figure 2.30: Magnetic field trajectories; (a) at time t_1 (standard RFP) (b) t_2 - narrow spectrum during transition (without m=0 modes) (c) t_3 - steady non-oscillatory state without m=0 modes. ($S = 5 \times 10^5$, R/a=1.6)

References

- M. R. Stoneking, J. T. Chapman, D. J. Den Hartog, S. C. Prager, and J. S. Sarff, Phys. Plasmas 5, 1004, (1998)
- [2] S. Capello and D. Biskamp, Nucl. Fusion **36**,571 (1996)
- [3] C. R. Sovinec, Ph.D. thesis, University of Wisconsin–Madison 1995
- [4] J. Scheffel and D. D. Schnack, Phys. Rev. Lett. 85, 322,(2000); J. Scheffel and D. D.
 Schnack, Nucl. Fusion 40, 1885, (2000)
- [5] A. Y. Aydemir and D. C. Barnes, Journal of Comput. Phys. 53,100, (1984)
- [6] A. Y. Aydemir, D. C. Barnes, E. J. Caramana, A. A. Mirin, R. A. Nebel, D. D. Schnack, and A. G. Sgro, Phys. Fluids 28, 898, (1985)
- [7] A. Y. Aydemir and D. C. Barnes, Journal of Comput. Phys. 59, 108 (1985)
- [8] L. Chacon, D. A. Knoll, J. M. Finn, Journal of Comput. Phys. 178(1) p.15 2002
- [9] D. D. Schnack, D. C. Barnes, Z. Mikic, D. S. Harned, and E. J. Caramana, J. Comput. Phys. 70, 330, (1987).
- [10] A. H. Glasser, C. R. Sovinec, R. A. Nebel, T. A. Gianakon, S. J. Plimpton, M. S. Chu,
 D. D. Schnack and the NIMROD team, Plasma Phys. Cont. Fusion 41, A747 (1999);
 NIMROD team website: http://nimrodteam.org
- [11] V. Antoni, D. Merlin, S. Ortolani, and R. Paccagnella, Nucl. Fusion 26, 1711, (1986)
- [12] H. P. Furth, J. Killeen, M. N. Rosenbluth, Phys. Fluids, 6, 459 (1963)
- [13] P. W. Fontana, D. J. Den Hartog, G. Fiksel, and S. C. Prager, Phys. Rev. Lett 85, 566 (2000)
- [14] Y. L. Ho, Ph.D. thesis, University of Wisconsin–Madison 1990

- [15] Private communication with Carl Sovinec
- [16] J. T. Chapman, Ph.D. thesis, University of Wisconsin–Madison 1998
- [17] K. Kusano, T. Sato, Nucl. Fusion, 30, 2075 (1990)
- [18] S. C. Prager, A. F. Almagri, S. Assadi et al Phys. Fluids **B2**, 1367, (1990)].
- [19] A. Y. Aydemir, Phys. Fluids **B4**, 3469, (1992)
- [20] H. Ji, S. C. Prager and J. S. Sarff, Phys. Rev. Lett. 74, 2945 (1995)
- [21] Y. L. Ho and G. G. Craddock, Phys. Fluids B 3, 721, 1991
- [22] Choi et. al. APS poster presentation, 2002
- [23] A. B. Rechester and M. N. Rosenbluth, Phys. Rev. Lett. 40, 38 (1978)
- [24] S. Ortolani and D. D. Schnack, Magnetohydrodynamics of Plasma Relaxation 1993
- [25] A. B. Rechester and M. N. Rosenbluth, Phys. Rev. Lett. 40, 38 (1978)
- [26] D. Biskamp, Nonlinear Magnetohydrodynamics 1993
- [27] J. P. Freidberg, Ideal Magnetohydrodynamics 1987
- [28] J. Wesson, Tokamaks, Oxford engineering science series (1987).
- [29] J. M Finn, R. A. Nebel, and C. Bathke, Phys. Fluids B 4, 1262 (1992)

3 One-dimensional classical response to the oscillating fields

3.1 Introduction

To determine the effectiveness of AC helicity injection as a steady-state current drive technique, 3-D nonlinear computations are required. The role of non-axisymmetric fluctuations in the current relaxation process can not be explained without 3-D nonlinear treatment of plasma. However, the plasma 1-D classical response in the absence of asymmetric fluctuations provides a benchmark for comparison to full 3-D plasma response. In this chapter, we study the classical plasma response to the applied oscillating electric field using both 1-D computations and quasilinear analytical calculations. Chapter 4 then covers the full 3-D nonlinear computations of AC helicity injection.

One-dimensional studies, in which all quantities depend on radius only, are performed to examine plasma behavior with OFCD, but in the absence of asymmetric MHD fluctuations. This allows us to evaluate the OFCD-driven current, concentrated in the outer region of the plasma, that occurs in the absence of MHD relaxation. The 1-D model demonstrates some interesting physics, such as the quasi-linear $(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}$ effect arising from the axisymmetric velocity and magnetic field oscillations. The 1-D calculations are also useful for comparison to 3-D computation to highlight the additional effect of relaxation. In Sec. 3.2 we present computational solutions to the 1-D MHD equations. Sec. 3.3 contains an analytic quasilinear treatment for a simple 1-D equilibrium. The dependence of the OFCD-driven current modulation amplitudes on the key parameters, Lundquist number *S*, driving frequency ω and driving amplitudes in a 1-D classical plasma is described in Sec. 3.4.

3.2 One-dimensional computations

We employ the DEBS code with all θ and z dependent fluctuations suppressed. To study the linear dynamic response of both the mean and oscillating fields, low oscillating field amplitudes have been imposed on a plasma that is initially current-free ($B_{\theta} = 0, B_z =$ constant). The time-averaged (over a cycle) magnetic field profiles in steady-state are shown in Fig. 3.1. The axial field is little affected by the small oscillating fields. The alteration in the azimuthal field results from the cycle-averaged current density, shown in Fig. 3.2(a). The current density is localized to the outer region of the plasma, penetrating a distance equal to the classical skin depth $\delta = (\eta/\omega)^{1/2}$. The time dependence of the current density throughout one cycle is shown in Fig. 3.2(b). The oscillatory current density is similar to the classical penetration that occurs for a solid metal. However, the cycle-averaged component arises from the cycle-averaged term $(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}$, a dynamo-like effect due to the classically penetrating oscillatory fields, similar to that reported in Ref. [1]. This effect is proportional to the helicity injection rate, ($\sim \varepsilon_z \varepsilon_{\theta}/\omega$), as seen in Fig. 3.3.

At high oscillating field amplitudes (about ten times larger), the oscillatory behavior of the fields change. The electric field contains both higher harmonics and sub-harmonics (low frequency) components, as seen in Figs. 3.4(a) and (b). The sub-harmonic component yields a non-zero cycle-averaged electric field that decays toward zero as the plasma approaches steady-state. The cycle-averaged dynamo-like effect, $(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}$, increases with the helicity injection rate; however its structure remains unchanged (Fig. 3.5).

3.3 Analytical calculation and quasi-linear effects

From the 1-D computation, we see that low amplitude oscillating fields penetrate into the plasma with the OFCD frequency while both higher and lower frequencies are generated for higher amplitudes (large forcing amplitudes). To understand the time dependence of the fields, 1-D linear, resistive MHD equations (Eq. (2.1)) are analytically solved in cylindrical geometry. The partial differential equations are solved for uniform magnetic field $\mathbf{B} = B_0 \hat{z}$, $\nabla p = 0$, no viscosity, with initial conditions $A_z^1(r, 0) = \text{const.}, A_{\theta}^1(r, 0) = 0$ and boundary conditions $A_z^1(a,t) = (-\varepsilon_{z0}/\omega) \cos(\omega t), A_{\theta}^1(a,t) = (-\varepsilon_{\theta0}/\omega) \sin(\omega t)$, where the "1" superscript denotes a linear oscillating quantity. The equations for the vector potential and velocity fields can be simplified as follows,

$$\frac{\partial \mathbf{A}^{1}}{\partial t} = \mathbf{V}^{1} \times \mathbf{B} - \eta \nabla \times \nabla \times \mathbf{A}^{1}$$
(3.1)



Figure 3.1: Time-averaged profiles for axial and azimuthal magnetic fields, obtained in steady-state from 1-D computation ($\varepsilon_z = 1.0, \varepsilon_{\theta} = 0.1, \omega = 600\tau_R^{-1}, S = 10^5$).

$$\rho \frac{\partial \mathbf{V}^{1}}{\partial t} = -\nabla (\mathbf{B} \cdot \mathbf{B}^{1}) \tag{3.2}$$

Using $\mathbf{B}^1 = \nabla \times \mathbf{A}^1$, $\mathbf{J}^1 = \nabla \times \mathbf{B}^1$, and $\mathbf{B} = B_0 \hat{z}$ we can combine equations (3.1) and (3.2) in the form of axial and azimuthal vector potential (A_z^1, A_θ^1)

$$\frac{\partial A_z^1}{\partial t} = \eta \left(\frac{\partial^2 A_z^1}{\partial r^2} + \frac{1}{r} \frac{\partial A_z^1}{\partial r} \right)$$
(3.3)

$$\frac{\partial^2 A^1_{\theta}}{\partial t^2} = \frac{S^2 B^2_0}{\rho} \left[\frac{\partial^2 A^1_{\theta}}{\partial r^2} + \frac{1}{r} \frac{\partial A^1_{\theta}}{\partial r} - \frac{A^1_{\theta}}{r^2} \right] + \eta \frac{\partial}{\partial t} \left[\frac{\partial^2 A^1_{\theta}}{\partial r^2} + \frac{1}{r} \frac{\partial A^1_{\theta}}{\partial r} - \frac{A^1_{\theta}}{r^2} \right]$$
(3.4)

The normalization of the equations is similar to the one used in Sec. 2.2. The partial differential equation (PDE) with non-homogeneous boundary condition for the toroidal vector potential (Eq. (3.3)) represents a driven resistive diffusion equation. The PDE for the poloidal vector potential (Eq. (3.4)) consists of Alfvén waves and resistively damped modes. To solve the PDEs, the Laplace transform method can be applied to Eqs. (3.3) and (3.4) (see Appendix A). The solution for A_z^1 and B_θ^1 can be written as an expansion of eigenfunctions (Bessel functions):



Figure 3.2: Radial profiles of (a) cycle-averaged parallel current density, $\overline{J_{||}}$. (b) parallel current density at different times during one cycle (1-D low amplitude computation).

$$A_{z}^{1}(r,t) = \frac{-\varepsilon_{z0}}{\omega}\cos(\omega t) + \sum_{n=1}^{\infty}b_{n}(t)J_{0}(\lambda_{n}r)$$

$$B_{\theta}^{1} = \sum_{n=1}^{\infty}\lambda_{n}b_{n}(t)J_{1}(\lambda_{n}r)$$
(3.5)

where,

$$b_n(t) = \alpha_n(\omega, \omega_n) [\omega_n \sin(\omega t) - \omega \cos(\omega t) + \omega \exp(-\omega_n t)]$$

$$\alpha_n(\omega, \omega_n) = \frac{2\varepsilon_{z0}}{\lambda_n \omega_n} \frac{1}{J_1(\lambda_n)(\omega^2 + (\omega_n)^2)}$$
(3.6)

 $\omega_n = \eta \lambda_n^2$, and λ_n are the zeros of J_0 . Here, we have assumed uniform density and resistivity



Figure 3.3: Cycle-averaged dynamo-like term $\overline{(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}}$ vs. radius, for the 1-D computation. The oscillation frequency ω is $200\tau_R^{-1}$ and $600\tau_R^{-1}$ for the solid and dashed lines, respectively. The solid line has three times higher helicity injection rate. For both cases $\hat{E}_z = 1.0 \sin(\omega t), \hat{E}_{\theta} = -0.1 \cos(\omega t), S = 10^5.$

profiles ($\rho = \eta = 1$). The solution for B^1_{θ} consists of an oscillating part at the OFCD frequency and a transient decaying part (Fig. 3.6(a)). Equation (3.4) can be solved for A^1_{θ} and subsequently for V^1_r as follows,

$$V_r^1(r,t) = \sum_{m=1}^{\infty} C_m(t)\phi_m(r)$$
(3.7)

where,

$$C_m(t) = \frac{2S\varepsilon_{\theta 0}}{(\omega^2 - \omega_m^2)} \left[\frac{-\omega_m \cos(\omega t)}{\omega} + \frac{\omega \cos(\omega_m t)}{\omega_m} \right]$$

$$\phi_m(r) = \frac{1}{J_1 \prime(\lambda_m)} \left[\frac{\lambda_m^2}{4} \left(J_3(\lambda_m r) - 3J_1(\lambda_m r) \right) + \frac{\lambda_m}{2r} \left(J_0(\lambda_m r) - J_2(\lambda_m r) \right) - \frac{J_1(\lambda_m r)}{r^2} \right]$$
(3.8)

 $\omega_m = SB_0/\sqrt{\rho}\lambda_m$, and λ_m are the zeros of J_1 , $(B_0 = \rho = 1)$. The cycle-averaged $(V_r^1 \times B_{\theta}^1)$ effect can be obtained from the analytical solutions, $V_r^1(r,t) \times B_{\theta}^1(r,t) = \sum_{m=1}^{\infty} C_m(t)\phi_m(r)$ $\times \sum_{n=1}^{\infty} \lambda_n b_n(t) J_1(\lambda_n r)$. Figure 3.6(b) shows $S(V_r^1 B_{\theta}^1)$ from the analytical calculations,



Figure 3.4: (a) Axial and (b) azimuthal electric fields vs. time at radius r=0.89, respectively.



Figure 3.5: Cycle-averaged dynamo effect $\overline{(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}}$ for high driving amplitudes, 1-D computation ($\varepsilon_z = 10.0, \varepsilon_{\theta} = 1.0, \omega = 600\tau_R^{-1}, S = 10^5$).

which agrees with the 1-D computation (Fig. 3.3). The sharp edge feature in Fig. 3.6(b) results from the uniform resistivity profile assumed in the analytical model and the absence of viscosity. In the 1-D computation of $(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}$ (Sec. 3.2), the resistivity profile is exponential and the viscosity is finite. At high S, for arbitrary frequency and amplitudes, the second term $\cos(\omega_m t)$ in $C_m(t)$ (Eq. (3.8)) represents high frequency oscillations. These high frequency oscillations are also present in 1-D computation (Sec. 3.2) for the field solutions but dissipate at finite viscosity, and also dissipate due to the fluctuations in 3-D computation.

To understand the time response of the plasma to large oscillating amplitudes, the quasilinear effect is investigated including $f(r,t) = V_r^1(r,t) \times B_{\theta}^1(r,t)$, as an inhomogeneous source to the homogeneous PDE for A_z^1 . The 1-D driven diffusion equation plus the quasi-linear term is solved numerically using the Crank-Nicholson method. As shown in Fig. 3.7 the time response is a combination of the OFCD frequency, higher harmonics and a lower frequency which arises from the product of the exponential decaying component and the oscillation. The inhomogeneous solution can be found analytically as well, by defining $A_z^1(r,t) = \sum_{n=1}^{\infty} d_n(t) J_0(\lambda_n r)$, where now $d_n(t)$ has a different time dependence, which is the combination of the OFCD frequency, the harmonics, transient decaying solutions and the product of exponential decaying and the oscillations, $\sin(2\omega t)$, $\sin((\omega \pm \omega_m)t)$, $\sin(\omega t) \exp(-\omega_n t)$, $\exp(-\omega_n t)$...

Through the $\sin(\omega t) \exp(-\omega_n t)$ combinations in time, a non-zero cycle-averaged electric field is generated mainly at high amplitudes when the contribution of the quasi-linear term becomes important. This electric field decays slowly on a resistive diffusion time scale. A non-zero mean electric field is similarly seen in large amplitude 1-D computations (Sec. 3.2) as well as the nonlinear 3-D computations below. However, this electric field becomes small as the plasma gets close to quasi steady-state.


Figure 3.6: (a) B_{θ}^{1} vs. time at radius r/a=0.8 ($\varepsilon_{z0} = 1.0, \varepsilon_{\theta0} = 0.1, \omega = 200.0\tau_{R}^{-1}, S = 10^{5}$). (b) $S(V_{r}^{1}B_{\theta}^{1})$ vs. radius calculated analytically in 1-D for the same parameter in Fig. 3.3 (solid line).

3.4 Parameter dependences

Here, we present the dependence of the OFCD-driven current modulation amplitude on the key parameters: Lundquist number S, driving frequency ω and driving amplitude in a 1-D classical plasma.

We obtained the analytical field solutions in a classical plasma. As discussed in Sec. 3.2, the azimuthal magnetic field B^1_{θ} is the solution of the 1-D driven resistive diffusion equation



Figure 3.7: B_{θ} at r/a=0.65 (dashed) and r/a=0.94 (solid) vs. time, calculated numerically for the 1-D model with the quasi-linear term.

and is plotted in Fig. 3.8(a). The solution from the 1-D computation shown in Fig. 3.8(b) agrees with the linear analytical B_{θ} solution in both the temporal behavior and the magnitude. The viscosity is zero in the solution shown in Fig. 3.8(a) resulting in a sharp edge feature. We showed that 1-D OFCD-driven current diffuses within the classical skin depth $\delta = (\eta/\omega)^{1/2}$ and hence that the penetration depends only on the frequency and resistivity. The cycle-averaged $(V_{00} \times B_{00})_{\parallel}$ dynamo effect shown in Fig. 3.9 does not change with S. The viscosity effect is seen in this figure. At high viscosity for magnetic Prandtl number ν =200, the sharp edge becomes smooth.

The magnitude of the OFCD-generated current is proportional to the helicity injection rate (~ $\varepsilon_z \varepsilon_{\theta} / \omega$). For the fixed driven mean current (i.e. fixed helicity injection rate) the modulation field amplitudes should decrease with S. According to the cycle-averaged parallel Ohm's law $\eta J_{\parallel} = S(V_{00} \times B_{00})_{\parallel}$ it is expected that the product of axisymmetric velocity and magnetic field oscillations decreases with S. The computation shows that the azimuthal magnetic field modulation amplitude does not change with S but the radial velocity modulations are reduced. It is also seen from the linear analytical solution of B_{θ} (Eqs. 3.5 and 3.6) that the azimuthal magnetic field is independent of S. However, the linear solution for V_r (see Eq. 3.7) shows that radial velocity modulations decrease with ω_m and consequently with S. Fig. 3.10 shows the reduction of the radial velocity modulation amplitudes (from the 1-D computations) at higher S, which agrees with the analytical solutions. This S dependency is also seen in the 3-D computations which will be investigated in chapter 4. Clearly the nonlinearity alters the dependence of the B_{θ} modulations on Sand for a fixed plasma current generated by both axisymmetric and asymmetric dynamos we observe the reduction of current modulations at high S.

The field modulation amplitudes scale with another key parameter: the driving frequency ω . The lower the frequency, the greater the classical penetration and the higher the helicity injection rate. On the other hand, the field modulation amplitudes increase at low frequency as is seen from the linear field solutions in Eqs. 3.5–3.7 ($B_{\theta}^1 \sim 1/\omega$). 1-D computations show a similar scaling of axial current modulations with ω . Figure 3.11 shows the peak to peak axial current modulations scaling with ω for two values of $\varepsilon_z \varepsilon_{\theta}$. The current modulations decrease with frequency. The axial current modulations depend linearly on the axial oscillating electric field amplitude ε_z (Eq. 3.6). In Fig. 3.11 the triangles and the diamonds correspond to $\varepsilon_z = 3.0$ and $\varepsilon_z = 1.0$, respectively. The current modulation amplitudes increase linearly only with ε_z , not with the product of azimuthal and axial oscillating electric field amplitudes. Thus, in a classical plasma the axial current modulations vary linearly both with the frequency ($\sim 1/\omega$) and the axial electric field ε_z .

The temporal behavior of the axial and azimuthal currents (I_z, I_θ) for two frequencies $\omega \tau_R = 200$ and $\omega \tau_R = 50$, are also shown in Fig. 3.12. Because of the large axial oscillating electric field amplitude, the axial current I_z has the dominant ω oscillations $[\eta J_z = S(V_r^1 B_\theta + V_r B_\theta^1 + V_r^1 B_\theta^1) + E_z^1$ and $B_\theta^1 \varepsilon_z]$. However, both 2ω and ω oscillations are present in the azimuthal current I_θ (the azimuthal electric field is ten times smaller than the axial one). The azimuthal current (I_θ) modulation also decreases at higher frequency (Fig. 3.12). The axial and azimuthal current density profiles (J_z, J_θ) are shown in Fig 3.13. Because of the small oscillations of both B_z and V_r (proportional to ε_θ), the amplitude of J_θ is much smaller than J_z , and the 2ω oscillation is seen in J_θ . The modulation of J_z



Figure 3.8: The solution for $B_{\theta}(r,t)$ (a) from the 1-D driven resistive diffusion equation (b) from the 1-D computation. ($\varepsilon_{z0} = 3.0, \varepsilon_{\theta 0} = 0.3, \omega = 200.0\tau_R^{-1}$).

(Fig. 3.13) is three times larger than the one shown in Fig. 3.2 because the frequency is three times smaller for this case.

3.5 Summary

We have examined 1-D computations and quasi-linear analytical solutions to study the classical plasma response to the applied oscillating electric fields. The 1-D results are later compared with the full 3-D MHD dynamics to understand the role of tearing fluctuations. We have used a simple 1-D equilibrium to analytically solve the linearized resistive MHD equations with time dependent oscillatory boundary conditions. The analytical solutions yield a cycle-averaged $(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}$ quasi-linear effect which agrees with the 1-D computations. This dynamo-like effect arises from the axisymmetric velocity and magnetic field oscillations and generates a steady-state current confined to within a resistive skin depth of the plasma surface. We also find that at large amplitude of the oscillating transient fields are generated that persist for about a resistive diffusion time. The dependence of the 1-D



Figure 3.9: The cycle-averaged $(V_{00} \times B_{00})_{\parallel}$. The viscosity smoothes out the sharp feature near the edge. The viscosity coefficient ν is 200 and 4 for the solid and dashed lines, respectively. ($\varepsilon_{z0} = 1.0, \varepsilon_{\theta 0} = 0.1, \omega = 200.0 \tau_R^{-1}$).



Figure 3.10: The radial velocity modulations from 1-D computation at different S. The modulation amplitudes of V_r reduce at high S ($\varepsilon_{z0} = 1.0, \varepsilon_{\theta0} = 0.1, \omega = 200.0\tau_R^{-1}$), but the modulation amplitudes of B_{θ} do not change with S which agree with the analytical results.



Figure 3.11: The peak to peak axial current oscillations calculated from the 1-D computations vs frequency.



Figure 3.12: The axial and azimuthal current vs time for frequencies $\omega \tau_R = 200$ and $\omega \tau_R = 50$ (thick line).



Figure 3.13: The axial and azimuthal current densities vs time and radius from 1-D computations, $(\varepsilon_{z0} = 1.0, \varepsilon_{\theta 0} = 0.1, \omega = 200.0\tau_R^{-1})$.

axisymmetric modulation amplitudes on Lundquist number, the driving amplitudes and the driving frequency has also been obtained using 1-D computations. The 1-D velocity modulation amplitudes decrease with S but the axial current modulations remains unchanged in agreement with the analytical solutions. The modulation amplitudes vary linearly with both the driving amplitudes and the inverse driving frequency. However, in the presence of the MHD asymmetric fluctuations the scaling of the modulation amplitudes with the key parameters will change. The 1-D model provides an approximate dependence of the modulation amplitudes on the key parameters.

References

- [1] P. M. Bellan, Phys. Rev. Lett. 54, 1381, (1985).
- [2] Partial differential equations for scientists and engineers by Tyn Myint-U (1987).

4 Three-dimensional computation of AC helicity injection

4.1 Introduction

Chapter 3 described the 1-D MHD plasma response to an applied oscillating electric field in the absence of non-axisymmetric fluctuations. It was shown that a steady-state current is generated by the cycle-averaged dynamo-like $(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}$ effect from the axisymmetric velocity and magnetic field oscillations. The current diffuses classically and is confined to the outer region of the plasma. However, the full nonlinear 3-D MHD treatment is required to determine the efficiency of the current drive, the resulting current profile and the accompanying magnetic fluctuations. Here, we employ nonlinear 3-D MHD computations to examine the full 3-D MHD dynamics of OFCD. The original studies of OFCD assumed that the plasma relaxes to a Taylor state [8] with J_{\parallel}/B spatially constant. [9] MHD computation in which the fluctuations are treated as a hyper-resistivity has been used to treat the 1-D behavior of the plasma during OFCD. [10, 11] 3-D MHD computation has been used to study spheromak formation by helicity injection [12] and to model electrostatic helicity injection in tokamaks. [13]

Here we study the complete dynamics of OFCD using the code DEBS (see Sec. 2.2). Oscillating axial and azimuthal electric fields are imposed at the wall, $\hat{E}_z = \varepsilon_z \sin(\omega t)$ and $\hat{E}_{\theta} = \varepsilon_{\theta} \sin(\omega t + \pi/2)$, where ε_z and ε_{θ} are the axial and azimuthal AC amplitudes, respectively. The oscillation period is required to be long compared to the plasma relaxation time (the hybrid tearing time scale $\tau_{\text{hybrid}} \sim \sqrt{\tau_R \tau_A}$), and short compared to resistive diffusion time τ_R ($\tau_{\text{hybrid}} < \tau_\omega < \tau_R$). [10, 11] Furthermore, the frequency should be low enough for sufficient current relaxation through tearing dynamo effect, but high enough to avoid current reversal. The resistivity profile has been chosen to resemble the experimental profiles (increasing near the plasma edge), $\eta = (1 + 9(r/a)^{20})^2$. As will be shown later, the OFCD technique relies upon magnetic fluctuations to relax the current density profile. Therefore, 3-D MHD modeling is needed to understand the full MHD dynamics of OFCD. Fluctuations are generated by the unstable OFCD-driven edge current, ($\mathbf{V}_{00} \times \mathbf{B}_{00}$)_{||}. Current is generated in the plasma core via the fluctuation-induced MHD dynamo term, $\langle \widetilde{\mathbf{V}} \times \widetilde{\mathbf{B}} \rangle_{\parallel}$.

We employ an aspect ratio of 1.66. We examine OFCD at two different Lundquist numbers, 10^5 and 5×10^5 . The magnetic Prandtl number, P = 10 is used for both cases. An assessment of OFCD requires information on scaling with Lundquist number. For example, it is expected that the oscillation of the total plasma current will decrease with S, as has been indicated by the relaxed-state modeling in Ref. [20, 21]. The relaxed-state model provides a description of OFCD sustainment assuming that plasma maintains a stationary relaxed-state current profile throughout an OFCD cycle. Using a simple argument, by equating the cycle-averaged AC helicity injection rate to the ohmic helicity injection rate, the fractional AC current modulation amplitude is predicted to scale as

$$\widehat{I}_z / \overline{I}_z \sim S^{-1/4} \omega_h^{-1/2} \xi^{1/2} (R/a)^{-1/2} ,$$
 (4.1)

where ω_h is the frequency normalized to the hybrid tearing time and ξ is the ratio of the driving oscillating voltages ($\xi = \hat{v}_z/\hat{v}_{\theta}$). The scaling of the modulation amplitudes with Lundquist number S, the drive frequency ω , aspect ratio R/a and the relative amplitudes of the axial and poloidal oscillating fields have also been obtained by the relaxed-state modeling and have been compared with the predicted scaling in Eq. 4.1. [20, 21] The reduction of the current oscillation with S in the full 3-D OFCD computations is consistent with the predicted S-scaling of relaxed-state model ($S^{-1/4}$). Using the same set of parameters in high-S 3-D case (section 4.3), the prediction of the relaxed-state model for the current is in good agreement with 3-D computation. However, because of the stationary feature of current profile in relaxed- state modeling, this model predicts lower modulation amplitudes.

For both values of Lundquist number $(10^5 \text{ and } 5 \times 10^5)$, we first evolve the plasma to a steady-state in the absence of OFCD. This standard RFP plasma (at pinch parameter $\Theta = 1.8$) is evolved in the presence of a constant boundary axial electric field ($E_z(a) =$ constant). It then forms the target plasma for OFCD. The radial profiles for this standard, relaxed plasma are shown in Fig. 4.1, which displays the parallel components of the current, electric field, and dynamo effect generated by tearing modes. As shown in chapter 2, the tearing modes essentially transfer current from the core to the edge, to counter the peaking of the current by the applied electric field.

The dynamics of OFCD are presented in details, both through the cycle-averaged quantities and the behaviors during a cycle. We discuss the results at $S = 10^5$ in Sec. 4.2 and $S = 5 \times 10^5$ in Sec. 4.3. The large oscillations of the axisymmetric profiles can cause the excitation of the edge-resonant modes. The linear and quasilinear behavior of these modes and S-scaling will be discussed in Sec. 4.4.

4.2 $S = 10^5$

At some time during the steady-state phase of the plasma, the time-independent axial electric field is set to zero, and the oscillating poloidal and toroidal electric fields that constitute OFCD are applied. We first examine the effect on the total current and magnetic helicity in Sec. 4.2.1. In Sec. 4.2.2, we then examine the cycle-averaged terms in Ohm's law, including the two dynamo effects – one arising from the axially and azimuthally symmetric fields (\mathbf{V} and \mathbf{B}) oscillating at the OFCD frequency and one from the tearing fluctuations. For a more detailed analysis, we then investigate the behavior of each of the terms in Ohm's law, and the magnetic fluctuation spectrum, through an OFCD cycle in Sec. 4.2.3.

4.2.1 The axisymmetric quantities

The target plasma for OFCD, shown in Fig. 4.1, was computed with 147 radial mesh points, poloidal mode numbers m=0 to 5, and axial mode numbers n=-21 to 21. The target plasma was sustained at $\Theta = 1.8$ with a helicity injection rate $\dot{K} = \phi_z v_z = 50$. If the axial electric field is suddenly set to zero (at t = $0.24\tau_R$ in Fig. 4.2) then the current decays in a fraction of a resistive diffusion time (the dashed curve). To study OFCD, at t = $0.24\tau_R$ we impose boundary conditions $\hat{E}_z = 80 \sin(\omega t)$, $\hat{E}_{\theta} = 8 \sin(\omega t + \pi/2)$. This provides a helicity injection rate of $\hat{v}_z \hat{v}_{\theta}/2\omega = 35$. which is lower than the helicity injection rate of the target. As seen in Fig. 4.2 OFCD sustains the cycle-averaged current at about 2/3 of its initial value. However, the oscillations in the current are greater than 100%, causing the current to reverse direction.

If the OFCD helicity injection rate is increased, the cycle-averaged current increases and the relative oscillations decrease. We observe in Fig. 4.3 that if the OFCD helicity injection rate is doubled, then the cycle-averaged current increases by 20% and the current oscillations decrease by 10%. The cycle-averaged helicity is also seen to be sustained (Fig. 4.4). However, the helicity reaches a value that is less than the initial (by about 30%), despite the fact that the OFCD helicity injection rate exceeds that of the target plasma (by about 35%). This implies that the total helicity dissipation rate $\overline{\eta \int \mathbf{J} \cdot \mathbf{B} d\mathbf{v}} \approx 67$, including both symmetric oscillation and asymmetric fluctuation contributions, increases with OFCD (Fig. 4.5). The two components of the helicity dissipation rate are shown in Fig. 4.5(b). In standard RFP surrounded by a close-fitting conducting shell, the time-averaged helicity dissipation due to the tearing fluctuations is negligible. As it is seen in Fig. 4.5(b) the tearing fluctuating part of the helicity dissipation increases with OFCD (shown by the thicker line), resulting in a cycle-averaged value of a few percent of the total helicity dissipation rate. Axial current (Fig. 4.3) decreases when the helicity dissipation due to the tearing fluctuations increases. Due to the nonlinear plasma response, both the axial current and the helicity dissipation rate are not sinusoidal in time (Figs. 4.3 and 4.5). The sudden rise of the helicity dissipation (Fig. 4.5) indicates large changes in the mean profiles during a cycle.

The choice of frequency is important for efficient current drive. The frequency should be low enough that edge current can be transported by the tearing fluctuations into the plasma core, but high enough to avoid change of direction of the total plasma current through a cycle. A full frequency scan for a given Lundquist number would therefore be of interest. A scan is presently infeasible due to the long computational time required. We investigate OFCD at two frequencies. At low frequency, when the driving period is much longer than the plasma relaxation time scale, the plasma current (and Θ) changes sign (Fig. 4.6). Whether the plasma maintains the reversal during the OFCD cycle depends upon the ratio of the poloidal and toroidal oscillating amplitudes. At higher helicity injection rates and $\varepsilon_{\theta}/\varepsilon_z$ in the range of 10-15%, the toroidal field reversal parameter, F, is less positive and plasma



Figure 4.1: Radial profiles of the three terms in parallel Ohm's law, $E_{\parallel} + S < \widetilde{\mathbf{V}} \times \widetilde{\mathbf{B}} >_{\parallel} = \eta \mathbf{J}_{\parallel}$ for a standard RFP plasma. The dynamo term includes contribution from the m=0 and m=1 tearing modes for all the axial mode numbers, n ($S = 10^5$).



Figure 4.2: Total axial current vs. time. The oscillating fields $\hat{E}_z = 80 \sin(\omega t)$, $\hat{E}_{\theta} = 8 \sin(\omega t + \pi/2)$ are applied at $t = 0.24\tau_R$ ($\tau_{\omega} = 1.05 \times 10^3 \tau_A$). The bold points indicate the cycle-averaged current. The dashed line is the exponentially decaying current that occurs in the absence of OFCD ($E_z(a)$ set to zero at $t = 0.24\tau_R$).



Figure 4.3: Total axial plasma current vs. time for $\varepsilon_z = 112$, $\varepsilon_\theta = 11$, $\tau_\omega = 1.05 \times 10^3 \tau_A$, $S = 10^5$. When plasma reaches quasi steady-state, the cycle-averaged current, $\overline{I_z} = 2$ is shown by the solid trace.



Figure 4.4: Helicity vs. time. The solid line with points shows the cycle-averaged helicity.



Figure 4.5: (a) Total helicity dissipation rate, $\dot{K}_{diss} = \eta \int \mathbf{J} \cdot \mathbf{B} d\mathbf{v}$ vs. time. The solid line is the helicity dissipation before OFCD (about 50). The bold points show the cycle-averaged total helicity dissipation rate, which at steady-state balances the OFCD helicity injection rate, $\overline{\eta \int \mathbf{J} \cdot \mathbf{B} d\mathbf{v}} \approx 67$. (b) The two terms contributing to the total helicity dissipation rate , the symmetric mean part $\eta \int \mathbf{J}_{00} \cdot \mathbf{B}_{00} d\mathbf{v}$. and the asymmetric fluctuating part $\eta \int \mathbf{J} \cdot \mathbf{B} d\mathbf{v}$ (m, n $\neq 0$) are shown. The thicker line indicates the fluctuating part.



Figure 4.6: $F - \Theta$ trajectories for two different periods. (a) $\tau_{\omega} = 1000\tau_A$, (b) $\tau_{\omega} = 1500\tau_A$, $(S = 10^5)$, where $\tau_{\omega} = 2\pi/\omega$.



Figure 4.7: $F - \Theta$ trajectories for $\varepsilon_z \varepsilon_{\theta} / \omega = 2.1$ (dashed) and $\varepsilon_z \varepsilon_{\theta} / \omega = 2.7$ (solid). The driving frequency is the same for the two cases. The toroidal field is more deeply reversed for higher helicity injection.

maintains the reversal (Fig. 4.7).

According to the helicity balance equation, the phase between the axial and poloidal voltages for maximal helicity injection is $\delta = \pi/2$. We have also examined $\delta = 0$ and $\delta = -\pi/2$. Fig. 4.8 shows that both the cycle-averaged helicity and the cycle-averaged current decay to zero as expected when $\delta = 0$. The dashed line in Fig. 4.8 shows helicity and current when the axial electric field is set to zero (no OFCD) and the solid line with bold points indicates the cycle-averaged current with OFCD with $\delta = 0$. The OFCD cycle-averaged current decays faster than the ohmic current (dashed line). The opposite phase $(\delta = -\pi/2)$ leads to helicity ejection and cycle-averaged helicity and current decay more rapidly during the early cycles.

4.2.2 The cycle-averaged quantities

A large time variation of the parallel current density, J_{\parallel} , occurs during an OFCD cycle, shown in Fig. 4.9 for maximum and minimum Θ . Current density is peaked in the interior of the plasma when Θ is maximum and F is most negative. The OFCD period is in the



Figure 4.8: (a) Helicity and (b) axial current vs. time when phase between axial and poloidal oscillating fields is set to zero ($\delta = 0$). The decay of K and I_z , when ohmic axial electric field is set to zero (without OFCD) are shown with the dashed line. The bold points are the cycle-averaged quantities (with OFCD).

range of the hybrid tearing time; thus, the current penetrates to the interior of the plasma. The cycle-averaged $\lambda(r)$ profile is shown in Fig. 4.10. Non-zero parallel current density on axis is evidence of the penetration of edge current into the core through the tearing mode dynamo effect. The time-averaged λ and J_{\parallel} profiles of the standard RFP plasma are also shown.

The dynamics of this current relaxation can be investigated by analyzing the dynamo terms (from both the symmetric oscillations $(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}$ and the tearing fluctuations



Figure 4.9: Parallel current density at two different times during a cycle, at maximum (solid line) and minimum Θ (dashed line). $\varepsilon_z = 112$, $\varepsilon_\theta = 11$ and $\tau_\omega = 1.05 \times 10^3 \tau_A$.

 $\langle \widetilde{\mathbf{V}} \times \widetilde{\mathbf{B}} \rangle_{\parallel}$) in the cycle-averaged parallel Ohm's law. As expected, the oscillations drive a cycle-averaged edge current (Fig. 4.11(a)). The core current is mainly sustained by the tearing dynamo (Fig. 4.11(b)).

4.2.3 Temporal behavior during a cycle

During one cycle, the plasma is driven to a state which is far from relaxed, with significant effect on fluctuations. In the standard RFP the current density is controlled by the core tearing modes, resonant within the reversal surface, with mode numbers m=1, n=-2 to -10, as shown in Fig. 4.12. The oscillating fields of OFCD broaden the q profile and excite additional modes. Edge modes, resonant outside the reversal surface, with m=1, n=2, are excited, as well as additional core modes with n=1, n=-2, as shown in Fig. 4.13. The edge-resonant mode develops the largest amplitude. The linear and quasilinear computations of the edge-resonant modes will be discussed in Sec. 4.4.

The plasma experiences two phases of the magnetic fluctuations, the helicity injection and ejection phases (Fig. 4.14). In the helicity injection phase ($\dot{K} > 0$), the total plasma current (or Θ) increases and core fluctuations transport edge current into the core. In



Figure 4.10: (a) Cycle-averaged $\lambda = J_{\parallel}/B$ and (b) cycle-averaged parallel current density, J_{\parallel} , profile without OFCD (dashed) and with OFCD (solid). Since the total current is smaller with OFCD (see Fig. 4.3), J_{\parallel} is smaller as well.



Figure 4.11: Cycle-averaged dynamo terms (a) from symmetric oscillations $(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}$ and (b) from the asymmetric tearing dynamo terms $\langle \widetilde{\mathbf{V}} \times \widetilde{\mathbf{B}} \rangle_{\parallel}$.

the helicity ejection phase, Θ decreases , and the global edge-resonant modes suppress the current density everywhere. The λ profiles at different times during one cycle, marked by the vertical lines in Fig. 4.14, are shown in Fig. 4.15. The first three profiles (a)-(c) are during the helicity ejection phase, while (d)-(f) show the λ profiles during the injection phase. As is seen, the λ profile varies from hollow (during the ejection phase) to peaked (during the injection phase) within a cycle. Radial dynamo profiles during a cycle can provide better understanding of current relaxation process from edge to the core region. Fig. 4.16 illustrates the surface average dynamo term of the dominant core modes, m=1,n=-2,-3,-4,-5, at different times marked by the vertical lines in Fig. 4.14. As seen, on average the $\langle \widetilde{\mathbf{V}} \times \widetilde{\mathbf{B}} \rangle_{||}$ term suppresses current in the core region during the ejection phase (Figs. 4.16(a) and (b)) and drives current on axis during the injection phase (Figs. 4.16(e))



Figure 4.12: Modal magnetic energy $(W_{m,n} = 1/2 \int \tilde{B}_{r(m,n)}^2 d^3r)$ vs time for a standard RFP. The (1,-4) and (1,-3) modes are the most dominant tearing modes ($S = 10^5$, R/a=1.66).

and (f)).

4.3 $S = 5 \times 10^5$

Although OFCD is able to sustain the plasma current at $S = 10^5$, the current oscillations are large. The relaxed state model predicts that the current oscillations decrease with Lundquist number. [21] To investigate the effect of higher Lundquist number on current oscillations and magnetic fluctuations, we have performed a computation at $S = 5 \times 10^5$. We have employed higher spatial resolution (260 radial mesh points, $0 \le m \le 5$ and $-41 \le n \le 41$) to allow for more localized features that accompany higher S values. Ohmic helicity injection is replaced by OFCD at t=0.035 τ_R , as shown in Fig. 4.17(a). The current is sustained and the current oscillations are indeed reduced by about 50% relative to $S = 10^5$. The corresponding $F-\Theta$ trajectory is shown in Fig. 4.17(b), where it is seen that the plasma maintains reversal for



Figure 4.13: Time histories of magnetic energy, $W_{m,n} = 1/2 \int \tilde{B}_{r(m,n)}^2 d^3r$ for the dominant tearing modes, (m,n)=(1,+2),(1,-3),(1,-4),(1,-2) in an OFCD-sustained plasma. The edge resonant mode m=1,n=+2, is excited by the oscillating fields and has the largest amplitude.



Figure 4.14: Time histories of helicity K, reversal parameter F, pinch parameter Θ , and magnetic fluctuation $\widetilde{\mathbf{B}}/B$ ($S = 10^5$).



Figure 4.15: λ profiles for different times during one cycle (for times marked with vertical lines in Fig. 4.14).



Figure 4.16: Profiles of the surface-averaged dynamo, $\sum_{m=1,n} \langle \widetilde{\mathbf{V}} \times \widetilde{\mathbf{B}} \rangle_{||}$ including n=-2,-3,-4,-5, at different times during one cycle (times marked with vertical lines in Fig. 4.14). most of the cycle.

The cycle-averaged λ profile is shown in Fig. 4.18. For the same helicity injection rate, the cycle-averaged parallel current density on axis is higher than the $S = 10^5$ case, indicating that current penetrates more effectively into the plasma core at higher S. Similar to the $S = 10^5$ case, there are two phases, the helicity injection (current drive phase) and helicity ejection phase. In the helicity injection phase, the positive dynamo term from the core tearing fluctuations, transfers the edge current into the core. Because of the excitation of the edge-resonant modes, magnetic fluctuations level are enhanced (about the same level of $S = 10^5$ case) during the ejection phase. The λ profiles during the injection and ejection phases are shown in Fig. 4.19. This profile varies from hollow (during the ejection phase) to peaked (during the injection phase) within a cycle.

Figures 4.20(a)-(d) illustrates the m=1 magnetic energy spectrum, at different times during the OFCD cycle. The corresponding q profiles are shown in Figs. 4.21(a)-(d), including the cycle-averaged q profile (shown by the thicker line) for comparison. The dominant core



Figure 4.17: (a) Toroidal plasma current I_z , and (b) $F - \Theta$ trajectory for OFCD-sustained plasma at $S = 5 \times 10^5$ ($\varepsilon_z = 140$, $\varepsilon_{\theta} = 16$ and $\tau_{\omega} = 2.85 \times 10^3 \tau_A$). The $F - \Theta$ limit-cycle is shown by the solid curve.



Figure 4.18: Radial profile of cycle-averaged λ ($S = 5 \times 10^5$).

modes m=1, n=-3, -4, -5, -6 can be seen in Fig. 4.20(a) with the magnetic fluctuation level about 0.1-2%. This spectrum is the typical spectrum during the maximum current drive, maximum Θ , and is similar to the standard inductive RFP spectrum. The q profile at this time is shown in Fig. 4.21(a). As discussed earlier, when the plasma reversal starts to deepen, edge-resonant modes become linearly unstable and the dominant modes move toward the positive part of the spectrum. The q profile on the edge becomes more negative (Fig. 4.21(b)). The linearly growing m=1, n=+2 mode is seen in Fig 4.20(b). This figure shows the magnetic spectrum during the growth of edge-resonant mode fluctuations. At this time the m=1, n=+2 fluctuation level is about 10% and the core mode (m=1,n=-3,-4,-5,-6) fluctuation level is about 0.1-1%. It can also been seen in Fig. 4.20(c) that the amplitudes of other edge-resonant modes m=1, n=+3,+4 start to increase to higher values (1-5 %) during the peak of the \mathbf{B}/B . The q profile for this spectrum is broader both on axis and on the edge (Fig. 4.21(c)). The spectrum after the decay of edge-resonant modes begins to return to the typical standard RFP spectrum with the core dominant mode m=1, m=-3. Figs. 4.20(d) and 4.21(d) show the spectrum and the q profile at a time during the injection phase.



Figure 4.19: λ profiles at four different times during OFCD cycle (ejection and injection phases), $S = 5 \times 10^5$.

4.4 The excitation of edge-resonant modes – linear and quasilinear computations

As shown in Sec. 4.2.3, edge-resonant mode with n=1, n=2 develops the largest amplitude. The edge modes become resonant as the reversal deepens through an OFCD cycle, with F reaching -2. To determine whether this mode is linearly unstable or nonlinearly driven we compute the linear drive terms in the equation

$$\frac{1}{2}\frac{\partial B_1^2}{\partial t} = S\mathbf{B}_1^*[(\mathbf{B}_0 \cdot \nabla)\mathbf{V}_1 - (\mathbf{V}_1 \cdot \nabla)\mathbf{B}_0] + \dots$$
(4.2)

where the "1" subscript indicates a perturbed m=1, n=2 quantity and a "0" subscript indicates a mean (0,0) quantity. We compute the volume integral of the LHS and RHS of Eq. (4.2). We observe that during the sudden growth phase, the two terms are equal (Fig. 4.22). Thus, the growth of m=1, n=2 mode is a linear instability and nonlinearity only affects the saturation and damping of this mode. A linear resistive MHD stability



Figure 4.20: The evolution of the magnetic energy $W_{m=1,n}=1/2\int B_{r,(m=1,n)}^2 d^3r$ spectrum during an OFCD cycle ($S = 5 \times 10^5$). The dominant (m,n) modes are marked in the figures.



Figure 4.21: (a)-(d) are the q profiles for the spectrums (a)-(d) in Fig. 4.20, respectively. The thicker profile is the cycle-averaged q profile.

analysis has also been performed to obtain the growth rate and spatial structure of this mode. Linear evolution of the mode is studied using the DEBS code (with all other modes suppressed). Equilibrium profiles are chosen to resemble those of the deeply reversed phase of OFCD (Fig. 4.23). The global eigenfunctions of the m=1,n=+2 mode are shown in Fig. 4.24. The growth rate of the mode , $\gamma \tau_A = 0.1$, is in the range expected for ideal MHD instability.

As shown in Sec. 2.4.2, edge-resonant tearing mode, resonant outside the reversal surface, have similar mode structure to core resonant tearing mode and their growth rates follow the linear tearing S-scaling ($\gamma \propto S^{-3/5}$). There is a spectrum of m=1 edge-resonant modes that can be excited linearly. With the equilibrium chosen in Sec. 2.4.2, the m=1, n=+6 mode has both resistive mode structure and resistive growth rate. However, if the current gradient around the resonant surface increases, the edge-resonant modes can be driven harder and approach the ideal regime with ideal growth rates (close to Alfvenic). Edgeresonant modes can particularly be excited by AC helicity injection. The large modulations of axisymmetric fields by OFCD in low-S plasmas and deep reversal cause edge-resonant modes to grow linearly. We therefore here analyze these modes using both linear and quasi-linear computations.

The edge-resonant modes excited in the 3-D computations with full current sustainment by OFCD, are mostly low-n modes (m=1, n=+2). The linear and quasi-linear stability analysis of m=1, n=+2 edge-resonant mode is investigated here. The equilibrium profiles are chosen to resemble the equilibrium profiles of the 3-D OFCD case. As discussed in section 2.3 the alpha equilibrium model with extreme reversal profiles has been used with $\alpha = 65$, $\theta = 1.75$. The linear eigenfunction of the mode shown in Fig. 4.24 is global and is different from the resistive edge-resonant localized mode structure shown in chapter 2. Because of the deeply reversed equilibrium profiles and large gradient around the resonate surface, this mode has growth rate close to the ideal regime ($\gamma \tau_A \sim 0.1$).

We have also studied the S-scaling of the linear edge-resonant modes. The S-scaling of two edge-resonant modes (1,+2) and (1,+6) is shown in Fig. 4.25. The (1,+6) mode with mode structure shown in section 2.3 is a resistive edge-resonant mode with growth rate that scales as $S^{-3/5}$. The (1,+2) mode however (with deeper reversed equilibrium profiles), has growth rate close to ideal and does not conform to the tearing S-scaling.

The quasi-linear computations are performed by allowing the same equilibrium profiles as in the linear cases evolve, but with an oscillating field imposed on the plasma boundary $(\varepsilon_{\theta} = 5.2, \varepsilon_z = 35 \text{ and } \omega \tau_R = 250)$. The single edge-resonant mode (1,+2) starts to grow linearly as the mode becomes resonant on the q profile. However, because the equilibrium can evolve in response to the mode, the mode amplitude saturates as shown in Fig. 4.26. This figure illustrates the sudden growth as F becomes very deep and the saturation of this mode in the quasi-linear OFCD simulation at $S = 5 \times 10^4$. Similar behavior in the fluctuation amplitude of the (1,+2) mode is seen in 3-D computations (see Fig. 4.13). Thus,



Figure 4.22: The m=1, n=+2 energy terms (integrated over radius) of Eq. (4.2) vs. time. The total energy (LHS) is shown by the solid line. The diamonds show the sum of the linear energy terms in the RHS. The growth period where the total energy (LHS) and linear energy (RHS) overlap, is marked by the shaded area.

we conclude that the increase in the total magnetic fluctuation in the 3-D computations is mainly due to the quasi-linear evolution of a single edge-resonant mode. To verify that this mode is linearly driven, we have suppressed all the tearing fluctuations in the 3-D OFCD computation case $S = 10^5$ except the dominant edge-resonant mode (1,+2). Under this condition, the mode amplitude of (1,+2) mode still starts to grow as F deepens as shown in Fig. 4.27, indicating that the mode growth does not depend upon the other modes (i.e. it is not driven nonlinearly).



Figure 4.23: Profiles of the equilibrium magnetic fields, B_z and B_{θ} , and q profile for the linear calculation of the m=1, n=+2 edge-resonant mode.



Figure 4.24: Linear radial eigenfunctions of the m=1, n=+2 mode.



Figure 4.25: The Lundquist number scaling of edge-resonant modes. The edge-resonant mode m=1, n=+2 has growth rate close to ideal. However, there is a slow decrease of the growth rate with S. The edge-resonant m=1, n=+6 mode is resistive with tearing S-scaling $(\gamma \propto S^{-3/5})$. The triangles and diamonds are the computational points.



Figure 4.26: Quasi-linear evolution of edge-resonant mode (1,+2) with OFCD boundary condition. (a) Modal magnetic energy $W_{m,n} = 1/2 \int \tilde{B}_{r(m,n)}^2 d^3r$ vs time (b) F vs time.



Figure 4.27: Quasi-linear evolution of edge-resonant mode (1,+2) when all the other tearing modes have been suppressed in the 3-D OFCD computation. (a) Modal magnetic energy $W_{m,n} = 1/2 \int \tilde{B}_{r(m,n)}^2 d^3r$ vs time (b) F vs time.

4.5 Summary

We have investigated the full nonlinear dynamics of OFCD, a form of AC helicity injection, using 3-D nonlinear MHD computation. 3-D plasma fluctuations and instabilities in large part determine the effectiveness of OFCD and its influence on confinement. The full MHD dynamics of OFCD can only be explained using 3-D nonlinear modeling where all the tearing fluctuations are present and can nonlinearly interact. The 3-D MHD computation provides understanding of current relaxation through the non-axisymmetric MHD fluctuations.

The 1-D relaxed state model with fixed current density profile reveals the scaling of the current modulations on the key parameters (see Ref. [20, 21]). Because of the large amount of CPU time and memory required, investigating the full 3-D scaling of the modulations of the both axisymmetric quantities and fluctuations is numerically challenging.

The 1-D OFCD-driven edge current excites plasma MHD instabilities and fluctuations which then drive current in the core through the dynamo effect that arises from nonaxisymmetric velocity and magnetic fluctuations. That is, magnetic relaxation causes the current to penetrate to the core. This physics is captured through 3-D MHD computation. We find that OFCD indeed can sustain the plasma current steady-state in the absence of a DC electric field. There are two causes for concern for the OFCD as a steady-state current drive technique. First, the effectiveness of the current drive and the oscillations of the axisymmetric quantities. Second, the effect of OFCD on the non-axisymmetric fluctuations and transport. The axisymmetric plasma quantities, such as the toroidal current, experience very large oscillations. For example, at $S = 10^5$ the current oscillates by 100%, a value likely unacceptable in an experimental plasma. However, we find that the current oscillation decreases to about 50% at $S = 5 \times 10^5$, consistent with the prediction of the 1-D relaxed state model that oscillations scale as $S^{-1/4}$. Thus, at the higher S values of experiments or a reactor, the current oscillation may be acceptably small. We have also optimized OFCD with regard to frequency and the relative phase. As expected, the optimum frequency is one that is sufficiently low to permit relaxation to occur and sufficiently high that the oscillation in the total current is minimized. We have examined three different phases, $-\pi/2, 0, \pi/2$ in
the 3-D modeling. As expected, the $\delta = \pi/2$ results in the maximum AC helicity injection and current. The zero and $-\pi/2$ phases yield no cycle-averaged helicity and current.

We have studied both the spatial and temporal variation of all the terms in parallel Ohms's law. We have examined the response of both the oscillating axisymmetric profiles and the non-axisymmetric fluctuations through a cycle as well as the cycle-averaged response. It has been shown that the resistive MHD fluctuations transfer the OFCD-driven edge current, $\langle \mathbf{V}_{00} \times \mathbf{B}_{00} \rangle_{\parallel}$, into the core of the plasma, generating a non-zero current density over the entire plasma cross-section. The profiles of the mean fields (such as J_{\parallel}/B) and the fluctuations vary significantly throughout a cycle. For example, the J_{\parallel}/B profile varies from hollow to peaked within a cycle. The profiles are such that the helicity dissipation is higher than for conventional current sustainment by a DC toroidal electric field. Hence, the helicity injection rate for an OFCD-sustained plasma is greater than that for standard Ohmic plasmas.

Plasma fluctuations (and transport) can be affected by OFCD. We identify two parts of the OFCD cycle. During the helicity injection phase, the current density profile peaks and the tearing mode dynamo drives current in the core (transporting current from edge to the core). The fluctuation level is roughly equal to that of the standard RFP. During the helicity ejection phase, new global modes appear that are resonant at the extreme plasma edge. These modes produce a "dynamo" effect that suppresses current everywhere. A linear stability analysis shows that these modes are unstable in plasmas with strong field reversal (large, negative toroidal magnetic field at the plasma surface). The calculation of the linear and total modal energy drives in the 3-D computation show that this mode is linearly driven under the extremely deep field reversal equilibrium condition. Therefore, the instability is suppressed in high S plasmas where the reversal is weak. Clearly, investigations at even higher S values, beyond the scope of the present computation, are needed.

References

- M. K. Bevir and J. W. Gray, in Proceedings of the Reversed-Field Pinch Theory Workshop, edited by H. R. Lewis and R. A. Gerwin (Los Alamos Scientific Laboratory, Los Alamos, NM, 1981), Vol. III, p. A-3.
- [2] T. H. Jensen and M. S. Chu, Phys. Fluids 27, 2881 (1984).
- [3] M. K. Bevir, C. G. Gimblett and G. Miller, Phys. Fluids 28, 1826 (1985).
- [4] T. R. Jarboe, I. Henins, A. R. Sherwood, C. W. Barnes, and H. W. Hoida, Phys. Rev. Lett. 51, 39 (1983).
- [5] M. Ono, G. J. Greene, D. Darrow, C. Forest, H. Park, and T. H. Stix, Phys. Rev. Lett. 59, 2165 (1987).
- [6] B. A. Nelson, T. R. Jarboe, D. J. Orvis, L. A. McCullough, J. Xie, C. Zhang, and L. Zhou, Phys. Rev. Lett. 72, 3666 (1994).
- [7] K. F. Schoenberg, J. C. Ingraham, C. P. Munson *et al.*, Phys. Fluids **31**, 2285 (1988).
- [8] J. B. Taylor, Phys. Rev. Lett. **33**, 1139 (1974).
- [9] K. F. Schoenberg, R. F. Gribble and D. A. Baker, J. Appl. Phys. 56, 2519 (1984).
- [10] H. R. Strauss and D. S. Harned, Phys. Fluids **30**, 164 (1987).
- [11] D. S. Harned, D. D. Schnack, H. R. Strauss and R. A. Nebel, Phys. Fluids **31**, 1979 (1988).
- [12] C. R. Sovinec, J. M. Finn, and D. del-Castillo-Negrete, Phys. Plasmas 8, 475, (2001).
- [13] C. R. Sovinec, S. C. Prager, Phys. Plasmas 3, 1038, (1996).
- [14] D. D. Schnack, D. C. Barnes, Z. Mikic, D. S. Harned, and E. J. Caramana, J. Comput. Phys. 70, 330, (1987).

- [15] P. M. Bellan, Phys. Rev. Lett. 54, 1381, (1985).
- [16] F. Najmabadi *et al.*, Final Report No. UCLA-PPG-1200, University of California, Los Angeles (1990).
- [17] M. R. Stoneking, J. T. Chapman, D. J. Den Hartog, S. C. Prager, and J. S. Sarff, Phys. Plasmas 5, 1004, (1998).
- [18] C. R. Sovinec, Ph.D. thesis, University of Wisconsin, Madison, (1995).
- [19] Y. L. Ho, D. D. Schnack, P. Nordlund, S. Mazur, H.-E. Sätherblom, J. Scheffel, and J. R. Drake, Phys. Plasmas 2, 3407, (1995).
- [20] F. Ebrahimi, S. C. Prager, J. S. Sarff and J. C. Wright, Phys. Plasmas 10, 999, (2003).
- [21] Private communication with John Sarff.

5 Current profile control by AC helicity injection

5.1 Introduction

In conventional RFP devices, a toroidal inductive electric field has been used to drive and sustain the plasma current. The edge magnetic field is dominantly poloidal in the RFP and poloidal current drive is required for parallel current profile control. Different techniques have the potential for current profile control and ultimately for steady-state current sustainment as the RFP configuration advances toward improved confinement conditions necessary for reactor operation. The main purpose of current profile control in RFPs is to suppress the magnetic fluctuations. In the past few years, the core tearing fluctuations have been reduced substantially through inductive current profile control. A surface poloidal inductive electric field has been applied experimentally to drive edge poloidal current and modify the current profile. [1] Recently toroidal loop voltage programming has also been added to optimize inductive current profile control and its effect on magnetic fluctuations and transport. [2] Non-inductive auxiliary current drive techniques, such as RF current drive can also be used for current profile control and fluctuation reduction and are currently being tested in the MST experiment. [3] AC helicity injection has been studied in the previous chapter as a method to sustain the current in RFP. It can also be used to modify the ohmic current profile. Here, we investigate current profile control via AC helicity injection.

In chapter 4, we examined steady-state current sustainment by OFCD using 3-D nonlinear MHD computations. We found that OFCD can sustain the plasma current steady-state in the absence of an ohmic toroidal loop voltage. We also showed that full current sustainment by OFCD leads to the excitation of the edge-resonant modes and large modulation amplitudes at low Lundquist number S. As a result the total magnetic fluctuations are increased. However, the core tearing fluctuations did not display a significant change. Here, we present 3-D MHD simulations of OFCD demonstrating both significant shaping of the ohmic current profile, partial current sustainment, and reduction of magnetic fluctuations.

Using the concept of magnetic helicity balance, the rate of change of magnetic helicity

is

$$\frac{\partial K}{\partial t} = 2(\phi_z v_z) - 2 \int \mathbf{E} \cdot \mathbf{B} d\mathbf{v} , \qquad (5.1)$$

For partial current sustainment by OFCD, the helicity injection rate $\phi_z v_z$ on the RHS of Eq. 5.1 consists of the contribution from both the ohmic helicity injection rate $(\phi_z v_z)_{dc}$ and the AC helicity injection rate $\hat{\phi}_z \hat{v}_z$ (the "hat" denotes an oscillating quantity). In steadystate, the dissipation rate (the second term on the RHS) balances the helicity injection rate. Electrostatic helicity injection, which requires the intersection of magnetic field lines with biased electrodes, has been simulated using a resistive MHD code and showed stabilization of the tearing modes. [4, 5]

To inject AC magnetic helicity we impose oscillating fields on a relaxed plasma (standard RFP) which is ohmically sustained by an axial time-independent electric field. In the present simulations about 50% of the DC magnetic helicity (ohmic helicity) is injected by oscillating fields. The computations are at Lundquist number $S = 10^5$ and aspect ratios R/a=2.88 (MST aspect ratio) and R/a=1.66. For high aspect ratio R/a=2.88, we have used resolutions 220 radial mesh points with 41 axial modes, -41 < n < 41, and 5 azimuthal modes, $0 \le m < 5$. Lower resolutions were sufficient for aspect ratio R/a=1.66.

In this chapter we will analyze the details of an OFCD case with significant current profile modification. In Sec. 5.2 the time-averages of both the axisymmetric quantities and the non-axisymmetric fluctuations are presented. The time variations of both axisymmetric fields and the asymmetric magnetic fluctuations throughout an OFCD cycle are discussed in Sec. 5.3. To understand the dynamics of OFCD for current profile control, we study, first oscillating poloidal current drive (OPCD) in which only the poloidal surface electric field is oscillated, then oscillating toroidal current drive (OTCD) in which only the surface toroidal electric field is oscillated. The detailed dynamics of OPCD and OTCD are presented in Secs. 5.3.1 and 5.3.2. The AC helicity injection rate decreases with oscillation frequency. However, the frequency should be low enough that edge OFCD-driven current can be relaxed by the tearing fluctuations into the plasma core to result in current modification, but high enough to avoid current reversal. The optimum balance between these two effects is the topic of Sec. 5.4.

5.2 Time-averaged quantities

In our simulations, oscillating fields are imposed on a relaxed plasma (standard RFP) with a DC axial electric field boundary condition providing a pinch parameter $\theta = 1.68$. The AC helicity injection rate, $\dot{K}_{inj} = \hat{V}_z \hat{V}_{\theta}/2\omega = 19$, is about 50-60% of the ohmic DC helicity. The oscillation period is required to be between the hybrid tearing time, τ_H , and the resistive diffusion time, τ_R . Therefore, we choose an OFCD frequency $\omega \tau_H = 0.16$ ($\tau_{\omega} = 12000\tau_A$), which is low enough for both the relaxation and modification of the current density. The total axial current for the 60% AC helicity injection is shown in Fig. 5.1. Because of the low frequency, the modulation amplitudes for this case are large (about 75% of the mean). The time-averaged total axial current is increased by 10-15%.

The cycle-averaged parallel current density is increased as shown in Fig. 5.2(a). The modification of the cycle-averaged $\lambda = J_{\parallel}/B$ profile with the partial OFCD can be seen in Fig. 5.2(b). OFCD makes the λ profile flatter around the point r/a=0.8 with the reduction of the gradient starting around r=0.5.

The dynamics of current sustainment can be investigated using the cycle-averaged Ohm's law,

$$\overline{\mathbf{E}_{\parallel}} + \overline{(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}} + \overline{\langle \widetilde{\mathbf{V}} \times \widetilde{\mathbf{B}} \rangle_{\parallel}} = \overline{\eta \mathbf{J}_{\parallel}} , \qquad (5.2)$$

where \mathbf{V}_{00} and \mathbf{B}_{00} are the oscillating velocity and magnetic fields with poloidal and toroidal mode numbers $\mathbf{m} = \mathbf{n} = 0$, $\mathbf{\tilde{V}}$ and $\mathbf{\tilde{B}}$ are the fields with \mathbf{m} , $\mathbf{n} \neq 0$, and $\langle \rangle$ denotes an average over a magnetic surface $[()_{\parallel} = () \cdot \mathbf{\bar{B}}/B$, where $\mathbf{\bar{B}}$ is the cycle-averaged mean (0,0) magnetic field]. The second and third terms are the dynamo terms generated by the axisymmetric oscillations and the non-axisymmetric tearing instabilities, respectively. The first term $\mathbf{\bar{E}}_{\parallel}$ is the ohmic toroidal electric field which is zero for the full current sustainment by OFCD in the absence of a DC loop voltage. [6] The second term is the OFCD dynamo term which represents the contribution of current driven by partial OFCD. Using $\mathbf{V}_{00} = \mathbf{E}_{00} \times \mathbf{B}/B^2$, the first and second terms can be combined and written as $(\mathbf{\bar{E}}_{00} \cdot \mathbf{\bar{B}}_{00})/B$. Therefore,



Figure 5.1: The total axial current with partial OFCD at the frequency $\omega \tau_H = 0.16$ vs time. The cycle-averaged current boost (shown with dotted line) by OFCD is about 15%.

we can consider the first two terms on the LHS of Eq. 5.2 as the time-averaged parallel component of the electric field which has both oscillating (AC) and DC components. We see that the cycle-averaged parallel current is sustained by all the three term on the LHS of Eq. 5.2. However, as we will discuss in more detail later, the time variation of the current density profile during a cycle is substantial. The electric field variations and the resulting parallel current gradients around the core resonant modes during a cycle affects the resistive MHD instabilities and the tearing fluctuation amplitudes. Thus, the significant effect is the reduction of the total fluctuation amplitudes. As is shown in Fig. 5.3 the fluctuation amplitude become zero during part of the OFCD cycle. The time average of the total rms fluctuation amplitudes decreases by a factor of 2–2.5. Below we present a detailed analysis of this case during a cycle.



Figure 5.2: Radial profiles of (a) the cycle-averaged parallel current density J_{\parallel} (b) the $\lambda = J_{\parallel}/B$. The dashed lines denote the same profiles for standard RFP (without OFCD).



Figure 5.3: The magnetic fluctuation amplitude $[rms(\tilde{B}/B)]$ with partial OFCD and without OFCD (standard RFP).

5.3 Time-dependence

One of the basic features of OFCD sustainment is that the oscillating fields cause large variations of the axisymmetric profiles during a cycle. As was shown above, large oscillations of the total axial current are observed. The axisymmetric magnetic and velocity fields also exhibit large variations throughout a cycle. Therefore, in this section we study the detailed dynamics of profile variations and magnetic fluctuations throughout a cycle. The time-averaged magnetic fluctuations exhibit a reduction by a factor of 2 by applying partial OFCD. The physics behind the modified current profile and the suppression of the magnetic fluctuations can be explained through detailed study of the profile variations during a cycle. The three terms in parallel Ohm's law: the current density, the electric field and the fluctuation-induced dynamo term will be studied during an OFCD cycle.

To understand the dynamics of OFCD for current profile control, we first investigate oscillating poloidal current drive (OPCD) and oscillating toroidal current drive (OTCD) separately. We study the separate effect of OPCD and OTCD on both current profile and magnetic fluctuations. Then, we present the OFCD dynamics in which both toroidal and poloidal electric fields are oscillated out of phase to inject a time-averaged magnetic helicity and to modify the current profile.

5.3.1 Oscillating poloidal current drive (OPCD)

An oscillating poloidal electric field $[\hat{E}_{\theta} = \varepsilon_{\theta} \sin(\omega t + \pi/2), \varepsilon_{\theta} = 2.4, \tau_{\omega} = 0.126\tau_R]$ is imposed at the plasma wall on a target standard RFP plasma (with $\Theta = 1.68$) at time $t = 0.7\tau_R$. The poloidal electric field oscillates around a zero mean value, causing the parallel electric field to become both positive and negative during a cycle. This is different from the pulsed poloidal current drive (PPCD, a technique for current profile control applied on MST) in which parallel electric field is experimentally programmed to always remain positive. The radial component of the total magnetic fluctuations \tilde{B}/B and field reversal F are shown in Fig. 5.4. As is seen, the total magnetic fluctuation level is higher than the standard case but it is lower during the other part of the cycle (see Fig. 5.4(a)). Thus, the time-averaged magnetic fluctuation level remains roughly the same as in the standard plasma. Since the frequency is low, the modulation amplitude of the symmetric quantities is large, as demonstrated by the modulation of the field reversal F shown in Fig. 5.4 (b). No mean helicity is injected by the oscillating poloidal electric field $(\vec{K}_{OPCD} = \overline{\phi_z V_z} = 0)$. However, because of the change of the axisymmetric profiles and reduction of the helicity dissipation, there is a slight increase in the time-averaged axial current and helicity. The time-averaged parallel current density J_{\parallel} and $\lambda(r) = J_{\parallel}/B$ profiles are shown in Fig. 5.5, indicating that the radial-averaged current does not change significantly (a small amount of current is driven near the plasma edge and the current on axis is reduced). However, the current density gradient is reduced with OPCD from $\mathbf{r} = 0.6$ out to the plasma edge.

Although the time-averaged effect of the oscillating poloidal electric field on both axisymmetric and asymmetric fields is insignificant, OPCD does affect the radial profiles during a cycle. Figure 5.6 shows the temporal variations of the modal magnetic energies (W_{mn}) with poloidal loop voltage (V_p) . It is seen that the core mode magnetic fluctuations are reduced during the positive phase of poloidal electric field $(V_p > 0)$ and enhanced during the negative phase $(V_p < 0)$. The three terms in parallel Ohm's law at times t₁, t₂ and t_3 shown in Fig. 5.7 reveal the profile variations during a cycle. The parallel electric field $(E_{\parallel} = E_z \cdot B_z + E_{\theta} \cdot B_{\theta})$ is positive everywhere at t₁ (while $V_p > 0, F < 0$) and a more stable current density profile with smaller gradient is formed (see Fig. 5.7(a)). The current is sustained by the positive electric field ($\langle \tilde{V} \times \tilde{B} \rangle_{\parallel} = 0$). Because the magnetic field is mainly poloidal near the edge, during the negative phase with $V_p < 0$ (t₂ is shown), the parallel electric field becomes peaked in the core and negative near the plasma edge (see Fig. 5.7(b)). Thus, the current density gradient becomes large which leads to the growth of core resonant modes [(1,-4),(1,-3)] shown in Fig. 5.6 at t₂. The dynamo term becomes large (both in the core and at the edge) to relax the unstable current density profile at t_2 (see Fig. 5.7(b)). As the poloidal loop voltage changes sign, a positive parallel electric field and consequently positive current density is generated over the entire plasma radius as seen in

	$\overline{\widetilde{B}/B_0}$	\overline{K}	$\overline{\dot{K}}_{diss}$	$\overline{I_z}$	$\frac{\widehat{I_z}}{2\overline{I_z}}$	\overline{F}	F_{p-p}
OPCD	1.23~%	6.6	47.4	2.84	8 %	-0.12	$0.9 \ (F_{min} = -0.7, F_{max} = 0.2)$
OTCD	1.1 %	5.1	40.6	2.3	60 %	0.0	2.0 $(F_{min} = -1., F_{max} = 1.)$
OFCD	0.6~%	8.6	72	3.0	45~%	-0.31	2.0 $(F_{min} = -1.5, F_{max} = 0.5)$
Standard	1.25~%	5.71	41	2.6	_	-0.12	$0.17 \ (F_{min} \approx -0.2, F_{max} \approx 0.03)$

Table 2: Time-averaged quantities.

Fig. 5.7(c). The positive poloidal electric field modifies the current density profile. As a result the tearing fluctuations are reduced and the cycle repeats. The modification of the $\lambda = J_{\parallel}/B$ profile is shown in Fig. 5.8. The λ profile is flattened in the core at t₁ during the positive phase (edge drive phase $V_p > 0$) and has larger gradient during the negative phase (edge anti-drive phase $V_p < 0$).

Fig. 5.9 illustrates the variation of the q profile at the three different times. Because of the low frequency, the q profile exhibits relatively large modulations on both axis and at the edge. The modal magnetic energies shown in Fig. 5.10 oscillate with the driving frequency and have large modulations but the time-averaged modal energies are comparable to the standard modal energies. The temporal variation of the total magnetic fluctuations is mainly in phase with variations of the core modal energies [(1,-3),(1,-4)] and the m=0 mode nonlinear growth follows after the rapid growth of the dominant core modes (Fig. 5.10). We conclude that OPCD drives an edge current during the positive phase with $V_p > 0$ and suppresses the magnetic fluctuations, and OPCD generates anti-drive near the edge during the negative phase with $V_p < 0$ and enhance core modal amplitudes.

5.3.2 Oscillating toroidal current drive (OTCD)

We have also examined the dynamics of oscillating toroidal field current drive (OTCD). An oscillating axial electric field $[\hat{E}_z = \varepsilon_z \sin(\omega t), \varepsilon_z = 15]$ is imposed on the plasma wall with the same initial conditions as for the OPCD case presented above. The axial electric



Figure 5.4: (a) Radial component of the total magnetic field fluctuations \tilde{B}/B (b) Field reversal parameter F. At time $t = 0.7\tau_R$ an oscillating poloidal field is imposed on a standard plasma. The period of the poloidal electric field is $\tau_{\omega} = 0.126\tau_R$.

field oscillates with large modulations and its time-averaged value is the standard axial electric field (standard loop voltage). During the part of the cycle with large negative electric field values the axial current decreases and the fluctuation amplitude increases. Since the axial flux is time-independent ($\hat{E}_{\theta} = 0$), OTCD does not inject mean helicity $(\bar{K}_{OTCD} = \overline{\phi_z \hat{V}_z} = 0, \widehat{\phi_z} = 0)$ and consequently does not drive mean current. However, the time-averaged helicity and axial current are reduced with OTCD as shown in Table 2. This is because of the large modulation amplitudes and negative axial electric field during part of the cycle, which lead to large variation of the axisymmetric profiles. The time-averaged parallel current density J_{\parallel} and $\lambda(r)$ are reduced in the plasma core as shown in Fig. 5.11.



Figure 5.5: Time-averaged $\lambda(r)$ and J_{\parallel} profiles for OPCD and standard (STD) cases.

The time-dependent axial electric field at the boundary causes a large variation in the current profile and magnetic fluctuations. The temporal behavior of the field reversal F and the total magnetic fluctuations \tilde{B}/B with the oscillation of toroidal voltage V_z are shown in Fig. 5.12. The modal magnetic energies $W_{m,n}$ for the core modes (1,-3), (1,-4) and (1,-2) and the m=0 mode (0,1) are also shown in Fig. 5.13. Similar to OPCD, the time-averaged magnetic fluctuation level does not change significantly, but the reduction and enhancement of the total magnetic fluctuations are larger than for OPCD. Large modulation amplitudes of the axisymmetric fields and q on axis cause the core mode (1,-2) to become resonant and develop a mode amplitude comparable to the dominant core mode [(1,-3) and (1,-4)] amplitudes. The terms in parallel Ohm's law are shown in Fig. 5.14 at the four different times marked in Fig. 5.12. At time t_1 the axial electric field and toroidal field reversal are positive ($V_z, F > 0$) yielding a positive parallel electric field everywhere ($E_{\parallel} = E_z \cdot B_z$,



Figure 5.6: Oscillating poloidal loop voltage V_p , magnetic modal energy $W_{m,n}=1/2\int B_{r,(m,n)}^2 d^3r$ for the (0,1) mode and the core modes (1,-3), (1,-4) and field reversal parameter F vs time.

 $E_{\theta} \sim 0$). At this time core dominant modes [(1,-3),(1,-4)] have small amplitudes (as seen in Fig. 5.13) and the fluctuation induced dynamo term is zero $[E_{\parallel} = \eta J_{\parallel}, \text{ Fig. 5.14(a)}]$. The core modal energies shown in Fig. 5.13 start to grow as E_{\parallel} becomes negative near the edge and the fluctuation amplitudes reach their largest level. The current density gradient increases as seen in Fig. 5.14(b). At time t₂, E_{\parallel} becomes negative near the edge and the dynamo term becomes large to relax the current profile toward a flatter profile by suppressing the current in the core and driving current at the edge, as seen in Fig. 5.14(b). The field reversal is maintained as the tearing fluctuations increase and energy is transfered



Figure 5.7: The three terms in parallel Ohm's law at times (a) t_1 , (b) t_2 and (c) t_3 (OPCD).



Figure 5.8: The $\lambda(r)$ profile during edge drive phase (t₁ and t₃) and edge anti-drive phase (t₂) (OPCD).



Figure 5.9: The q profile at the three different times t_1 , t_2 and t_3 (OPCD).



Figure 5.10: Magnetic modal energy for the modes (1,-3), (1,-4) and (0,1) without OPCD (standard case) and with OPCD vs time.

to the small scale fluctuations and the m=0 modes grow through nonlinear mode coupling (Fig. 5.13). During the second part of the phase when axial electric field is negative (V_z) , the parallel electric field $E_{\parallel} = E_z \cdot B_z$ can become positive again since F is negative. The positive parallel electric field at time t₃ is shown in Fig. 5.14(c). The core tearing mode amplitudes decrease at t₃ as seen in Fig. 5.13 and the dynamo term is weaker due to the positive edge E_{\parallel} . As the axial electric field reaches its minimum negative value, the field reversal becomes positive yielding a negative parallel electric field near the plasma edge as shown in Fig. 5.14(d). The dynamo term becomes strong again to relax the current density. As is seen in Fig. 5.14(d), the current density in the core is fairly flat which causes the reduction of core tearing modes at later times when E_{\parallel} begins to become positive again near the edge. The cycle repeats and returns back to the profiles shown at time t₁. Thus,



Figure 5.11: Time-averaged $\lambda(r)$ and J_{\parallel} profiles shown for OTCD and standard (STD) cases.

we find that the modification of the current profile is significant by OTCD accompanied by large modulation amplitudes. During part of the OTCD cycle, a positive E_{\parallel} profile is generated and an edge current is driven. OTCD also flattens the current density profile in the core out to the radius r=0.9. The latter effect is not produced by OPCD.

5.3.3 The combination of the oscillating fields – OFCD

Through the separation of oscillating poloidal field and oscillating toroidal field, we learned that the time-averaged magnetic fluctuation level remains unchanged in both cases, and the parallel electric field tends to modify the current density profile toward a more stable profile (when $E_{\parallel} > 0$) or toward a more unstable profile (when $E_{\parallel} < 0$). However, in OFCD by oscillating both poloidal and toroidal electric fields out of phase, the time-averaged



Figure 5.12: The oscillating toroidal loop voltage V_z , field reversal parameter F and total magnetic fluctuations \tilde{B}/B . The period of the toroidal electric field is $\tau_{\omega} = 0.126\tau_R$

magnetic fluctuations are reduced and a time-averaged magnetic helicity is also injected and partial current can be maintained as shown in Sec. 5.2 (Fig. 5.1). Thus, the net effect is mainly because of the combination of the two oscillating fields. Here, we study the OFCD dynamics during a cycle, i.e. the effect of the oscillating fields on the axisymmetric profiles and asymmetric fluctuations.

The toroidal and poloidal loop voltages V_z and V_p , field reversal parameter F, and the total magnetic fluctuation \tilde{B}/B are shown in Fig. 5.15. The variations of current profile and dynamo term with regard to parallel electric field are studied during a cycle. The three terms in parallel Ohm's law are shown in Fig. 5.16 at different times marked in Fig. 5.15. Because V_z and F are both negative, the parallel electric field E_{\parallel} is positive over the entire



Figure 5.13: The modal magnetic energy W_{mn} for (a) (1,-3),(1,-4) and (b) (0,1),(1,-2) (OTCD).

radius at time t_1 as shown in Fig. 5.16(a). As is seen, an edge current is driven by E_{\parallel} , the core current density is still fairly peaked even though it is partially suppressed by the dynamo term. The magnetic fluctuation level is about the same as standard plasma without OFCD. As the toroidal field loses its reversal ($F > 0, V_z < 0$ and $V_p < 0$), the parallel electric field ($E_{\parallel} = E_z \cdot B_z + E_{\theta} \cdot B_{\theta}$) becomes negative near the edge which is shown in Fig. 5.16(b). This causes the magnetic fluctuations to increase as seen in Fig. 5.15 at time t_2 . The dynamo term tends to relax the current density profile by suppressing the current in the core and driving current near the edge. The current density profile is flat in most of the core region. This current flattening in the core causes the core resonant mode amplitudes to reduce at a later time when a positive E_{\parallel} is generated as the axial voltage V_z



Figure 5.14: The three terms $\eta J_{\parallel}, E_{\parallel}, S < \tilde{V} \times \tilde{B} >$ in parallel Ohm's law at times t₁-t₄ during a cycle (OTCD).



Figure 5.15: The toroidal oscillating loop voltage V_z , the poloidal oscillating voltage V_{θ} , field reversal parameter F, and the total magnetic fluctuation \tilde{B}/B vs time.

becomes positive (Fig. 5.15 at t₃). The positive parallel electric field is shown in Fig. 5.16(c) at t₃. The dynamo term at this time is zero and $E_{\parallel} = \eta J_{\parallel}$. The current density on axis increases as helicity in injected into the plasma as shown in Fig. 5.16(d) at time t₄ (E_{\parallel} increases and $\dot{K} > 0$). The current density starts to peak in the core and the core tearing modes start to grow again as seen in Fig. 5.15 at t₅ and the cycle repeats.

Two phases during a cycle can be distinguished, injection and ejection. During the ejection phase, the helicity injection rate is negative ($\dot{K} < 0$) and the total axial current decreases (t₁, t₂ and t₅ in Fig. 5.15). The magnetic fluctuation amplitudes are about or slightly higher than the standard (without OFCD) fluctuations in this phase. Oscillating fields flatten the current density in the core and the fluctuation level starts to decrease



Figure 5.16: The three terms E_{\parallel} , ηJ_{\parallel} and $S < \tilde{V} \times \tilde{B} >$ in parallel Ohm's law at different times during an OFCD cycle marked in Fig. 5.15.



Figure 5.17: The $\lambda = J_{\parallel}/B$ profiles at times t_1 and t_3 .

toward zero during the second part of the cycle, the injection phase. The helicity injection rate is positive ($\dot{K} > 0$) and the total axial current increases during the injection phase (t₃, t₄ in Fig. 5.15). The current profile is mainly sustained by positive E_{\parallel} in the injection phase. However, during the ejection phase, the gradient in the parallel current density profile drives the tearing instabilities. The fluctuation induced tearing dynamo term is negative in the core, suppressing the current. Therefore, both the tearing dynamo and the parallel electric field shape the $\lambda = J_{\parallel}/B$ profile. Figure 5.17 shows the modification of the $\lambda(r)$ profile at t₁ (during the ejection phase) and t₃ (during the injection phase). The current density is hollow near the edge at t₁ and is flattened at t₃. The gradient of these profiles changes during a cycle. For instance during the injection phase (after t₃) the current on axis increases and the λ profile peaks. However, these λ profiles are snapshots taken at the time when the OFCD current profile modification, including current flattening in the core, is maximal.

To complete the analysis of the OFCD cycle, we next discuss the modal activities based on the resonant condition on the q profile. As shown before, the time-averaged magnetic fluctuations are reduced by OFCD. In Fig. 5.18 the effect of oscillating fields on the mode amplitudes can be seen. The volume-averaged modal magnetic amplitudes $(B_{m,n}/B)$ for the dominant modes is zero during part of the OFCD cycle and is comparable to the standard mode amplitudes during the other part of the cycle. The mode amplitudes for the standard case without OFCD are also shown for comparison. The dominant modes without OFCD are (1,-3), (1,-4). Because of the large variation of the axisymmetric profiles with OFCD another core mode (1,-2) reaches an amplitude comparable to the core modes without OFCD. Since the mode amplitudes shown in Fig. 5.18 are normalized to the mean magnetic field on axis, the normalized (1,-2) mode amplitude is larger than the other modes.

The magnetic modal activity changes significantly with the q profile variations during an OFCD cycle. Fig. 5.19 shows the q profiles at times $t_1 - t_4$. The q profile at time t_1 is a typical q profile for the standard RFP. At time t_2 the q profile is positive everywhere and m=1, n=-2 and m=1, n=-3 are the core dominant modes with the mode amplitudes shown in Fig. 5.18. At a later time (t_3) the mode amplitudes of (1,-3) and (1,-4) are suppressed and m=1, n=-2 mode is resonant (Fig. 5.19). The q profile on axis drops again at a later time t_4 . The core mode (1,-4) grows linearly when the current density profile peaks in the core at the time the total fluctuation level is minimum. This linear growth is seen in the total magnetic fluctuation B/B (Fig. 5.15 at time t₅) and in the mode amplitude of (1,-4) shown in Fig. 5.18. Thus a single helicity state is formed (after t_4). The single helicity mode grows until it reaches an amplitude high enough to cause nonlinear coupling. The field line trajectory during the single helicity state is shown in Fig. 5.20(c). Because of the nonlinear coupling of this mode with other modes and a cascading process, the magnetic energy spectrum becomes broad again. The stochasticity of the magnetic field lines increases to the level of the standard RFP shown in Fig. 5.20(a). As is seen in Fig. 5.20, there is a transition from stochastic magnetic field lines to ordered and then to the single helicity state.

The comparison of the oscillation of the single components of electric field (OPCD and OTCD) with the oscillation of the both components (OFCD) indicates that E_{\parallel} is negative near the edge region for almost half of the OPCD and OTCD cycles causing the enhancement



Figure 5.18: The volume-averaged mode amplitudes for (a) standard case (without OFCD) (b) with OFCD.

of the magnetic fluctuations, but E_{\parallel} remains positive for three quarter of the OFCD cycle. This makes OFCD more effective than OPCD or OTCD.

In summary, the current profile shapes significantly during an OFCD cycle. During the ejection phase the current profile is peaked in the core and has a hollow shape closer to the edge because of the oscillations of the OFCD-driven current near the edge region. During this phase the tearing dynamo term distributes the current density by suppressing the current in the core and driving current near the edge. The q profile and the modal activity are also similar to the standard RFP. As the total current decreases, the parallel electric field is modified in the core and also becomes positive near the edge. As a result, the current density is relaxed to a flat profile. The flattening of the current density profile results in the suppression of the magnetic fluctuations and the tearing dynamo term vanishes. The cycle repeats when the current profile peaks.



Figure 5.19: The q profiles at times $t_1 - t_4$.



Figure 5.20: Field line trajectory (Poincare plots) at times a) t_1 , b) t_4 , and c) t_5 .

5.4 The frequency dependence

In the previous sections we showed that current can be partially sustained by OFCD and also that current profile control is possible with OFCD. The partial current sustainment by OFCD depends on the AC helicity injection rate which is proportional to the ratio of the AC driving voltages and the oscillation frequency. However, the penetration of the OFCD-driven current into the plasma and the OFCD modification of the current density profile also depends upon the frequency. As mentioned before, the oscillation frequency should be low enough for sufficient current relaxation by the tearing fluctuations, but high enough to avoid current reversal. Additionally, the frequency should be calibrated to result in a flattening of the current profile by the oscillating fields. Here, the results of 3-D MHD computations at different OFCD frequencies are presented when the helicity injection rate is fixed.

The oscillating fields with frequencies $\omega \tau_H = 1.2$, $\omega \tau_H = 4.7$, and $\omega \tau_H = 9.8$ are imposed on a relaxed RFP with a constant axial electric field boundary condition with pinch parameter $\Theta = 1.68$ and aspect ratio R/a=2.88. The oscillating field with the frequency $\omega \tau_H = 1.6$ (discussed in the previous section), and $\omega \tau_H = 0.8$ are also imposed on a target plasma with the same current but with the aspect ratio R/a=1.66. Fig. 5.21(a) shows that the oscillating fields inject helicity into the standard RFP plasma at time t=0.34 τ_R with frequency $\omega \tau_H = 1.2$ and with the helicity injection rate of 50% of the ohmic helicity rate ($\dot{K}_{inj} = \hat{V}_z \hat{V}_{\theta}/2\omega = 40$). As shown in Fig. 5.21(b), total axial current is increased by 10%. The peak to peak current modulation amplitude is about 35% of the mean total axial current and it is much smaller than the modulation amplitudes shown in Fig. 5.1 at $\omega \tau_H = 0.16$. The mean helicity dissipation , $\dot{K}_{diss} = \eta \int J \cdot BdV$, is increased with OFCD (Fig. 5.22) and balances the total helicity injection rate (AC and ohmic injection) as the plasma get close to the steady-state. However, the fluctuating helicity dissipation, $\eta \int \tilde{J} \cdot \tilde{B}dV$, remains small (similar to the standard RFP surrounded by a conducting wall).

Table 3 summarizes the results of the OFCD simulations with the same helicity injection rate but with different frequencies. The current modulation amplitudes $\hat{I}_z/2I_z$ and the peak-to-peak modulation of the field reversal parameter F are reduced at higher frequency. The reduction of the modulation amplitudes with frequency obtained here is consistent with the results from the linear 1-D calculations and the relaxed-state scaling of full current sustainment by OFCD. [6] However, a similar frequency-scaling study using numerically demanding 3-D computations would require more data points than what is currently feasible. The time-averaged total magnetic fluctuation \tilde{B}/B_0 is suppressed by a factor of two at $\omega \tau_H = 0.16$ as shown in Sec. 5.2. However, $\overline{\tilde{B}/B_0}$ is about the same as the standard fluctuation level at higher OFCD frequency. As discussed before, the reduction of the tearing fluctuations is mainly due to the modification of the current density profile by the oscillating electric fields. Here, using 3-D computations we show that the modification of the current profile depends on the penetration of the OFCD-driven dynamo term – the OFCD edge driven current – and hence the oscillation frequency range.

The cycle-averaged symmetric OFCD-driven dynamo term $(V_{00} \times B_{00})_{\parallel}$ obtained from the 3-D simulations is shown in Fig. 5.23 for different frequencies. As is seen, the classical penetration $[\delta = (\eta/\omega)^{1/2}]$ for a fixed helicity input rate increases with the OFCD period. At lower frequency, $\omega \tau_H \sim 1$, the OFCD-driven current penetration is deeper into the plasma. Therefore, the OFCD-driven peak can be further into the cycle-averaged current density profile depending on the frequency. Figure 5.24 shows the cycle-averaged current density profile for different frequencies. The ohmic current density profile is modified by OFCD. At higher frequencies $\omega \tau_H >> 1$ the OFCD-driven current is mostly peaked near the plasma edge, but at lower frequency ($\omega \tau_H \sim 1$) the OFCD-driven current is further into the plasma. At frequency $\omega \tau_H = 1.2$ the current density J_{\parallel} is increased everywhere but mainly near the edge region. We should also note that there is an exponentially growing resistivity profile near the plasma edge which causes current dissipation near the plasma edge at high frequencies ($\omega \tau_H >> 1$).

Figure 5.25 illustrates the temporal variation of $\lambda(r)$ profiles with oscillating fields for frequencies $\omega \tau_H \sim 1$ and $\omega \tau_H >> 1$. As is seen the OFCD-driven current is more localized near the edge region for $\omega \tau_H >> 1$. The modifications of the time-averaged $\lambda(r)$ and q



Figure 5.21: a) The magnetic helicity and b) total axial current at the frequency $\omega \tau_H = 1.2$.



Figure 5.22: The helicity dissipation $\dot{K}_{diss} = \eta \int J \cdot B$ vs time. The total helicity dissipation is balanced by the helicity injection as the plasma get close to steady-state at time t= 0.5 τ_R . The fluctuating helicity dissipation is almost zero.

profiles with OFCD at frequencies $\omega \tau_H \lesssim 1$ are shown in Fig. 5.26. The current density gradient between r=0.4 and r=0.8 is smaller in the $\omega \tau_H = 0.16$ case leading to a lower time-averaged magnetic fluctuation level. The field reversal modulations are higher at lower frequencies (Table 3) and field reversal is lost during part of the cycle. Thus, the time-averaged q at the edge is smaller for $\omega \tau_H = 0.16$ than for $\omega \tau_H = 0.8$ as shown in Fig. 5.26(b).



Figure 5.23: The cycle-averaged axisymmetric dynamo-like term, $(V_{00} \times B_{00})_{\parallel}$, for partial OFCD sustainment at three different frequencies.



Figure 5.24: The cycle-averaged parallel current density profiles of standard RFP (solid), OFCD with the frequency $\omega \tau_H \sim 1$ (dashed), and OFCD with the high frequency $\omega \tau_H >> 1$ (dash-dotted).



Figure 5.25: The $\lambda(r)$ profile vs time (a) $\omega \tau_H = 9.8$ (b) $\omega \tau_H = 1.2$.



Figure 5.26: The cycle-averaged (a) $\lambda(r)$ profile (b) q profile for standard case and OFCD with $\omega \tau_H \lesssim 1$.

		$\overline{\widetilde{B}}/B_0$	\overline{K}	$\overline{\dot{K}}_{diss}$	$\overline{I_z}$	$\frac{\widehat{I_z}}{2\overline{I_z}}$	F_{p-p}
Case I	Standard $(R/a=1.66)$	1.25~%	5.71	41	2.6	_	0.17
	$\omega \tau_H = 0.16$	0.6~%	8.6	72	3.0	45~%	2.0
	$\omega \tau_H = 0.8$	$1.5 \ \%$	7.3	71	2.82	21~%	1.58
Case II	Standard $(R/a=2.88)$	1.27~%	9.8	70	2.6	_	0.09
	$\omega \tau_H = 1.2$	1.0%	12.2	120	2.8	16~%	1.4
	$\omega \tau_H = 4.7$	1.44%	10.2	112	2.6	$11 \ \%$	0.95
	$\omega \tau_H = 9.8$	1.46%	9.8	111	2.6	8 %	0.9

Tabl	ما	3.
Lau	LC.	J.

We also examine the penetration of OFCD oscillations into the plasma core. We investigate the penetration for OPCD and OTCD at the same frequency. We study the penetration of the oscillations in the plasma core for three cases, OPCD, OTCD and OFCD. At high frequency, $\omega \tau_H = 9.8$, three simulations have been performed: OPCD ($\varepsilon_{\theta} = 12$), OTCD ($\varepsilon_z = 90$) and OFCD($\varepsilon_{\theta} = 12, \varepsilon_z = 90$). The AC component of the axisymmetric parallel current density, J_{\parallel} , vs time and radius is shown in Fig. 5.27. It is seen that at this frequency the oscillations penetrate into the core for both OTCD and OFCD, but the penetration is only to r=0.8 for OPCD. All the axisymmetric fields oscillate at the OFCD frequency far into the plasma core. There is no penetration for the poloidal magnetic field B_{θ} in OPCD case ($\varepsilon_z = 0$). This can also be expected from the classical penetration indicating that the poloidal magnetic field oscillation is proportional to ε_z (see Eq. 3.5). However, because of the large toroidal driving electric field in OTCD, the penetration of OTCD and consequently OFCD is global into the core as observed in Fig. 5.27. Since the OFCD oscillations penetrate further into the plasma core at this high frequency, the OFCD penetration at lower frequency is also clearly into the core region.

We also examine the OPCD penetration a two different frequencies. As shown above, the oscillation penetration is only into r=0.8 at high frequency $\omega \tau_H = 9.8$. Fig. 5.28 illustrates the penetration for, $\omega \tau_H = 9.8$ and $\omega \tau_H = 1.9$. The AC component of the axial magnetic

field B_z vs time and radius is shown in this figure. At high frequency, the penetration is almost classical and is confined to the edge region, but the penetration at frequency $\omega \tau_H = 1.9$ is global into the core. Thus, from the 3-D computations we conclude that at lower frequency ($\omega \tau_H \leq 2$) the oscillations penetrate into the core for all three cases (OPCD, OTCD and OFCD).

5.5 Summary

We have examined OFCD current profile control using 3-D MHD computations. In chapter 4, it is shown that time-averaged total current can be sustained in an OFCD plasma. A separate application of OFCD is the modification of the ohmic current profile. It is shown that OFCD can control the current profile density and a substantial reduction of the core tearing fluctuations can be obtained. The effect of OFCD on both the axisymmetric fields and the asymmetric fluctuations are investigated using 3-D modeling. The 3-D fluctuations are required to understand the full MHD dynamics.

To better understand the detailed dynamics, OFCD, OPCD and OTCD are examined separately. The effect of OPCD and OTCD on the axisymmetric profiles and the nonaxisymmetric fluctuations have also been studied. We find that the time-averaged magnetic fluctuation level remains unchanged in both cases, and the parallel electric field (when $E_{\parallel} > 0$) tends to modify the current density profile toward a more stable profile or toward a more unstable profile (when $E_{\parallel} < 0$). However, in OFCD by oscillating both poloidal and toroidal electric fields out of phase, the time-averaged magnetic fluctuations are reduced and a time-averaged magnetic helicity is also injected and partial current can be maintained. Thus, the net effect is mainly because of the combination of the two oscillating fields. Through the combination of poloidal and toroidal oscillating fields, a more favorable parallel electric field results which causes the reduction of magnetic fluctuations for most part of the cycle.

We distinguish two phases during an OFCD cycle. During the ejection phase the current profile is peaked in the core and there is an OFCD-driven current near the edge region.



Figure 5.27: The AC component of axisymmetric parallel current density, J_{\parallel} . (a) OPCD (b) OTCD (c) OFCD. The penetration of the oscillations during five cycles are seen. For all the cases $\omega \tau_H = 9.8$.



Figure 5.28: The AC component of axisymmetric axial magnetic field, B_Z for OPCD at two different frequencies. (a) $\omega \tau_H = 9.8$ (b) $\omega \tau_H = 1.9$

During this phase the tearing dynamo term distributes the current density by suppressing the current in the core and driving current near the edge. The q profile and the modal activity are also similar to the standard RFP. As the total current decreases, the parallel electric field is modified in the core and also becomes positive near the edge. As a result, the current density is relaxed to a flat profile. The flattening of the current density profile results in the suppression of the magnetic fluctuations and the tearing dynamo term vanishes. The cycle repeats when the current profile peaks.

The effectiveness of the current profile control by OFCD depends largely upon the OFCD frequency, and the effect of the relative phase between the toroidal and poloidal oscillating electric fields yet to be shown. For the Lundquist number used in these simulations, we have found the optimum frequency range where the relaxation is sufficient to modify the parallel electric field and consequently the current density. The modulation amplitudes depend on the Lundquist number and the computations at higher S should suppress the oscillations. It is also shown that the penetration of the oscillating fields depends on the frequency. This
affects the cycle-averaged edge current induced by the dynamo-like $(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}$ effect. A thorough phase scan has also yet to be performed.

References

- J. S. Sarff, S. A. Hokin, H. Ji, S. C. Prager, and C. R. Sovinec, Phys. Rev. Lett. 72, (1994).
- [2] B. E. Chapman, A. F. Almagri, J. K. Anderson *et al.*, Phys. of Plasmas, 9, 2061 (2002); B. E. Chapman, J. K. Anderson, T. M. Biewer, *et al.*, Phys. Rev. Lett, 87, 205001-1 (2001)
- [3] C. B. Forest *et al.*, Phys. Plasmas **7**, 1352 (2000).
- [4] Y. L. Ho, Nucl. Fusion **31**, 341 (1991).
- [5] C. R. Sovinec, Ph. D. thesis, University of Wisconsin-Madison, 1995.
- [6] F. Ebrahimi, S. C. Prager, J. S. Sarff and J. C. Wright, Phys. Plasmas 10, 999, (2003).
- [7] M. K. Bevir and J. W. Gray, in Proceedings of the Reversed-Field Pinch Theory Workshop, edited by H. R. Lewis and R. A. Gerwin (Los Alamos Scientific Laboratory, Los Alamos, NM, 1981), Vol. III, p. A-3.
- [8] T. H. Jensen and M. S. Chu, Phys. Fluids 27, 2881 (1984).
- [9] M. K. Bevir, C. G. Gimblett and G. Miller, Phys. Fluids 28, 1826 (1985).
- [10] D. D. Schnack, D. C. Barnes, Z. Mikic, D. S. Harned, and E. J. Caramana, J. Comput. Phys. 70, 330, (1987).

6 Conclusions and future work

6.1 Conclusions

AC helicity injection is a technique to sustain current in configurations where the current distribution relaxes by internal processes. Magnetic helicity injection has been tested in various configurations such as spheromaks, RFPs and spherical tokamaks. In this thesis, we have investigated the 3-D MHD dynamics of OFCD, a form of AC helicity injection, in the RFP configuration. In OFCD, toroidal and poloidal surface voltages are oscillated out of phase to inject magnetic helicity into the plasma. This technique is considered one of the candidates for driving steady-state current in high-*S* plasmas such as reactors. OFCD relies upon magnetic fluctuations to relax the current density profile. Therefore, 3-D MHD fluctuations and instabilities are required to determine the effectiveness of current drive and the accompanying magnetic fluctuations and transport. We have employed 3-D nonlinear MHD computation to capture the magnetic relaxation physics. We have investigated the two key concerns regarding OFCD as a steady-state current drive technique. First the physics of the resulting current profile and the oscillations of the axisymmetric quantities. Second, the effect of OFCD on the non-axisymmetric fluctuations important to transport.

We have first examined simplified 1-D computations and quasi-linear analytical solutions. 1-D models are compared with the 3-D results to understand the role of nonaxisymmetric fluctuations. In the absence of tearing fluctuations, an edge steady-state current is generated through the cycle-averaged dynamo-like effect, $(\mathbf{V}_{00} \times \mathbf{B}_{00})_{\parallel}$, from the oscillations of axisymmetric velocity and magnetic field. This current is localized to the outer region of the plasma, penetrating a distance equal to the classical skin depth. The edge OFCD-driven current excites MHD instabilities and fluctuations. These magnetic fluctuations then transport the current into the plasma core through the fluctuation-induced dynamo $\langle \tilde{\mathbf{V}} \times \tilde{\mathbf{B}} \rangle_{\parallel}$ effect. 3-D MHD computations show that the OFCD can sustain plasma current steady-state in the absence of the ohmic toroidal loop voltage. OFCD causes large modulation amplitudes of the axisymmetric profiles. We obtain current modulations about 100% of the mean value at $S = 10^5$. However, we find that the current oscillation decreases to about 50% at $S = 5 \times 10^5$, consistent with the prediction of the 1-D relaxed state model that oscillations scale as $S^{-1/4}$. Thus, at the higher S values of experiments or a reactor, the current oscillation may be acceptably small. The large modulation amplitudes at low S cause very deep toroidal field reversal at the edge and the excitation of the edge-resonant modes. These modes are linearly driven and can be avoided in high-S plasmas with smaller modulation amplitudes and weaker field reversal. The core tearing fluctuations did not display a significant change. The 3-D computations at higher S remain numerically challenging. We should also note that the effectiveness of current drive largely depends on the key parameters such as driving frequency and the relative phase. We have found the optimum frequency range between the hybrid tearing time scale and resistive diffusion time ($\tau_{hybrid} < \tau_{\omega} < \tau_R$), for sufficient current relaxation while avoiding current reversal.

We have also studied current profile control by OFCD as a separate application. Various techniques for controlling the current profile in the poloidal field dominated RFP configuration have been suggested. The main purpose of current profile control in RFPs is to suppress the core tearing fluctuations to improve the confinement. In this thesis, we have computationally investigated AC helicity injection as an alternative for current profile control. We have examined the detailed MHD dynamics of current modification by OFCD using 3-D MHD computations. The current profile control by OFCD is complex. To better understand the OFCD, we separate the dynamics into oscillating poloidal current drive (OPCD) in which only poloidal surface electric field is oscillated, and oscillating toroidal current drive (OTCD) in which only toroidal surface electric field is oscillated. The effect of OPCD and OTCD on the axisymmetric profiles and the non-axisymmetric fluctuations have been studied. We find that the time-averaged magnetic fluctuation level remains unchanged in both cases, and the parallel electric field tends to modify the current density profile toward a more stable profile (when $E_{\parallel} > 0$), or toward a more unstable profile (when $E_{\parallel} < 0$). However, in OFCD by oscillating both poloidal and toroidal electric fields out of phase, the time-averaged magnetic fluctuations are reduced and a time-averaged magnetic helicity is also injected and partial current can be maintained. Thus, the net effect is mainly because of the combination of the two oscillating fields. Through the combination of poloidal and toroidal oscillating fields, a more favorable parallel electric field results which causes the reduction of magnetic fluctuations for most part of the cycle. During part of the cycle, an edge current is driven by OFCD near the plasma edge and current is peaked in the plasma core. The magnetic fluctuations during this phase are still as high as in a standard plasma. During the other part of the cycle, the current density is relaxed to a flat profile. The flattening of the current density profile results in the suppression of the magnetic fluctuations and the tearing dynamo term vanishes. The effectiveness of current modification by OFCD largely depends upon the driving frequency, and the effect of the relative phase between the toroidal and poloidal oscillating electric fields yet to be shown. We have found the optimum frequency for current modification through relaxation during part of the cycle. A thorough phase scan has also yet to be performed.

Throughout this thesis, we have performed MHD computations at the highest Lundquist numbers to date for RFP computations. Using high Lundquist numbers is crucial for determining the viability of OFCD. It is also important for a more realistic picture of the MHD dynamics in the standard RFP. MHD computations at high Lundquist number provide more regular and pronounced oscillations similar to the experimental observations of sawtooth oscillations. We have also performed high-S computations for the standard RFP. We obtain behavior closer to the experimental observations, such as more regular sawtooth oscillations. High-S computations also allows studying and understanding the dynamics of sawtooth oscillations and the associated m=0 fluctuations. The effect of m=0 and m=1 nonlinear mode coupling on the sawtooth oscillations are not observed without m=0 modes, indicating the important role of m=0 modes in the sawteeth relaxation dynamics. In the absence of the m=0 nonlinear mode coupling the plasma transitions to a non-oscillatory steady state. However, the total magnetic fluctuation level is not reduced.

6.2 Future work

As the RFP moves toward improved confinement conditions and high-beta plasmas, finite pressure effects become important. The linear and nonlinear pressure-driven instabilities at high beta and techniques to control the pressure profile are physics issues that need to be further studied. We have performed linear MHD stability analysis for the pressure-driven instabilities in conditions exceeding the Suydam limit. We found that the transition from the resistive to ideal pressure-driven modes occurs only at high beta values, several times the Suydam limit. The mode structure of both high-n and low-n pressure driven modes has also been studied. The nonlinear behavior of the small-scale and large-scale fluctuations at finite pressure remains a topic for future investigations.

We have shown that in an OFCD plasma the core tearing fluctuation level is roughly equal to that of the standard RFP. Thus, in an OFCD plasma, important concerns about the transport and confinement will be raised, just as for the standard RFP. Different techniques to improve the confinement in high-S plasmas (such as future reactors) sustained by OFCD should thus be investigated. The application of other complementary current drive methods might suppress the magnetic fluctuation level and improve the confinement. Inductive or non-inductive current profile control techniques might be applicable in combination with OFCD to both sustain steady-state current and suppress the fluctuations. In this thesis, we did not consider the finite pressure effect in the MHD computations of OFCD. To further investigate the OFCD MHD dynamics, the finite pressure effect and transport should be included in future studies.

Most of the important features of RFP physics, such as magnetic fluctuations, tearing dynamo relaxation and axisymmetric profiles have been understood through MHD computations. High-S MHD computations provide an even more detailed picture of experimental observations such as sawtooth oscillations. However, some features, such as the Hall dynamo and the effect of energetic particles on fluctuations and transport can only be explained using models beyond MHD. Therefore, physics beyond MHD, such as two-fluid and kinetic effects are of great interest for future investigations.

Appendix A

The driven diffusion equation with time-dependent boundary condition can be solved using the method of eigenfunctions [1]. In this method it is assumed that a solution of the homogeneous problem $L[u] = cu_t$ can be represented by a series of eigenfunctions of the associated eigenvalue problem $L[\phi] + \lambda c\phi = 0$ with ϕ satisfying the boundary conditions given by u. We assume the solution $A_z^1(r, t) = \sum_{n=1}^{\infty} b_n(t)\phi_n(r)$, and substitute this solution into Eq. 3.3, then the eigenvalue problem is

$$\frac{\partial b_n(t)}{\partial t} + \eta \lambda_n^2 b_n(t) = 0$$

$$\frac{\partial^2 \phi_n(r)}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_n(r)}{\partial r} + \lambda_n^2 \phi_n(r) = 0$$
(A.1)

The corresponding eigenfunction $\phi_n(r)$ is the zeroth order Bessel function, and we obtain $A_z^1(r,t) = \sum_{n=1}^{\infty} b_n(t) J_0(\lambda_n r)$, where λ_n s are the zeros of J_0 . We assume a solution the form $A_z^1(r,t) = A_{z1}^1(r,t) + A_{z2}^1(r,t)$, where A_{z2} satisfies the time-dependent boundary condition such that $A_{z2}^1(a,t) = (-\varepsilon_{z0}/\omega) \cos(\omega t)$. Thus the new problem to solve is

$$\frac{\partial A_{z1}^1}{\partial t} + \varepsilon_{z0} \sin \omega t = \eta \left(\frac{\partial^2 A_{z1}^1}{\partial r^2} + \frac{1}{r} \frac{\partial A_{z1}^1}{\partial r} \right) \,. \tag{A.2}$$

After we substitute a solution of the form $A_z^1(r,t) = \sum_{n=1}^{\infty} b_n(t) J_0(\lambda_n r)$ into Eq. A.2 the result is

$$J_0(\lambda_n r)\frac{\partial b_n(t)}{\partial t} + \eta \lambda_n^2 J_0(\lambda_n r) b_n(t) = -\varepsilon_{z0} \sin \omega t .$$
(A.3)

We multiply Eq. A.3 by $J_0(\lambda_n r)$ and integrate over radius, and using the Bessel integrals (ref.qq),

$$\frac{\partial b_n(t)}{\partial t} + \eta \lambda_n^2 b_n(t) = \frac{-2\varepsilon_{z0} \sin \omega t}{\lambda_n J_1(\lambda_n)} . \tag{A.4}$$

The special solution of this ordinary differential equation is obtained by evaluating the following integral

$$b_n(t) = \frac{-2\varepsilon_{z0}}{\eta\lambda^3 J_1(\lambda_n)} \int_0^t \sin\omega\tau \exp[-\eta\lambda_n^2(t-\tau)]d\tau .$$
 (A.5)

Therefore, the solution is given by,

$$A_z^1(r,t) = \frac{-\varepsilon_{z0}}{\omega}\cos(\omega t) + \sum_{n=1}^{\infty} b_n(t)J_0(\lambda_n r) , \qquad (A.6)$$

where $b_n(t)$ is given in Eq. 3.6.

The Laplace transform method can be applied to Eqs. (3.3) and (3.4). By defining $\Omega^2 = S^2 B_0^2 / \rho$, and performing Laplace transformation on Eq. 3.4, we obtain

$$\frac{\partial^2 A^1_{\theta}(r,s)}{\partial r^2} + \frac{1}{r} \frac{\partial A^1_{\theta}(r,s)}{\partial r} - \left[\frac{1}{r^2} + \frac{s^2}{(\Omega^2 + \eta s)}\right] A^1_{\theta}(r,s) = 0 , \qquad (A.7)$$

where $A_{\theta}^{1}(r, s)$ is the Laplace transform of $A_{\theta}^{1}(r, t)$. The equation A.7 now is an ODE. The solution of Eq. A.7 can be written as $A_{\theta}^{1}(r, s) = bI_{1}(kr)$, where $k = s/(\Omega^{2} + \eta s)^{1/2}$. The coefficient *b* is obtained by $b = f(s)/I_{1}(k)$, where we have used the boundary condition at the wall (r = 1), $A_{\theta}^{1}(1, s) = bI_{1}(k) = f(s)$ [where f(s) is the Laplace transform of the $f(t) = A_{\theta}^{1}(1, t) = (-\varepsilon_{\theta 0}/\omega)\sin(\omega t)$]. We write the solution as the convolution $A_{\theta}^{1}(r, s) =$ f(s)g(s), where $g(s) = J_{1}(ikr)/J(ik)$. The real space solution can be found by performing the inverse Laplace transform on the convolution

$$A^{1}_{\theta}(r,t) = L^{-1}[f(s) \cdot g(s)] = \int_{0}^{t} f(\tau)g(r,t-\tau)d\tau$$
(A.8)

and the inverse transform of g(s) is calculated by evaluating the following integral

$$g(t) = \int exp(st) \frac{J_1(ikr)}{J_1(ik)} ds .$$
(A.9)

The poles of this integral are $\lambda_n = is/(\Omega^2 + \eta s)^{1/2}$. The solution of g(t) is obtained from the sum of residues of the two poles $s_1 = -i\lambda_n\Omega$ and $s_2 = i\lambda_n\Omega$ ($\Omega^2 >> \eta s$). Using $\lim_{s\to s_1}(s-s_1)exp(s_1t)\frac{J_1(\lambda_n r)}{J_1(\lambda_n)}$, the residue of s_1 is $-i\Omega exp(-i\lambda_n\Omega)\frac{J_1(\lambda_n r)}{J_1(\lambda_n)}$, and similarly for s_2 we get, $i\Omega exp(i\lambda_n\Omega)\frac{J_1(\lambda_n r)}{J_1(\lambda_n)}$. Therefore, using Eq. A.8 the solution can be written as follows

$$A_{\theta}^{1}(r,t) = \sum_{n=1}^{\infty} C_{n}(t) \frac{J_{1}(\lambda_{n}r)}{J_{1}'(\lambda_{n})} , \qquad (A.10)$$

where

$$C_n(t) = -\frac{2\varepsilon_{\theta 0}\Omega[\Omega\lambda_n \sin\omega t - \omega\sin(\lambda_n\Omega t)]}{\omega(\omega^2 - \lambda_n^2\Omega^2)}$$
(A.11)

and to obtain the temporal coefficient $C_n(t)$ we have to evaluate the integral $2\Omega \frac{\varepsilon_{\theta 0}}{\omega} \int_0^t \sin \omega \tau \sin[\lambda_n \Omega(t-\tau)] d\tau$. Now that we have calculated the solution for the azimuthal vector potential, the solution for the radial flow V_r can also be obtained using the following equation:

$$\frac{\partial V_r^1(r,t)}{\partial t} = -\frac{SB_0}{\rho} \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (rA_\theta^1(r,t)) \right) \right] . \tag{A.12}$$

The solution for V_r^1 is then given by Eqs. 3.7 and 3.8.

References

[1] Partial differential equations for scientists and engineers by Tyn Myint-U (1987).

Appendix B

The linear stability of ideal and resistive pressure-driven interchange modes is an old subject that has received extensive analysis. Its relevance today is somewhat heightened, as experiments with unfavorable magnetic curvature, such as reversed field pinches (RFP) and stellarators, are operating with pressure at or above the ideal interchange stability limit. In stellarators beta values above the Mercier limit are obtained in experiment, with no observation of instability. [1] Investigation of global resistive modes have been examined in stellarators in currentless equilibria applicable to the Heliotron DR device. [2] In the RFP, control of the current density profile has succeeded in substantially reducing current-driven tearing instability and increasing beta to the point that pressure-driven modes may begin to be consequential. [3] Here, we examine the behavior of the linear resistive interchange instability in current-carrying cylindrical plasmas, as beta varies from less than the ideal stability (Suydam) limit to much larger than the ideal limit.

The ideal interchange instability in a cylinder has been examined in some detail, following the calculation by Suydam that a localized pressure-driven instability in a bad curvature region, is excited if the stability parameter $D_S = -(8\pi p'/r)(q/B_z q')^2|_{r_s} > 0.25$ [4], where q is the safety factor, p is the pressure and ()' = d/dr. Subsequently, the dependence of the analytic growth rate on D_S (in the limit of large wave number, k) has been treated by several authors. [5, 6] In many of these treatments the inertial term is included in a layer around the resonant surface only. Eigenfunction solution in the outer region is matched to that obtained in the layer. [7, 8, 9] The result is that the growth rate depends on D_S (which is proportional to beta) as $\gamma_{max} \approx Cexp(-2\pi/\sqrt{\sigma})$, where $\sigma = D_S - 0.25$. Thus, the growth rate is exponentially small near the ideal limit ($D_S = 0.25$), becoming large for D_S values well above this limit. Numerical values for the growth rate of ideal interchange modes have also been obtained in a diffuse linear pinch. [10, 11]

The addition of small resistivity defeats the shear stabilization and resistive interchange modes become always unstable in a cylinder. [12] Matching the outer solution to a layer that includes resistivity yields an analytical growth rate that scales with Lundquist number, S, as $\gamma \sim S^{-1/3}$. [13] Numerical studies of the growth rate have been accomplished using eigenmode analysis (matrix shooting). [14, 15, 16]

In the present work, we employ initial value computation to evaluate the growth rate and radial structure, for arbitrary wave number, of the resistive pressure-driven instability. We find two results. First, for a rather wide range of beta, from zero to several times the Suydam limit, the high-k interchange mode is resistive. It is resistive in its radial structure (which results in reconnection), and its growth rate, which is small and scales as $S^{-1/3}$ at low D_S , and more weakly with S as D_S increases. The instability transitions to an ideal mode at very high beta values (D_S), several times the Suydam limit. Only at these very high beta values is the mode ideal in its radial structure and its growth rate (which becomes independent of S and scales with D_S as described by ideal MHD). Second, we find that for the RFP global pressure-driven modes are important. These modes transition from resistive to ideal as beta increases, similar to that of the interchange.

The three dimensional nonlinear Debs code [17] is used to solve the following set of compressible resistive MHD equations in cylindrical geometry in the linear regime,

$$\begin{aligned} \frac{\partial \overline{A}}{\partial t} &= S\overline{V} \times \overline{B} - \eta \overline{J} \\ \rho \frac{\partial \overline{V}}{\partial t} &= -S\rho \overline{V} . \nabla \overline{V} + S\overline{J} \times \overline{B} + \nu \nabla^2 \overline{V} - S \frac{\beta_0}{2} \nabla P \\ \overline{B} &= \nabla \times \overline{A} \\ \overline{J} &= \nabla \times \overline{B} \\ \frac{\partial P}{\partial t} &= -S \nabla . (P\overline{V}) - S(\gamma - 1) P \nabla . \overline{V}, \end{aligned}$$

where time and radius are normalized to resistive diffusion time $\tau_R = 4\pi a^2/c^2\eta_0$ and minor radius a, $S = \frac{\tau_R}{\tau_A}$ is the Lundquist number, ν is the viscosity coefficient, which measures the ratio of characteristic viscosity to resistivity (the magnetic Prandtl number), and $\beta_0 = 8\pi P_0/B_0^2$ is the beta normalized to the axis value. The mass density ρ is assumed to be uniform in space and time. The equations are fully compressible and describe both shear and compressional Alfvén waves, as well as resistive instabilities. To resolve the ideal and resistive interchange modes in the linear computation, the maximum timestep has been examined for convergence. The growth rate solutions are converged in timestep and spatial resolution to the level of 2% and 1% respectively. The code uses the finite difference method for the radial coordinate.

To isolate the pressure driven modes, an equilibrium which is stable to resistive current driven modes is chosen (by the \triangle' criterion). The equilibrium parallel current profile and pressure profile are $\lambda(r) = J \cdot B/B^2 = 2\theta_0(1 - r^{\alpha})$ and $p(r) = p_0(1 - p_1r^{\delta})$ respectively, where $\alpha, \theta_0, \delta, p_0$ and p_1 are free constants. Other equilibrium quantities can be computed from the θ and z components of $\nabla \times \overline{B} = \lambda(r)\overline{B} + \beta_0\overline{B} \times \nabla p(r)/2B^2$ (see Fig. B.1).

First, we examine highly localized interchange modes by choosing modes with high axial wave number, k. The dependence of growth rate on $D_S = -(8\pi p'/r)(q/B_z q')^2|_{r_s}$ is shown in Fig. B.2. The mode selected (azimuthal mode number m=1, k=10.5) is resonant at r/a= 0.78. We see that the growth rate is always non-zero and increasing with D_S , but follows the analytical ideal value only at $D_S > 1.0$. The growth rate at lower D_S values is much greater than the ideal growth rate. It increases smoothly through the Suydam limit ($D_S = 0.25$), which plays no role for resistive instability. As expected, the growth rate depends on D_S only, rather than its constituents, β_0 or magnetic shear, separately.

The radial structure of instability also indicates that a transition from a resistive to ideal interchange mode occurs at $D_S \sim 1.0$ (for this particular m, k and S). Ideal and resistive instabilities can be distinguished by the magnitude of the radial magnetic field B_r . The radial field is non-zero at the resonant surface only for a resistive mode. We see that the mode structure is resistive for $D_S < 0.9$ (Fig. B.3 a, b) and ideal for $D_S > 0.9$ (Fig. B.3 c, d), in agreement with the growth rates of Fig. B.2.

The transition from resistive to ideal modes is also evident in the S dependence of the growth rate γ (Fig. B.4). At low D_S , γ scales as $S^{-1/3}$ (resistive scaling), whereas at very high D_S , γ is roughly independent of S (ideal scaling). The D_S value at which the mode transitions from a resistive to an ideal mode depends upon S. The transition value for D_S decreases with S. This can be inferred from Fig. 4. The triangles are resistive modes

(from the radial structure) and the square boxes are ideal. For values of S $(10^6 - 10^7)$ of present experiments, the transition occurs at $D_S \sim 0.7 - 1.0$ (or $\beta_0 \sim 40 - 60\%$) well above experimental beta values.

High-k localized modes can be stabilized by finite Larmor radius effects. [8],[18] Thus, global, low-k pressure-driven modes may be more important for the RFP. The ideal stability of global pressure-driven modes have been examined in the past and it has been shown that these modes become unstable with the violation of Suydam criterion as well and have kinklike behavior. [11] Prior calculation of the growth rate for the resistive global pressure-driven modes also show an explicit dependence on the local parameter, D_S (as well as the global parameters). [12] Here, we have examined the growth rate and radial structure of global modes, and find that they also display a transition from resistive to ideal instability as beta increases. The growth rate for the m=1, k=1.8 mode is shown in Fig. B.5. The triangles correspond to resistive modes (as judged from the radial structures), while the boxes correspond to ideal modes. The mode is unstable at low beta values (less than the Suydam limit) and transitions to ideal modes at high beta (several times the Suydam limit). The radial structure for low and high D_S values (Fig. B.6) shows the change from a resistive to an ideal structure. These modes differ from the localized modes in their parity. The global mode structure for the radial magnetic and velocity fields (Fig. B.7 a,b) show tearing mode parity (B_r even about the resonant surface, v_r odd). The parity is opposite for the localized interchange modes (Fig. B.7 c,d). The k spectrum of the growth rate of all pressure-driven modes (Fig. B.8) illustrates the transition from tearing parity modes (depicted by triangles) to interchange parity (boxes) as k increases. We also observe that the growth rate for the global modes is about equal to that of the localized interchange.

The resistive-ideal transition of the localized modes is similar to that calculated for the stellarator. [2] However, there are significant differences between the behavior of global modes. In the currentless stellarator, the global, low-k modes have interchange parity and do not display a transition to a distinct ideal structure. In contrast, for the current-carrying plasmas examined here, modes with tearing parity are the most unstable and evolve from resistive to ideal at high beta.

In summary, motivated by the advance of present day experiments toward high beta regimes, we have revisited the behavior of linear local and global resistive pressure-driven MHD instabilities over a wide range of beta and resistivity (Lundquist number). We find that the Suydam criterion is not very relevant, in agreement with earlier analytical calculation of ideal growth rates. The localized interchange is resistive (in growth rate and radial structure) at beta values up to several times the Suydam limit, transitioning to an ideal mode at extremely high beta. No sudden changes in growth rate occur at the Suydam limit. This result may be consistent with the apparent absence of localized instability onset in experiments operating at or above the Suydam (or Mercier) stability limit. [1] For the RFP, we find that global pressure-driven modes (of tearing spatial parity) are equally unstable and have a similar transition from resistive to ideal as beta increases. Since the localized modes are more subject to stabilization mechanisms beyond MHD (such as finite Larmor radius stabilization), the global modes will likely be more influential in the reversed field pinches at high beta. In future studies we will examine the nonlinear behavior of these instabilities.



Figure B.1: Equilibrium magnetic field and pressure profiles $(B_Z,B_\theta$, p).



Figure B.2: The growth rate, $\gamma \tau_A$, of the m=1, k=10.5 mode vs. D_S . $S = 10^6$, $\theta_0 = 1.6$, $\alpha = 4$, $\delta = 3$, $p_1 = 0.9$. The triangles are computational results corresponding to resistive modes and the square boxes correspond to pure ideal modes. The solid line is the analytical growth rate of ideal interchange modes. The transition from resistive to ideal interchange modes occurs at high $D_S \sim 1.0$. The dashed vertical line is the Suydam limit.



Figure B.3: Radial magnetic field magnitude vs. radius for a) $D_S = 0.23$, $\gamma \tau_A = 6.5 \times 10^{-3}$, b) $D_S = 0.756$, $\gamma \tau_A = 3.3 \times 10^{-2}$, c) $D_S = 0.95$, $\gamma \tau_A = 5.4 \times 10^{-2}$, d) $D_S = 1.72$, $\gamma \tau_A = 0.35$. For all cases $S = 10^6$, m=1, k=10.5.



Figure B.4: Growth rate scaling of localized interchange mode m=1, k=10.5, with Lundquist number S, for various values of D_S . At low D_S (< 0.25) this scaling is resistive and at high D_S (high β) is ideal.



Figure B.5: Growth rate of low-k pressure driven mode, m=1 k=1.8 mode vs. β_0 . The triangles denote resistive modes and the square boxes denote pure ideal modes. Some of the points are computed at $S = 10^4$ (dashed curve), and while some are at $S = 10^5$ (solid curve).



Figure B.6: Radial magnetic field B_r vs. radius for global modes (m=1, k=1.8) at $S = 10^5$ for a) $D_S = 0.24$, $\gamma \tau_A = 3.4 \times 10^{-3}$, b) $D_S = 1.3$, $\gamma \tau_A = 7.3 \times 10^{-2}$



Figure B.7: Radial magnetic field (B_r) and radial velocity (v_r) eigenfunctions for global kink (m=1, k=2) and localized interchange (m=1, k=45) modes in the ideal limit $(S = 10^6, D_S = 0.9)$. a) B_r for k=2, b) v_r for k=2, c) B_r for k=45, d) v_r for k=45.



Figure B.8: Wave number spectrum of ideal pressure-driven modes at $D_s \sim 1.0$, $S = 10^6$. Triangles denote modes with a radial structure with tearing mode parity; boxes denote interchange parity.

References

- S. Okamura, K. Matsuoka, K. Nishimura, et al., in Proceedings of the 15th International Conference on Plasma Physics and Controlled Nuclear Fusion Research (International Atomic Energy Agency, Vienna, 1995), Vol. 1, p.381.
- K. Ichiguchi, Y. Nakamura, M. Wakatani, N. Yanagi, S. Morimoto, Nucl. Fusion, 29, 2093, (1989)
- [3] J. Sarff, N. Lanier, S.C. Prager, and M. R. Stoneking, Phys. Rev. Lett, 78, 62 (1997).
- [4] B. R. Suydam, in Proc. of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy (Geneva 1958), Vol.31, p. 152.
- [5] H. Grad, Proc. Nat. Acad. Sci. USA, Vol. 70, No. 12, 3277, (1973)
- [6] Y. P. Pao, Nucl. Fusion, **14**, 25, (1974)
- [7] R. M. Kulsrud, Phys. Fluids, 6, 904, (1963)
- [8] T. E. Stringer, Nucl. Fusion, 15, 125, (1975)
- [9] S. Gupta, C. C. Hegna, J. D. Callen, "Violating Suydam criterion produces feeble instabilities" submitted to Phys. Plasmas.
- [10] J. P. Goedbloed, H. J. Hagebeuk, Phys. Fluids, 15, 1090, (1972)
- [11] J. P. Goedbloed, P. H. Sakanaka, Phys. Fluids, 17, 908, (1974)
- [12] B. Coppi, J. M. Greene, J. L. Johnson, Nucl. Fusion, 6, 101, (1966)
- [13] H. P. Furth, J. Killeen, M. N. Rosenbluth, Phys. Fluids, 6, 459, (1963)
- [14] J. P. Freidberg, D. W. Hewett, J. Plasma Physics, 26, 177, (1981)
- [15] D. Merlin, S. Ortolani, R. Paccagnella, M. Scapin, Nucl. Fusion, 29, 1153, (1989)
- [16] D. H. Liu, Nucl. Fusion, **37**, 1083, (1997)

- [17] D. D. Schnack, D. C. Barnes, Z. Mikic, D. S. Harned, E. J. Caramana, J. Comput. Physics, 70, 330, (1987)
- [18] M. N. Rosenbluth, N. Krall, Nucl. Fusion Suppl. A1 143, (1962)