# Suppression of Transport Cross Phase by Strongly Sheared Flow

P. W. Terry,[1] D. E. Newman,[2] and A. S. Ware[3]
[1]*Department of Physics, University of Wisconsin–Madison, Madison, Wisconsin 53706*
[2]*Department of Physics, University of Alaska at Fairbanks, Fairbanks, Alaska 99775*
[3]*Department of Physics and Astronomy, University of Montana–Missoula, Missoula, Montana 59812*
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A generic model for advection of a scalar by $\mathbf{E} \times \mathbf{B}$ flow with a linearly varying mean shows that the cross phase factor in the transport flux is strongly reduced in the strong shear regime (shearing rate > eddy turnover rate), leading to significant transport suppression. The cross phase scales much more strongly with shear strength than do fluctuation amplitudes, allowing significant transport reduction even if fluctuations increase, or decrease only slightly. Cross-phase suppression thus can be the dominant transport-reduction mechanism in transport barriers.

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Transport barriers are widely used in fusion plasmas to improve confinement [1,2]. The suppression of transport by flow shear is a key feature of virtually all barriers that operate under strong flow shear, i.e., whose mean flow shear straining rate exceeds the nonlinear decorrelation rate. By definition, a transport barrier is a region of reduced transport. Because turbulent transport is proportional to fluctuation intensity, it is natural to attribute reduced transport to reduced fluctuations. However, transport rates also depend on the phase difference, or cross phase, between the two fluctuations whose correlation drives a given transport flux. Significantly, measurements examining the effect of flow shear on the transport cross phase have found that the cross phase is more strongly suppressed than the fluctuation amplitudes [3–9]. This is true in the Ohmic $H$ mode of DIII-D, where the sine of the density-potential cross phase is driven to a near-zero value, while the amplitudes decrease only modestly, or in some places actually increase [3]. Similar behavior is observed in flow-shear layers generated by biased probes in a variety of devices [4–9].

The bulk of theoretical work on the suppression of transport by flow shear has dealt with the suppression of fluctuation amplitude. This work has been successful in explaining fluctuation suppression as a remarkably general effect, in providing a compelling physical picture and in identifying the strong shear criterion for barriers. Theoretical work on the response of the cross phase to flow shear is much less well developed. Indeed, prior work has been restricted to resistive pressure gradient driven turbulence [10–12] and to the weak shear limit [11,12]. Unlike the observations, the fluctuation amplitudes and cross phase are predicted to experience comparable reductions. The experimentally relevant strong shear regime has not been studied analytically. Nor has there been any investigation of generic, non-mode-specific models that might provide a general understanding like that obtained for amplitude suppression using generic models [13].

We report here the general properties of cross-phase suppression in the strong shear regime, using the generic, non-mode-specific model of Biglari, Diamond, and Terry (BDT) [13]. The model treats the advection of an arbitrary scalar fluctuation by an $\mathbf{E} \times \mathbf{B}$ flow whose mean component has linear shear and whose fluctuating component is taken as known, either through measurement or a theoretical calculation. As shown below, this response is largely independent of whether the scalar is passive or active and of the collective properties of the fluctuation spectrum. This generality of the model is important because observations indicate that cross-phase suppression, like amplitude suppression, occurs under a wide range of conditions. In the strong shear limit, the cross phase has an inverse scaling with flow shear strength that is two powers stronger than that of the amplitude (absolute value) of the flux. The stronger suppression of cross phase is sufficient to enable the flux to decrease with flow shear strength *even if the fluctuations do not decrease, or actually increase.* The stronger suppression of cross phase reflects its proportionality to the real part of the inverted advective resonance operator. In the collisionless strong shear limit this goes as the resonance width, or turbulent diffusivity, divided by the flow shear strength. However, the diffusivity is governed by the same operator as the advected scalar and thus is itself suppressed like the advected scalar.

Consider Cartesian coordinates aligned so that $z$ is in the direction of the mean magnetic field, $y$ is in the direction of the mean flow, and $x$ is in the direction of the inhomogeneity of the mean flow and mean scalar $\chi_0$. The mean scalar is subjected to transport under advection of scalar fluctuations by the fluctuating flow. The evolution equation describing this transport is $\partial \chi_0/\partial t - \partial/\partial x\langle \tilde{\chi} c B_0^{-1} \nabla \phi \times \mathbf{z} \cdot \mathbf{x} \rangle = 0$, where the tilde indicates the fluctuating component, the brackets indicate an average over $y$ and $z$, and $-cB_0^{-1}\nabla\phi \times \mathbf{z}$ is the fluctuating $\mathbf{E} \times \mathbf{B}$ flow expressed in terms of the electrostatic potential $\phi$. The bracketed quantity is the fluctuation-induced flux $\Gamma$,

$$\Gamma = -\mathrm{Re}\langle \tilde{\chi} c B_0^{-1} \nabla \phi \times \mathbf{z} \cdot \mathbf{x} \rangle = \mathrm{Re} \sum_{\mathbf{k},\omega} i c B_0^{-1} k_y \tilde{\chi}_{\mathbf{k},\omega}(x) \phi_{-\mathbf{k},-\omega}(x) \equiv -\sum_{\mathbf{k},\omega} c B_0^{-1} k_y |\tilde{\chi}_{\mathbf{k},\omega}| |\phi_{-\mathbf{k},-\omega}| \sin\delta_{\mathbf{k},\omega} , \quad (1)$$

  

where $\omega$ is the Fourier frequency and $\mathbf{k}$ is the wave vector of the $y$ and $z$ directions. The factor $|\tilde{\chi}_{\mathbf{k},\omega}||\phi_{-\mathbf{k},-\omega}|$ represents the amplitude dependence of the flux. The last equality defines the cross phase $\delta_{\mathbf{k},\omega}$ as the difference between the *phases* of the scalar and the electrostatic potential fluctuation. The coherence, which generally appears as an additional factor in the last expression, is assumed to be unity. The equation for $\tilde{\chi}_{\mathbf{k},\omega}$ is

$$[-i\omega + ik_y x v_0']\tilde{\chi}_{\mathbf{k},\omega} + \sum_{\mathbf{k}',\omega'} \frac{c}{B_0} \mathbf{k}' \times \mathbf{x} \cdot \mathbf{k} \phi_{\mathbf{k}',\omega'} \tilde{\chi}_{\mathbf{k}-\mathbf{k}',\omega-\omega'} = \frac{c}{B_0} ik_y \phi_{\mathbf{k},\omega} \frac{d\chi_0}{dx}, \tag{2}$$

where the mean flow has a linear variation with slope $v_0'$ and $x$ is defined as the distance from the rational surface of the mode $k$. The frequency $\omega$ is understood to include the Doppler shift induced by the mean flow at the rational surface. The Doppler shift enters the frequency in the other equations which combine with the scalar equation to specify the eigenmode. When the summation of Eq. (1) is performed, $\omega$ becomes the mode frequency evaluated in the frame of the flow at the rational surface. The linear advective response $\omega - k_y v_0' x$ vanishes at a unique location for each mode $k$, defining a set of Kelvin-neutral layers associated with the scalar advection. The eigenmode problem that determines $\omega$ also specifies the electrostatic potential $\phi_{\mathbf{k},\omega}$. The eigenmode problem is mode specific; hence to keep our treatment as general as possible we leave the form of $\phi_{\mathbf{k},\omega}$ unspecified. As shown below, the eigenmode intensity $|\phi_{k,\omega}|^2$ enters with equal measure in both the flux amplitude factor $|\tilde{\chi}_{\mathbf{k},\omega}||\phi_{k,\omega}|$ and the cross phase. When we determine the relative sensitivity of each quantity to flow shear by taking the ratio, the unspecified eigenmode intensity (which can either increase or decrease in the presence of flow shear [1,14,15]) cancels out.

In the limit that the shearing rate $k_y v_0' x$ becomes arbitrarily large relative to dissipative processes (turbulent or collisional), the scalar fluctuation is in phase with the potential, and the flux is zero. This explains why strong flow shear suppresses the cross phase. For a large but finite shearing rate (small but finite dissipation), the flux is nonzero, and the degree of suppression depends on the magnitude of the effective dissipation rate relative to the shearing rate. If the system is turbulent, collisional dissipation is negligible in the energy-containing scales, and dissipation is governed by the scattering of scalar fluctuations by the turbulent flow. The scattering rate is quantified by renormalizing Eq. (2). Using an eddy damped quasinormal Markovian (EDQNM) closure [16,17], Eq. (2) becomes

$$\left[-i\omega + ik_y x v_0' - \frac{\partial}{\partial x} D_{\mathbf{k},\omega} \frac{\partial}{\partial x} - k_y^2 d_{\mathbf{k},\omega}\right]\tilde{\chi}_{\mathbf{k},\omega} = \frac{c}{B_0} ik_y \phi_{\mathbf{k},\omega} \frac{d\chi_0}{dx}, \tag{3}$$

where the turbulent diffusivities $D_{\mathbf{k},\omega}$ and $d_{\mathbf{k},\omega}$ are given by

$$D_{\mathbf{k},\omega} = \sum_{\mathbf{k}',\omega'} \frac{c^2}{B_0^2} k_y'(k_y' - k_y)\phi_{\mathbf{k}',\omega'}(x) \times R_{\mathbf{k}-\mathbf{k}',\omega-\omega'} \phi_{-\mathbf{k}',-\omega'}(x), \tag{4}$$

$$d_{\mathbf{k},\omega} = \sum_{\mathbf{k}',\omega'} \frac{c^2(k_y' - k_y)\partial\phi_{\mathbf{k}',\omega'}}{B_0^2 \quad k_y \quad \partial x} R_{\mathbf{k}-\mathbf{k}',\omega-\omega'} \frac{\partial\phi_{-\mathbf{k}',-\omega'}}{\partial x},$$

and $R_{\mathbf{k}-\mathbf{k}',\omega-\omega'}$ is the advective response at wave number $k - k'$:

$$R_{\mathbf{k}-\mathbf{k}',\omega-\omega'} = \Bigg[-i(\omega - \omega') + i(k_y - k_y')x v_0' - \frac{\partial}{\partial x} D_{\mathbf{k}-\mathbf{k}',\omega-\omega'} \frac{\partial}{\partial x} - (k_y - k_y')^2 d_{\mathbf{k}-\mathbf{k}',\omega-\omega'}\Bigg]^{-1}. \tag{5}$$

Note that the two diffusivities $D$ and $d$ are effectively the same up to a factor parametrizing the anisotropy of turbulent eddies. The operator inversion required to solve Eq. (3) allows the quantitative determination of the scalar phase (and hence flux magnitude) set by the diffusivities. However, the same operator inversion also governs the diffusivities themselves. This is a crucial feature of cross-phase suppression.

Equation (3) is inverted using a Green function, yielding $\tilde{\chi}_{\mathbf{k},\omega}(x) = \int dx' G_{\mathbf{k},\omega}(x\,|\,x')cB_0^{-1}ik_y\phi_{\mathbf{k},\omega}(x')\,d\chi_0/dx'$, where $G_{\mathbf{k},\omega}(x\,|\,x')$ is the solution of $R^{-1}_{\mathbf{k},\omega}G_{\mathbf{k},\omega}(x\,|\,x') = \delta(x - x')$. The Green function is obtained from a WKB expansion of the homogeneous problem in the asymptotic limit of large shear. In this limit the operator of Eq. (5) develops a singular layer because its highest derivative is nominally of lower order. On the scale of the layer, the diffusivities are smooth. As discussed in Ref. [16], they, like $\tilde{\chi}_{\mathbf{k},\omega}(x)$, have an integral over $x'$ from the inversion of $R$, making their scale length of lower order than that of the operator. The sum over wave number also smooths the diffusivities, as illustrated in Fig. 3a of Ref. [17]. Treating the diffusivities as uniform, and matching solutions satisfying $G \to 0$ at $x \to \pm\infty$ across the singularity, we obtain

$$G_{\mathbf{k},\omega}(x\,|\,x') \sim \frac{-i^{3/2}\Delta x}{2D_{\mathbf{k},\omega}} \varepsilon^{1/2} A^{-1/4}(x_<)A^{-1/4}(x_>) \times \exp\left\{-\frac{2}{3} i^{1/2} \frac{[A^{3/2}(x_>) - A^{3/2}(x_<)]}{\varepsilon^{1/2}}\right\}, \quad (\varepsilon \to 0), \tag{6}$$

where $A(x) = [x/\Delta x - (\omega + ik_y^2 d_{\mathbf{k},\omega})/k_y v_0' \Delta x]$ is the structure function of the neutral layer, $\varepsilon = (D_{\mathbf{k},\omega}/k_y v_0' \Delta x^3)$ is the flow shear parameter, $\Delta x$ is the fluctuation scale of the $v_0' = 0$ reference case, and $x_<$ ($x_>$) is the smaller (larger) of $x$ and $x'$. The WKB solution is uniformly valid provided $(\mathrm{Re} d_{\mathbf{k},\omega}/\mathrm{Re} D_{\mathbf{k},\omega})k_y^2\Delta x^2 > \exp[-4(\mathrm{Re}\varepsilon)^{-1/2}]$. This condition is always met for sufficiently strong shear; otherwise WKB breaks down in a layer around the neutral surface defined by $\mathrm{Re}A(x) < \mathrm{Re}\varepsilon^{1/2}\exp[-4(\mathrm{Re}\varepsilon)^{-1/2}]$. The amplitude suppression theory is based on moments of the Green function in the large shear limit [13]; hence Eq. (6) reflects the radial scale reduction of BDT. Assuming the shearing rate exceeds frequency and decorrelation rates, the width of $G$, obtained by setting the argument of the exponent to unity, is given by $\Delta x \varepsilon^{1/3}$. This is the reference mode width $\Delta x$, reduced by precisely the BDT factor.

The integral in the Green function solution of $\tilde{\chi}_{\mathbf{k},\omega}(x)$ can be evaluated asymptotically yielding an analytic expression. Substituting into Eq. (1), the leading order flux is

$$\Gamma = \mathrm{Re}\sum_{\mathbf{k}} \frac{ic^2 k_y^2 \Delta x^2 |\psi_{\mathbf{k}}(x)|^2}{B_0^2 \quad k_y v_0' \Delta x^3} \frac{d\chi_0}{dx} \frac{1}{A(x)} + \mathrm{O}(\varepsilon^2). \quad (7)$$

Here, the fluctuation spectrum has been represented as the product of a spatial eigenfunction and a Lorentzian frequency spectrum, $|\phi_{\mathbf{k},\omega}(x)|^2 = |\psi_{\mathbf{k}}(x)|^2 \gamma_k [(\omega - \omega_k)^2 + \gamma_k^2]^{-1}$, and the sum over $\omega$ has been carried out in the continuous limit. Any $\omega$ dependence in Eq. (7) is thus understood to be evaluated at $\omega_k + i\gamma_k$, representing the peak (nonlinear mode frequency) and linewidth of the frequency spectrum at fixed $k$. In Eq. (7) the spatial structure of the eigenmode and the neutral layer of scalar advection are separated mathematically in the functions $\psi_{\mathbf{k}}(x)$ and $A(x)$, respectively. Previous calculations of eigenmode behavior in flow shear indicate that the primary effect is a shift of the eigenmode off the rational surface by an amount proportional to $v_0'$ [1,17]. Changes in the mode width are weaker and are often ignored altogether. However, even if the mode width is reduced to the BDT width (an upper limit on the degree of reduction), it is far broader than the width of the neutral layer. The latter goes as $\varepsilon$ to the first power, while the BDT width goes like $\varepsilon$ to the 1/3 power. Thus the neutral layer width is much narrower than any other structure in the turbulence. This is significant. In the neutral layer the effect of flow shear effectively vanishes, and there is strong mixing of $\chi$ across the neutral sheet. Only outside the layer does the differential motion of the shear flow impede transport by reducing the cross phase. Hence, an observed reduction of flux is necessarily a manifestation of the extreme narrowness of the neutral layer in the strong shear limit, allowing, for example, a probe that samples a region of overlapping fluctuation structures to weight preferentially the exteriors of neutral layers.

Outside the layer, the flux is

$$\Gamma = -\sum_{\mathbf{k}} \frac{c^2}{B_0^2} \frac{\pi k_y^2 |\psi_{\mathbf{k},\omega}(x)|^2}{k_y v_0' \Delta x} \frac{d\chi_0}{dx} \times \left(\frac{k_y^2 \mathrm{Re} d_{\mathbf{k},\omega} + \gamma_k}{k_y v_0' \Delta x}\right)\left(\frac{x}{\Delta x} - \frac{\omega_k}{k_y v_0' \Delta x}\right)^{-2}, \quad (8)$$

where $k_y^2 \mathrm{Re} d_{\mathbf{k},\omega} + \gamma_k \ll k_y v_0' \Delta x$, consistent with strong shear. This result validates the fundamental assertion of this Letter, that cross-phase suppression is stronger than amplitude suppression. The cross-phase factor is $\sin\delta_{\mathbf{k},\omega} = [(k_y^2 \mathrm{Re} d_{\mathbf{k},\omega} + \gamma_k)/k_y v_0' \Delta x][x/\Delta x - \omega_k/k_y v_0' \Delta x]^{-1}$; the amplitude factor is everything else. Assuming $\omega_k < k_y v_0' \Delta x$, each factor has a scaling of $(k_y v_0' \Delta x)^{-1}$; however, the cross-phase factor is proportional to $k_y^2 \mathrm{Re} d_{\mathbf{k},\omega} + \gamma_k$, which itself is strongly suppressed due to dependence on the spectral energy and additional powers of $1/v_0'$. To determine these powers we note first that the spectral linewidth $\gamma_k$ is induced by nonlinearity in the steady state and assume that it scales like $k_y^2 \mathrm{Re} d_{\mathbf{k},\omega}$. The latter is defined by Eq. (4), indicating that $\mathrm{Re}d$ is governed by $\mathrm{Re}R_{\mathbf{k}-\mathbf{k}',\omega-\omega'}$. This, however, is the same shearing operator that governs $\Gamma$, i.e., $\Gamma = \mathrm{Re}\sum c^2 k_y^2 B_0^{-2} \phi_{-\mathbf{k},-\omega} R_{\mathbf{k},\omega} \phi_{\mathbf{k},\omega} d\chi_0/dx$. Consequently, $\mathrm{Re}d$ has the reduction factors of $\Gamma$, both those of amplitude and phase, making the cross-phase factor of Eq. (8) much more sensitive to shear strength than the amplitude factor. This result requires $\omega_k < k_y v_0' \Delta x$; otherwise all explicit $v_0'$ dependence in Eq. (8) cancels out for $x < \Delta x$. We conclude therefore that suppression of flux beyond any reduction of the fluctuation energy $|\psi_{\mathbf{k},\omega}(x)|^2$ implies that the spectrum is dominated by low frequency modes. (The fluctuation energy is usually reduced in shear flow, but in some situations is observed to increase [3,5,7] and in others is predicted to remain unchanged or increase [1,14,15].)

It is possible to evaluate $\mathrm{Re} d_{\mathbf{k},\omega}$, solving Eq. (4) with the Green function used for Eq. (3). In terms of the Green function, $d_{\mathbf{k},\omega} = \sum_{k'} c^2 B_0^{-2} (k_y' - k_y) k_y^{-1} (d\phi_{\mathbf{k}',\omega'}/dx) \times \int dx' \, G_{\mathbf{k}-\mathbf{k}',\omega-\omega'}(x\,|\,x')\,(d\phi_{-\mathbf{k}',-\omega'}/dx')$. Applying the asymptotic procedures used to determine $\chi$, $\mathrm{Re} d_{\mathbf{k},\omega}$ is given by

$$\mathrm{Re} d_{\mathbf{k},\omega} = \sum_{\mathbf{k}'} \frac{c^2}{B_0^2} \frac{\pi |d\psi_{\mathbf{k}'}/dx|^2}{k_y v_0' \Delta x} \times \left(\frac{(k_y - k_y')^2 \mathrm{Re} d_{\mathbf{k}-\mathbf{k}',\omega-\omega'} + \gamma_k - \gamma_{k'}}{(k_y - k_y') v_0' \Delta x}\right) \times \left(\frac{x}{\Delta x} - \frac{\omega_k - \omega_{k'}}{(k_y - k_y') v_0' \Delta x}\right)^{-2}, \quad (9)$$

where $\omega$ and $\omega - \omega'$ are evaluated at $\omega_k + i\gamma_k$ and $\omega_k - \omega_{k'} + i(\gamma_k - \gamma_{k'})$. The strong shear limit in Eq. (9), $(D_{\mathbf{k}-\mathbf{k}',\omega-\omega'}/(k_y - k_y') v_0' \Delta x^3) \ll 0$, excludes $k_y - k_y' \rightarrow 0$. Equation (9) is not a closed form for $d_{\mathbf{k},\omega}$, but a recurrence relation that follows from the recursive definitions of diffusivities intrinsic to large Reynolds number closures such as the EDQNM. It expresses the nature of spectral coupling, which links the diffusivity at scale $k$ to dissipation at scale $k - k'$. The recursive nature of Eq. (9) makes it impractical for the evaluation of $d_{\mathbf{k},\omega}$ except in certain limits. One of these limits is relevant to experiment and represents an upper bound on the flux. The limit applies to a restricted range of strongly turbulent (nondissipated) scales coupled by a direct cascade to dissipated scales at a somewhat higher wave number. This is the type of situation suggested by microscale fluctuation bispectra in tokamaks, which indicate the close proximity of driven and dissipated ranges [18]. In this circumstance the scale $k - k'$ is dominated by dissipation and $(k_y - k_y')^2 \mathrm{Re} d_{\mathbf{k}-\mathbf{k}',\omega-\omega'} + \gamma_k - \gamma_{k'}$ can be replaced by a collisional diffusion $(k - k')^2 \mu$. Substituting Eq. (9) into Eq. (8) yields

$$\Gamma \approx -\sum_{\mathbf{k},\mathbf{k}'} \frac{c^4}{B_0^4} \frac{\pi^2 k_y^2 |\psi_{\mathbf{k},\omega}(x)|^2}{k_y v_0' x} \frac{d\chi_0}{dx} \times \left( \frac{|d\psi_{\mathbf{k}'}/dx|^2 k_y^2 (k - k')^2 \mu}{k_y^2 (k_y - k_y')(v_0' x)^3} \right). \quad (10)$$

If the nondissipated turbulent scales are part of a wider inertial range, the recursion of Eq. (9) cannot be broken. Equation (9) then suggests that the inverse scaling of Re$d$ with $v_0'$ is even stronger. Equation (10) is thus an upper bound on the flux in a strong shearing regime. The cross-phase contribution (in parentheses) has the factor $(v_0' \Delta x)^{-3}$; the amplitude contribution (everything else) has the factor $(v_0' \Delta x)^{-1}$. Both are proportional to spectral intensities $|\psi_{\mathbf{k},\omega}(x)|^2$. The spectral intensity in the cross-phase factor involves the radial derivative squared, versus $k_y^2$ in the amplitude factor. Given the weak changes in eigenmode width noted in other studies [1,10,17], this difference is not likely to be significant. The intensity factors have important consequences. If the driving instability is completely stabilized by flow shear, the spectral intensity factors will vanish, and with them the flux. More importantly, Eqs. (8)–(10) describe experiments in which the flux decreases despite fluctuation intensities that *increase* with flow shear. Fluctuation intensities can respond in this fashion for a variety of reasons [1,14,15], including the increase in driving gradients caused by flux reductions [see Eq. (2.21) of Ref. [1]]. For example, taking $\chi_0$ as density, if the steepening of $d\chi_0/dx$ due to the particle flux reduction is stronger than $v_0'^{-1}$, the density fluctuation $\psi_{\mathbf{k},\omega}(x)(k_y v_0' x)^{-1} d\chi_0/dx$ will increase, as observed in several experiments [5,7]. However, the additional three factors of $v_0'^{-1}$ in the cross-phase contribution can easily overcome this increase and yield a flux reduction sufficient to make the steepening of $d\chi_0/dx$ stronger than $v_0'$.

Equations (8) and (10) represent the flux of arbitrary advected scalars and are generic up to the eigenmode details that specify the forms and scalings of $\psi_{\mathbf{k},\omega}(x)$ and the eigenfrequency. For low frequency collisionless fluctuations in the strong flow shear regime, the fluxes of advected scalars like density and temperature are strongly suppressed by flow shear, with cross-phase suppression as the dominant effect and amplitude suppression as secondary. The result assumes that the flux is measured in a region external to the narrow neutral layer. The result is consistent with direct measurements in a variety of devices [3–9]. It is also consistent with fluctuation measurements that show residual fluctuations in transport barriers whose diffusivity is so low that neoclassical estimates exceed the measured value [2]. We emphasize that the present work applies to advected scalars, and not to the advection of vector quantities, such as the flow itself, which governs momentum transport. The latter is expected to behave very differently. The modeling of turbulent momentum transport in shear flows has a long history, dating back to Prandtl's mixing length hypothesis, and embodied in the present-day $k$-$\varepsilon$ closures [19]. In these models flow shear leads to an anomalous momentum flux expressible with a flux-gradient relationship. Indeed observed rotation rates under momentum input are consistent with a momentum flux that exceeds the heat flux by an appreciable amount [20].

The present work, while showing how the transport cross phase behaves in strong flow shear, is obviously not complete. The effect of the eigenmode, which in general introduces a very difficult problem, needs to be assessed. In certain experiments [4], it appears that the flow is localized to a layer so narrow that the flow profile, which is not linear, plays a major role in the fluctuation structure. These experiments also report that the sign of the flux can actually reverse in the region of the shear flow, an effect not predicted by the present theory [4,8]. Numerical simulations of scalar advection in sheared flow will be carried out for comparison with the present theory. These, and the addition of details such as the eigenmode physics should allow more quantitative comparison with experiment in the future.

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