Nonlinear Damping of Plasma Zonal Flows Excited by Inverse Spectral Transfer

P.W. Terry, R. Gatto, and D. A. Baver
Department of Physics, University of Wisconsin–Madison, Madison, Wisconsin 53706
(Received 9 April 2002; published 29 October 2002)

Plasma zonal-flow excitation and saturation in fluid electron-drift-wave turbulence are studied spectrally. The zonal flow is a spectral condensation onto the zero-frequency linear-wave structure. In the representation diagonalizing the wave coupling that dominates interactions at long wavelengths, nonlinear triad interactions involving zero-frequency waves are greatly enhanced. Zonal modes are excited on both unstable and purely stable eigenmode branches. Coupling to the latter introduces robust, finite amplitude-induced damping of zonal flows, providing saturation.

DOI: 10.1103/PhysRevLett.89.205001 PACS numbers: 52.30.–q, 52.35.Kt, 52.35.Ra

To probe important aspects of the excitation and saturation of zonal flows in plasma turbulence it is helpful to adopt the vantage point offered by related systems in fluid dynamics. Consider rotating 3D turbulence. Numerical simulations show an inverse spectral energy transfer process that drives slow, large-scale, quasi-2D cyclonic vortical columns from an energy input at short scale [1]. In 3D turbulence the normal cascade is forward, i.e., from large to small scale. Reference [1] provides a key insight into the creation of the columns by noting that the ratio of inertial wave frequency to rotation frequency tends to zero for wave vectors associated with these structures. We formulate these results as a set of general principles which might apply to a wider class of systems, including plasma zonal flows: (1) A global-scale, symmetry-breaking fluctuation structure corresponding to a vanishing Fourier wave number component ($\boldsymbol{k} \cdot \hat{\boldsymbol{n}} = 0$) is driven by turbulent spectral energy transfer. (2) For this wave number, waves induced by the symmetry breaking have a vanishing frequency. Because the symmetry-breaking $\boldsymbol{k} \cdot \hat{\boldsymbol{n}} = 0$ anisotropy resides in the wave dynamics, the structure is a zero-frequency wave, subjected to nonlinear interactions. (3) For the systems considered here wave physics becomes important relative to the nonlinearity at large scale. Hence the global-scale anisotropic structure is driven by inverse energy transfer. This occurs even if the nonlinearity normally produces a forward cascade.

Rotating 3D flow is only one example of a general phenomenon intrinsic to a class of turbulent systems. Another example appears to be 3D rotating, stratified turbulence [2]. Here we examine a third case, collisionless trapped electron mode (CTEM) turbulence in magnetically confined plasmas, and show that in addition to the above properties the symmetry-breaking fluctuation structure is saturated by finite-amplitude-induced damping. The anisotropic global structure in CTEM turbulence [3] is referred to as a zonal flow. Numerical studies have shown that when zonal flows are present in other models of drift wave turbulence, fluctuation levels are markedly lower [4]. In simple CTEM models, zonal flows are $k_y = 0$ CTEM eigenmodes that build up under inverse energy transfer (k_y represents the poloidal wave number). The anisotropy associated with $k_y = 0$ reflects the anisotropy of wave dynamics, which become important at large scales and break the isotropy of the nonlinearity. For $k_y = 0$ the drift wave frequency vanishes. To replicate the cascade of 3D rotating flows, we examine the long wavelength spectral subrange where the cascade is forward.

We present an analytical procedure that uncovers the dynamics of the inverse energy transfer process. The procedure is analogous to the helical decomposition employed for rotating fluids [5,6]. In this procedure the original system of coupled dynamical equations is transformed to the representation that diagonalizes the coupling matrix of the wave physics [7]. In this eigenmode decomposition the wave frequencies are *good quantum numbers;* i.e., eigenfrequencies are not mixed. This is the only representation in which the zero-frequency wave is properly isolated as a wave condensate of nonlinear transfer. However, nonlinear transfer to wave eigenfrequencies is different from the transfer to Fourier modes of the original fields: the transformation that diagonalizes wave eigenfrequencies mixes the nonlinearities. As shown below, the coupling between interacting modes, one of which is a zero-frequency eigenmode, becomes greatly enhanced relative to interactions with no zero-frequency eigenmode. Energy is transferred into zero-frequency eigenmodes from unstable modes at higher wave numbers.

Finite-amplitude-induced damping of the zonal flow arises from the mixed nonlinear transfer. CTEM turbulence is driven by a linear instability. However, eigenmodes on a second, purely stable branch of the dispersion relation (i.e., on which every Fourier wave number is damped), and with $k_y = 0$, are also strongly driven, reaching amplitudes comparable to the linearly unstable modes. Therefore, at finite amplitude the eigenmode is nonlinear. Its projection onto the basis set of linear eigenmodes yields approximately equal weights on the two branches (unstable and purely stable). Because modes on the latter are damped, the zonal flow is subjected to robust nonlinear (finite-amplitude-induced) damping. The nonlinear damping is truly dissipative, arising from a change in the fluctuation cross phase that governs dissipation. Nonlinear damping is the saturation mechanism of the zonal flow. Spectral transfer on the unstable branch is not available to provide saturation because it excited the zonal flow. For the CTEM model there is no direct damping of zonal flows.

We employ a 2D fluid model for CTEM turbulence [7–10],

$$\frac{\partial n_{\boldsymbol{k}}}{\partial t} + \alpha_{11}(\boldsymbol{k}) n_{\boldsymbol{k}} + \alpha_{12}(\boldsymbol{k})\phi_{\boldsymbol{k}} = b_1(\boldsymbol{k}) = -\sum_{\boldsymbol{k}'} \boldsymbol{k}' \times \boldsymbol{z} \cdot \boldsymbol{k} \phi_{\boldsymbol{k}'} n_{\boldsymbol{k}-\boldsymbol{k}'}, \tag{1}$$

$$\frac{\partial \phi_{\boldsymbol{k}}}{\partial t} + \alpha_{21}(\boldsymbol{k}) n_{\boldsymbol{k}} + \alpha_{22}(\boldsymbol{k})\phi_{\boldsymbol{k}} = 0, \tag{2}$$

where $\phi_{\boldsymbol{k}}$ is the turbulent electrostatic potential at wave vector $\boldsymbol{k}$, $n_{\boldsymbol{k}} = \varepsilon^{-1/2} n_{e_k} + \phi_k$, n_{e_k} is the electron density, consisting of passing and trapped species, $\varepsilon^{1/2}$ is the electron trapping fraction, $\alpha_{ij}(\boldsymbol{k})$ is the matrix of linear coupling coefficients, given below, and $b_1(\boldsymbol{k})$ is the $E \times B$ nonlinearity, or nonlinearity of electron density. This nonlinearity does not conserve enstrophy, and the energy cascade is forward [9]. A second nonlinearity, normally appearing in Eq. (2), is the advection of vorticity by $E \times B$ flow. This nonlinearity has an inverse energy cascade, and has been set to zero to demonstrate the existence of inverse energy transfer in the presence of a forward cascade. Neglecting the vorticity nonlinearity is a consistent approximation, valid for a spectrum limited to long wavelengths [9]. The linear coupling matrix is

$$A = \begin{pmatrix} \alpha_{11}(k) & \alpha_{12}(k) \\ \alpha_{21}(k) & \alpha_{22}(k) \end{pmatrix} = \begin{pmatrix} \upsilon & i k_y \nu_D \hat{\alpha} + \upsilon \\ \frac{-\varepsilon^{1/2}\upsilon}{1+k^2-\varepsilon^{1/2}} & \frac{i k_y \nu_D \hat{\alpha} + \varepsilon^{1/2}\upsilon}{1+k^2-\varepsilon^{1/2}} \end{pmatrix}, \tag{3}$$

where υ is the rate of pitch angle scattering for electron detrapping, $\nu_D = cT_e/eBL_n$ is the diamagnetic drift velocity, $L_n = (d \ln n_0/dx)^{-1}$ is the density gradient scale length, $\hat{\alpha} = 1 + (3/2)\eta_e$, η_e is the ratio of the density gradient scale length to the temperature gradient scale length, and $k^2 = k_x^2 + k_y^2$. Equations (1)–(3) are similar to a model for zonal-flow generation in Rayleigh-Taylor unstable turbulence [11].

The linear coupling matrix elements depend on lower powers of wave number than the nonlinearity. Consequently wave properties govern the structure of the fluctuation spectrum at long wavelengths. Wave frequencies are obtained by linearizing Eqs. (1) and (2) and applying a normal mode analysis. The linear eigenfrequencies, found from solutions of the characteristic equation, are

$$\omega_1 = \frac{\nu_D k_y (1 - \varepsilon^{1/2}\hat{\alpha})}{(1 + k^2 - \varepsilon^{1/2})} + \frac{i\upsilon\varepsilon^{1/2}[\hat{\alpha}(1 + k^2) - 1]}{(1 + k^2 - \varepsilon^{1/2})(1 - \varepsilon^{1/2}\hat{\alpha})} + O\left(\frac{\upsilon^2}{k_y \nu_D}\right), \tag{4}$$

$$\omega_2 = \frac{-i\upsilon}{(1 - \varepsilon^{1/2}\hat{\alpha})} - \frac{\upsilon^2\varepsilon^{1/2}[\hat{\alpha}(1 + k^2) - 1]}{k_y \nu_D (1 - \varepsilon^{1/2}\hat{\alpha})^3} + O\left(\frac{\upsilon^3}{k_y^2 \nu_D^2}\right), \tag{5}$$

where an expansion in the small collisionality parameter $\upsilon/k_y\nu_D$ has been performed. The frequency ω_1 belongs to the unstable CTEM eigenmode. This mode is unstable at long wavelengths and stable at short wavelengths if viscous or hyperviscous damping is added. The ω_2 eigenmode is stable for all wave numbers, and is referred to as the purely stable branch. The expansion of Eqs. (4) and (5) is valid only for modes that are not zonally averaged ($k_y \neq 0$). Zonally averaged modes have $k_y = 0$, and are necessarily in the collisional regime, even if all other modes are collisionless. For zonally averaged modes, $\omega_1 = 0$ and $\omega_2 = -i\upsilon(1 + k_x^2)/(1 + k_x^2 - \varepsilon^{1/2})$. The eigenmode corresponding to ω_1 has $n_k = \phi_k$ for $k_y = 0$, making the electron density, n_{ek}, zero. Consequently, the zonally averaged mode of the unstable (ω_1) branch yields the zonal flow when multiplied by ik_x. The zonally averaged eigenmode of the purely damped branch is $n_k = [1 + i(1 + k_x^2)/\varepsilon^{1/2}]\phi_k$. This eigenmode combines a zonally averaged density with the zonally averaged potential. Note that both zonal mode eigenfrequencies are zero to lowest order in the small collisionality expansion parameter.

We transform Eqs. (1) and (2) to the eigenmode decomposition. The time dependent projection of the $\phi(\boldsymbol{x}, t)$ and $n(\boldsymbol{x}, t)$ onto the eigenmode basis set is

$$\begin{pmatrix} n_k(t) \\ \phi_k(t) \end{pmatrix} = \beta_1(k,t)\begin{pmatrix} R_1(k) \\ 1 \end{pmatrix} + \beta_2(k,t)\begin{pmatrix} R_2(k) \\ 1 \end{pmatrix} = \begin{pmatrix} R_1(k) & R_2(k) \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \beta_1(k,t) \\ \beta_2(k,t) \end{pmatrix} = \boldsymbol{M}\begin{pmatrix} \beta_1(k,t) \\ \beta_2(k,t) \end{pmatrix}, \tag{6}$$

where $(R_j(k), 1)$ is the eigenvector corresponding to the eigenfrequency $\omega_j(k)$, normalized so that its component along ϕ_k is unity, $\boldsymbol{M}$ is the eigenvector matrix, and $\beta_j(k, t)$ are the nonlinearly evolving projection amplitudes. The function $R_j(k)$ is obtained by linearizing Eq. (2), replacing $\partial/\partial t$ with $-i\omega_j(k)$, and solving for n_k, yielding $R_j(k) = [i\omega_j(k) - \alpha_{22}(k)]/\alpha_{21}(k)$. To obtain the evolution equations for $\beta_j(k, t)$ we invert Eq. (6), take the time derivative, and substitute Eqs. (1) and (2) for the vector $(\dot{n}_k, \dot{\phi}_k)$. Expressing n_k and ϕ_k in terms of β_1 and β_2 using Eq. (6), we obtain

$$\begin{pmatrix} \dot{\beta}_1(k,t) \\ \dot{\beta}_2(k,t) \end{pmatrix} = -\boldsymbol{M}^{-1}\boldsymbol{A}\boldsymbol{M}\begin{pmatrix} \beta_1(k,t) \\ \beta_2(k,t) \end{pmatrix} + \frac{1}{R_1 - R_2}\begin{pmatrix} b_1 \\ -b_1 \end{pmatrix}\Bigg|_{\substack{n_k = R_1\beta_1 + R_2\beta_2 \\ \phi_k = \beta_1 + \beta_2}}. \tag{7}$$

The matrix $\boldsymbol{M}^{-1}\boldsymbol{A}\boldsymbol{M}$ is diagonal with the eigenfrequencies $i\omega_1(k)$ and $i\omega_2(k)$ as diagonal elements. If both nonlinearities were nonzero each equation would evolve under a combination of the nonlinearities. With $b_2 = 0$, both equations are driven by b_1, but with opposite sign.

From these expressions nonlinear coupling to wave eigenmodes is governed by $(R_1 - R_2)^{-1} = \alpha_{21}/i(\omega_1 - \omega_2)$. For coupling to zonal modes, $\omega_1 - \omega_2$ vanishes to lowest order (i.e., is of order v) while for coupling to other modes, $\omega_1 - \omega_2$ is finite (i.e., of order $k_y \nu_D \gg v$). Consequently, the coupling to zonal modes is greatly enhanced relative to that of other modes. Substituting from Eqs. (3)–(5), the coupling strengths are $(R_1 - R_2)^{-1} = \varepsilon^{1/2}/(1 + k_x^2)$ for zonal modes and $(R_1 - R_2)^{-1} = v\varepsilon^{1/2}/ik_y\nu_D(1 - \varepsilon^{1/2}\hat{\alpha})$ for nonzonal modes.

It is straightforward to show that this nonlinear coupling excites the zonal modes. In the linear growth phase of the instability, nonlinear transfer from linearly unstable, nonzonal modes causes linearly stable zonal modes to grow exponentially. Because their drive is nonlinear, β_2 (for both zonal and nonzonal wave numbers) and β_1 ($k_y = 0$) are smaller. This is the classic situation for parametric excitation. Following the standard procedure, Eq. (7) for $k_y = 0$ is solved by dropping β_2 in the nonlinearity and replacing β_1 with its exponentially growing, linear solution, yielding

$$\beta_j(k,t)\big|_{k_y=0} \cong (-1)^j \sum_{k'} \left[\frac{\hat{\alpha}\varepsilon^{1/2} k'_x k'_y k_x^2 \beta'_1 \beta''_1}{(1 + k_x^2)(1 - \varepsilon^{1/2}\hat{\alpha})} \frac{\{\exp[-i\omega_1(k')t - i\omega_1(k-k')t]\}}{\{\frac{v(1+k_x^2)}{(1+k_x^2-\varepsilon^{1/2})} - i[\omega_1(k') + \omega_1(k-k')]\}} \right]\Bigg|_{k_y=0}, \tag{8}$$

where $\beta'_1 = \beta_1(k', t = 0)$ and $\beta''_1 = \beta_1(k - k', t = 0)$. The zonal modes grow exponentially under the coupling to modes with growth rates $-i\omega_1(k')$ and $-i\omega_1(k - k')$. Equation (8) has limited validity because it breaks down as the growth of zonal modes brings their amplitudes into approximate parity with the linearly unstable modes. However, it demonstrates that the zonal modes are excited by nonlinear energy transfer from nonzonal modes. Moreover, this excitation applies to both the unstable and purely stable branches of the wave dispersion relation. Because $k_y = 0$, zonal modes are driven by an inverse energy transfer process, despite the fact that the electron nonlinearity b_1 drives a forward cascade in nonzonal modes. The excitation of β_2 is favored by b_1, and b_1 is the dominant drive of β_2 even when $b_2 \neq 0$.

A solution of the steady state amplitudes of all modes in saturation can be obtained by applying the eddy damped quasi-non-Markovian closure to Eq. (7). This closure was carried out and solved in Ref. [10] for nonzonal modes with mean wave number k' in the range of the instability. In the asymptotic collisionless limit $v/k'\nu_D \ll 1$, dominant balances among the various terms lead to spectrum-averaged saturation levels whose scalings with respect to the instability parameter $v/k'\nu_D$ are given by $|\beta_1|^2 \sim k'^2\nu_D^2$, $|\beta_2|^2 \sim v^2$, $\mathrm{Im}\langle\beta_1 * \beta_2\rangle \sim vk'\nu_D$, and $\mathrm{Re}\langle\beta_1 * \beta_2\rangle \sim v^2$, for $v/k'_y\nu_D \ll 1$. The terms that enter the asymptotic balances reveal the spectral energy transfer processes determining saturation. For unstable modes two processes enter the lowest order balance. These are a classical cascade from unstable modes to stable high k modes, all on the β_1 branch, and energy transfer from unstable modes on the β_1 branch to the damped eigenmodes $\beta_2(k)$. The later is mediated by the cross correlation $\langle\beta_1 * \beta_2\rangle$. Nonlinear energy transfer into β_2 balances the linear damping, making β_2 a sink for energy introduced by the instability.

We extend this analysis to zonal modes. The saturation levels of nonzonal modes given above remain in force. Zonal modes in β_2 are linearly damped. For this damping to remain in balance with nonlinear transfer into β_2 modes from the unstable branch, it is necessary for the levels $|\beta_2(k_y = 0)|^2$ and $\mathrm{Re}\langle\beta_1 * \beta_2\rangle$ to increase relative to their nonzonal counterparts. In the equation for $|\beta_1(k_y = 0)|^2$, there is no linear damping or drive. The saturation balance is among nonlinear terms describing transfer into $|\beta_1(k_y = 0)|^2$ from nonzonal β_1 modes, and transfer out of $|\beta_1(k_y = 0)|^2$ to damped β_2 modes. The saturation levels that allow these balances are $|\beta_1(k_y = 0)|^2 \sim k'^2\nu_D^2$, $|\beta_2(k_y = 0)|^2 \sim k'^2\nu_D^2$, $\mathrm{Re}\langle\beta_1 * \beta_2\rangle \sim k'^2\nu_D^2$, and $\mathrm{Im}\langle\beta_1 * \beta_2\rangle \sim vk'\nu_D$, for $v/k'_y\nu_D \ll 1$. The analysis leading to these conclusions follows by substituting $k_y = 0$ into the coupling coefficients and nonlinear response times of the closure equations of Ref. [10] [Eqs. (32)–(36)]. Zonal modes on the unstable branch $[\beta_1(k_y = 0)]$ saturate at the same level as the linearly unstable (nonzonal) modes, even though the zonal modes are marginally stable at infinitesimal amplitude. Zonal modes on the purely stable branch, which are linearly damped at a rate proportional to v, also saturate at the same level as the linearly unstable modes. Thus there is a dramatic spike in the β_2 spectrum for $k_y = 0$. There is a similar feature in the spectrum of electrostatic fluctuations in the TEXT tokamak that has recently been attributed to *stable* geodesic acoustic modes [12]. The latter have also been observed in the DIII-D tokamak [13].

The excitation of the damped eigenmode β_2 opens a significant dissipative sink accessible only at finite amplitude. The rate of dissipation associated with this sink, as well as other dissipative processes including the drive of the linear instability, is a finite-amplitude (nonlinear) growth rate given by $E^{-1}\partial E/\partial t$ [10], where E is the energy. The growth rate can be either positive or negative because $\partial E/\partial t = k_y\nu_D\hat{\alpha}\varepsilon^{1/2}\,\mathrm{Im}\langle n_k^*\phi_k\rangle - v\varepsilon^{1/2}|(n_k - \phi_k)|^2$ can have either sign. At infinitesimal amplitude with β_1 growing and β_2 decaying ($\beta_2 \ll \beta_1$), $E^{-1}\partial E/\partial t$ is the linear growth rate. Instability resides in the first term, and the second is a negative definite damping. When β_2 is excited, n_k and ϕ_k are no longer given by the unstable linear eigenmode; rather $n_k = R_1\beta_1 + R_2\beta_2$

and $\phi_k = \beta_1 + \beta_2$ constitute the nonlinear eigenmode, changing $E^{-1}\partial E/\partial t$. Focusing on zonal modes, the nonlinear growth rate can only be zero or negative because $k_y = 0$ eliminates the first term of $\partial E/\partial t$. The marginal stability of zonal modes in linear theory is recovered because, for $k_y = 0$, $R_1 = 1$, and, for the unstable eigenmode ($\beta_2 = 0$), $n_k = \phi_k$, making the damping term $v\varepsilon^{1/2}|(n_k - \phi_k)|^2$ zero. When β_2 is excited $n_k \neq \phi_k$, and the damping term becomes significant. Zonal modes are thus damped at saturation, regardless of the value of β_2. Using the saturation levels given above,

$$\frac{1}{E}\frac{\partial E}{\partial t}\bigg|_{k_y=0} \cong -\frac{v\varepsilon^{1/2}(1 + k_x^2)}{(1 + k_x^2 - \varepsilon^{1/2})}. \tag{9}$$

This nonlinear zonal-flow damping rate arising from the excitation of the damped eigenmode is comparable to the instability growth rate and the damping rate of the purely unstable branch.

This work has a variety of implications. For plasmas with weakly collisional trapped electrons, the nonlinear process that drives zonal flows also drives a purely stable eigenmode branch. Coupling to this mode saturates the zonal flows, and is a major saturation channel for higher wavelength turbulence. Its effect on zonal flows provides an answer to a puzzle arising in the modulational instability approach widely used in calculations of zonal-flow excitation: if direct flow damping is absent (as in the CTEM model), how do zonal flows saturate, given that the role played by mode coupling in the modulational instability approach is wholly that of a drive? The procedure used herein captures new aspects of mode coupling associated with the excitation of a damped branch, allowing mode coupling between unstable modes to excite the zonal flow, and mode coupling involving the damped branch to saturate it. Note also that it is not necessary to assume scale separation or weak turbulence, two restrictions of the modulational instability approach.

The inverse spectral energy transfer that drives zonal-flow modes and the purely damped branch acts as an energy sink to unstable, smaller scale CTEM fluctuations. Shear straining of smaller scale fluctuations by zonal flows, often depicted as enhancing a forward cascade process, in fact incorporates elements of an inverse cascade that accesses an energy sink. Because the energy transfer to nonlinearly damped zonal modes is significantly enhanced relative to other modes, the artificial removal of nonlinearly damped zonal modes from the coupling dynamics results in a higher fluctuation level. This is evident in Fig. 1 of Ref. [8], which shows an order of magnitude increase in saturation level when the electron nonlinearity is turned off. As noted in Ref. [10], the electron nonlinearity strongly excites the damped eigenmode branch. This significant mechanism for limiting the amplitude of unstable modes has not been identified previously.

Interesting implications also follow from recognizing behavior common to different systems where large-scale, anisotropic, quasistationary structures are driven by turbulence, including CTEM turbulence, rotating turbulence, geostrophic turbulence, and 3D MHD turbulence. For example, in MHD turbulence an anisotropy develops in which fluctuation scales parallel to a mean magnetic field become much longer than perpendicular scales, $k_\parallel \ll k_\perp$. For $k_\parallel = 0$, the frequency of shear Alfvén waves vanishes. Analogy with rotating flows, quasigeostrophic turbulence, and CTEM turbulence indicates that the anisotropy will build up slowly until the spectrum peaks at $k_\parallel = 0$. This contradicts a popular notion in interstellar turbulence [14], which posits that the spectrum peaks at the value of $k_\parallel$ where the wave frequency and nonlinear frequency are equal. In geostrophic turbulence the analogous notion [15] has been shown to be invalid [16], precisely because the cascade carries energy to zero wave number. However, the process takes place on very long time scales. Hence a key to observing the buildup of anisotropic $k = 0$ structures in large simulations is very long time integration. As a way of addressing the separate issue of the possible role of purely damped eigenmodes in large simulations, the nonlinear growth rate, Eq. (9), can be measured from the rate of change of total energy [10] and compared with the linear growth rate. Regions of enhanced damping are associated with damped eigenmodes.

This work was supported by the U.S. Department of Energy under Grant No. DE-FG02-89ER-53291. The authors acknowledge useful conversations with Leslie Smith.

[1] L. M. Smith and F. Waleffe, Phys. Fluids **11**, 1608 (1999).
[2] L. M. Smith and F. Waleffe, J. Fluid Mech. (to be published).
[3] A. Hasegawa and K. Mima, Phys. Rev. Lett. **39**, 205 (1977).
[4] Z. Lin *et al.*, Science **281**, 1835 (1998).
[5] F. Waleffe, Phys. Fluids A **5**, 677 (1993).
[6] C. Cambon, L. Jacquin, and J. L. Lubrano, Phys. Fluids A **4**, 812 (1992).
[7] D. A. Baver, P. W. Terry, and E. Fernandez, Phys. Lett. A **267**, 188 (2000).
[8] G. G. Craddock *et al.*, Phys. Plasmas **1**, 1877 (1994).
[9] D. E. Newman *et al.*, Phys. Plasmas **1**, 1592 (1994).
[10] D. A. Baver, P. W. Terry, R. Gatto, and E. Fernandez, Phys. Plasmas **9**, 3318 (2002).
[11] A. Das *et al.*, Phys. Plasmas **8**, 5104 (2001).
[12] P. M. Schock (private communication).
[13] M. Jakubowski *et al.* (to be published).
[14] P. Goldreich and S. Sridhar, Astrophys. J. **438**, 763 (1995).
[15] P. B. Rhines, Annu. Rev. Fluid Mech. **11**, 401 (1979).
[16] A. Chekhlov *et al.*, Physica (Amsterdam) **98D**, 321 (1996).