Plasma heating in reversed field pinches at the fundamental ion cyclotron frequency

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The possibility of plasma heating in reversed field pinches (RFP) by radio-frequency (rf) waves at $\omega \approx \omega_{ci}$ is studied. A simple cylindrical RFP equilibrium which is symmetric in poloidal and axial directions is considered. RF fields are excited with given poloidal and axial wave numbers by a model antenna. The plasma dielectric properties are described by a hot plasma dielectric tensor in the limit of zero Larmor radius. Collisionless absorption by plasma ions is assumed. The power deposition distribution in the plasma is examined for different plasma parameters. Results demonstrate that effective heating of the central part of the plasma at $\omega \approx \omega_{ci}$ is possible for some range of parameters when the $m = -1$ poloidal mode is excited. This is a hot plasma effect; it disappears in the cold plasma limit. © *2002 American Institute of Physics.*
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I. INTRODUCTION

The heating of fusion plasmas with waves in the ion cyclotron range of frequencies is successfully employed on tokamaks and stellarators. Several scenarios are identified for efficient plasma heating on these machines. They include the second or higher harmonic scenario, or the scenario involving ion–ion hybrid resonance (especially with a minority species that is fundamentally heated); see reviews by Swanson[1] and by Adam,[2] see also Refs. 3, 4.

In this paper we investigate the possibility of plasma heating in reversed field pinches (RFP) by rf waves at the fundamental ion cyclotron resonance. Early experiments demonstrated that this method of plasma heating is inefficient in tokamaks; see Ref. 5. Group velocity of the slow Alfvén wave (resonant at $\omega = \omega_{ci}$) is nearly parallel to the magnetic field for an arbitrary direction of the phase velocity (see Chap. 2 in Ref. 6). Due to this the wave is not accessible radially and its excitation efficiency by an antenna located outside the plasma is low. The wave energy supplied by an antenna goes into the fast Alfvén wave, which is not resonant at $\omega = \omega_{ci}$.

This method of plasma heating is somewhat efficient in stellarators. Plasma was heated efficiently at $\omega = \omega_{ci}$ on the L-2 Stellarator.[7] Some heating was observed in recent experiments on the large helical device.[8] This effect is explained by coupling between fast and slow waves in the stellarator magnetic field; see Refs. 9, 10.

In an RFP the poloidal component of magnetic field is comparable with the toroidal component. There is a strong variation of the direction of magnetic field with radius. This may result in a significant difference between the rf wave structures in RFPs and in tokamaks. Another important distinctive feature of an RFP is that it operates at low magnetic field.

A simple analysis with a cold plasma model in plane geometry is made to study whether the rapid change of the magnetic field direction leads to a resonant wave-plasma interaction at the location of the resonance. This result is briefly discussed in the paper. We demonstrate that this property of magnetic field does not lead to the wave-plasma resonance and by itself cannot lead to plasma heating.

The present investigation concentrates on the question whether the global properties of the wave structure (which is not resonant at $\omega = \omega_{ci}$) combined with the hot plasma effects can result in effective plasma heating near the location where $\omega = \omega_{ci}$. We consider a simple RFP equilibrium in cylindrical geometry with poloidal symmetry. Field equations are Fourier analyzed in θ and z directions. The model antenna induces rf fields with the wave numbers m and k_z. Because the higher harmonics effects are not important in our case, a hot plasma dielectric tensor is derived in the limit $\rho_L \rightarrow 0$ (where ρ_L is the Larmor radius). We consider collisionless wave damping by plasma species. The heating efficiency is examined for different plasma parameters and different mode numbers of the wave. Effective heating of the plasma center is found for some range of parameters in the case of the $m = -1$ mode. RFP result may also be applicable for other low field configurations such as spherical tokamaks.

Section II contains a detailed description of the model used and of the approximations made in the analysis. In Sec. III the results of the analysis are discussed. We summarize in Sec. IV.

II. DESCRIPTION OF THE MODEL AND FIELD EQUATIONS

We assume a cylindrical geometry with vessel radius a. We consider the Bessel function equilibrium (the Taylor state) given by[11,12]

$$\frac{B_z}{B_0} = J_0(\mu r), \quad \frac{B_\theta}{B_0} = J_1(\mu r), \quad B_r = 0, \tag{1}$$

where B_0 is the toroidal field on axis, J_0 and J_1 are Bessel functions. For this state B_z reverses direction with radius if $\mu a > 2.4$. The plasma density profile is of general form $n = n_0 \cdot n(r)$, where n_0 is the density on axis.

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This equilibrium is symmetric in θ and z directions. In RFP experiments the nonuniformity of magnetic field intensity in the poloidal direction is not strong; it is about 30% at $r/a = 0.6$ in Madison Symmetric Torus (MST) experiment.[13] Also we concentrate on heating near magnetic axis where the fraction of trapped particles is minimal. Thus the results of the rf study found in our 1-D model should be applicable to the real toroidal experiments.

We perform Fourier transform of θ and z coordinates of Maxwell equations in cylindrical coordinates. We study plasma heating at $\omega \approx \omega_{ci}$ but not at the harmonics of this frequency, so that the limit of zero ion Larmor radius ρ_{Li} is considered. In this limit the relation between the Fourier components of rf current and electric field is local with respect to r. Assuming time dependence $\propto \exp(-i\omega t)$ the Maxwell equations are

$$\left(\frac{m^2}{r^2} + k_z^2\right)E_r + \frac{im}{r}\left(E_\theta' + \frac{E_\theta}{r}\right) + ik_z E_z' = \frac{\omega^2}{c^2}(\mathsf{T}\mathbf{E})_r,$$

$$\frac{im}{r^2}[-(rE_r)' + 2E_r] + E_\theta'' + \frac{1}{r}E_\theta' - \left(\frac{1}{r^2} + k_z^2\right)E_\theta + \frac{mk_z}{r}E_z$$

$$= -\frac{\omega^2}{c^2}(\mathsf{T}\mathbf{E})_\theta, \qquad (2)$$

$$-\frac{ik_z}{r}(rE_r)' + \frac{mk_z}{r}E_\theta + E_z'' - \frac{m^2}{r^2}E_z + \frac{1}{r}E_z' = -\frac{\omega^2}{c^2}(\mathsf{T}\mathbf{E})_z,$$

where E_r, E_θ, E_z are the Fourier components of rf electric fields with the azimuthal and axial wave numbers m and k_z, and the dielectric tensor T is calculated in cylindrical coordinates. In the above equations the derivatives are with respect to r.

The hot plasma dielectric tensor in Eqs. (2) is calculated from linearized Vlasov equation by integrating along unperturbed particle trajectories in a Maxwellian plasma with $T_e = T_i$ in the limit $\rho_L \to 0$. We consider simple trajectories along the magnetic field lines without drifts in which the components of the ion velocity in cylindrical coordinates with respect to the direction of the local magnetic field are $v_\parallel = \text{const}$, $v_\perp = \text{const}$, $\phi = -\omega_{ci}t$. The angle ϕ is measured from the local radial direction $\mathbf{e}_r$. This calculation leads to the expected result

$$T_{11} = K_1, \quad T_{12} = \cos\theta_0 K_2, \quad T_{13} = -\sin\theta_0 K_2,$$

$$T_{21} = -T_{12}, \quad T_{22} = \cos^2\theta_0 K_1 + \sin^2\theta_0 K_3,$$

$$T_{23} = \cos\theta_0 \sin\theta_0 (K_3 - K_1), \qquad (3)$$

$$T_{31} = -T_{13}, \quad T_{32} = T_{23}, \quad T_{33} = \cos^2\theta_0 K_3 + \sin^2\theta_0 K_1,$$

where θ_0 is the angle between the magnetic field and z axis,

$$K_1 = 1 + \frac{1}{2}\sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}\zeta_\alpha[Z(\zeta_{\alpha+}) + Z(\zeta_{\alpha-})],$$

$$K_2 = -\frac{i}{2}\sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}\zeta_\alpha[Z(\zeta_{\alpha+}) - Z(\zeta_{\alpha-})], \qquad (4)$$

$$K_3 = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}\zeta_\alpha^2 Z'(\zeta_\alpha),$$

where α corresponds to ions and electrons,

$$\zeta_\alpha = \frac{\omega}{|k_\parallel|v_{T\alpha}}, \quad \zeta_{\alpha\pm} = \frac{\omega \pm \omega_{c\alpha}}{|k_\parallel|v_{T\alpha}}, \quad v_{T\alpha} = \sqrt{\frac{2T_\alpha}{m_\alpha}}, \qquad (5)$$

$\omega_{p\alpha}$, $\omega_{c\alpha}$ are the plasma and cyclotron frequency, Z is the plasma dispersion function and

$$k_\parallel = \frac{m\sin\theta_0}{r} + k_z\cos\theta_0. \qquad (6)$$

Equations (4) are the components of hot plasma dielectric tensor in a uniform magnetic field (see, e.g., Ref. 6) in the limit $\rho_L \to 0$ in which $k_\parallel$ of Eq. (6) is used in place of k_z.

Similar description involving $k_\parallel$ of Eq. (6) is often used for the rf analysis in tokamaks when the poloidal component of equilibrium magnetic field is included in the model.[14] Equations (3)–(6) provide an accurate description for $r \sim a$. One can see, however, that these equations are not accurate near $r = 0$. At $r = 0$ the magnetic field is parallel to the z axis, $\theta_0 = 0$. In this case the dielectric tensor is correctly described by Eqs. (3)–(5) with $k_\parallel = k_z$. If $r \to 0$ then for magnetic field of Eq. (1) the ratio $\sin\theta_0/r$ has a finite value and $k_\parallel$ given by Eq. (6) is not equal to k_z. This discrepancy is due to the fact that in the derivation of the dielectric tensor the angle ϕ in the velocity space is measured from the local radial direction $\mathbf{e}_r$ at a point in which particle is located. This direction changes rapidly when the particle moves along field line at small r which leads to an inappropriate description of the particle motion.

The rate of absorption of rf energy in a hot plasma is sensitive to the value of $k_\parallel$ in Eqs. (5). It appears that the exact calculation of the dielectric tensor even with the simple motion of particles and in the limit of zero Larmor radius would require additional numerical treatment which is beyond the scope of our analysis. This necessity of an accurate treatment of the particle's trajectory in our simple model indirectly supports the statement made in Ref. 15 that such exact calculation is necessary for a proper description of the plasma dielectric properties in the resonance region.

For a more accurate description we modify $k_\parallel$ by introducing the multiplyer $\sin^2\theta_0$ in the first term in Eq. (6):

$$k_\parallel = \frac{m\sin\theta_0}{r}\sin^2\theta_0 + k_z\cos\theta_0. \qquad (7)$$

When this $k_\parallel$ is used in Eqs. (5) it gives the correct result for $r \approx 0$ at which $\theta_0 \approx 0$ and for $r \approx a$ at which $\theta_0 \approx \pi/2$. From this point of view the use of $k_\parallel$ given by Eq. (7) is adequate for the analysis of absorption rates near the axis (which is the region of our interest) in our simple model. Some inaccuracy in the dielectric tensor for the intermediate r should not change the conclusions of our study. Similar results are obtained from a collisional cold plasma model. We assume a uniform temperature distribution in the plasma and find the power absorbed per unit volume as $P = \langle \mathbf{j}\cdot\mathbf{E}\rangle$, where $\mathbf{j}$ is the wave-induced plasma current density calculated using the dielectric tensor T. Because of the high thermal conductivity of the plasma along the magnetic field lines, the absorbed power should be averaged over a magnetic surface; see Sec. 38 in Ref. 16. In our simple analysis the antenna drives rf
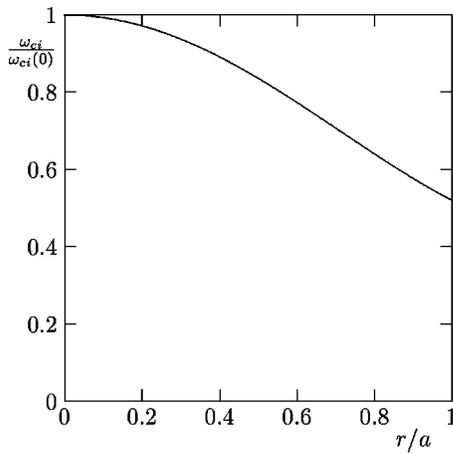
FIG. 1. $\omega_{ci}/\omega_{ci}(0)$ vs r/a.

fields with uniform amplitudes in θ and z directions. Thus the power P defined by the above equation is a surface averaged quantity.

For simplicity we assume that the antenna induces an electric field of a unit amplitude with time dependence $\propto \exp(-i\omega t)$ and wave numbers m, k_z on the inner surface of the vessel parallel to the z direction so that

$$\mathbf{E}^a = \mathbf{e}_z e^{im\theta + ik_z z}.$$

As a particular equilibrium we consider the equilibrium without field reversal. We take $B_z(a) = 0$ which corresponds to $\mu a = 2.4$ in Eq. (1). For this equilibrium the antenna electric field is perpendicular to the equilibrium magnetic field. In real experiments there is some, typically small, reversal of the B_z component near the edge. Our calculations show that such a reversal does not change the conclusions of our analysis.

Equations (2) in which $k_{\parallel}$ is given by Eq. (7) are solved numerically by a finite difference method. The condition $\nabla \cdot \mathbf{TE} = 0$ is included in the difference equations for the consistency of the method.

III. RESULTS AND DISCUSSION

Our goal is to examine plasma heating efficiency near the axis by the excited antenna fields with different wave numbers m and k_z. Heating is due to the presence of ion cyclotron resonance near the axis. The radial profile of normalized ω_{ci} found using Eq. (1) is shown in Fig. 1. Ion heating arises from a finite amount of the left-handed polarization (which coincides with the ion gyromotion) near the resonance. The only modes for which E_r, E_θ are nonzero at $r = 0$ are with $m = \pm 1$ (it is a general property of fields in cylindrical coordinates). We consider rf fields only with these wave numbers.

First we examined a simple model of a plane geometry (with a cold plasma) analytically to see whether there is a resonant interaction of the wave and plasma at $\omega = \omega_{ci}$ when the wave is launched away from the resonance and is propagating in the direction perpendicular to the magnetic field. In this model the magnetic field intensity and its direction are allowed to change along the direction of wave propagation. The result of this analysis is that neither a strong gradient of the magnetic field intensity nor a rapid change of the magnetic field direction (field rotation of $\pi/2$ over one wave length) result in the wave resonance at $\omega = \omega_{ci}$. The wave is right-handed at the location of the resonance.

Absence of the resonance in the magnetic field with a spatial gradient relates to a poor heating efficiency of tokamaks at $\omega = \omega_{ci}$. An initial motivation for this research was to see whether there is a resonance in the case of a rapid change of the direction of magnetic field. This rapid change takes place in an RFP in which the magnetic field is toroidal on axis and is poloidal near the edge. The absence of a resonance in this simple analysis leads to the similar conclusion about poor heating efficiency of an RFP (with a cold plasma) at $\omega = \omega_{ci}$.

Now we consider the problem of plasma heating from a different perspective. In the cold plasma limit the polarization is right-handed at the location of the resonance. If the global mode structure is such that in the vicinity of the resonance there is some amount of left-handed polarization then in the case of a hot plasma when the resonance is broadened heating is possible near the location where $\omega = \omega_{ci}$. We examine the heating efficiency for a hot plasma using the equations of the previous section.

The plasma density profile used in our calculations is

$$n(r) = \left\{ 0.1 + \frac{1}{2}[1 + \cos(\pi r/a)] \right\}/1.1.$$

First we fix the parameters in the model as $a = 50$ cm, $B_0 = 1$ kG, $n_0 = 10^{13}$ cm^{-3} and we consider a hydrogen plasma. These parameters are not far from the parameters of the MST experiment.

The electric field on axis is represented as a sum of left and right-handed polarizations (with respect to the direction of magnetic field) $\mathbf{E} = \mathbf{E}_L + \mathbf{E}_R$. Let $E_{\max}$ be the maximum absolute value of the amplitude of electric field in the cross-section. Then we find the ratio $|\mathbf{E}_L(0)|/E_{\max}$ as a function of k_z for the modes with $m = 1$ and $m = -1$ for the frequencies close to ω_{ci} on axis. The result shows that this ratio is practically zero for the $m = 1$ mode for the parameters of interest. This means that the $m = 1$ mode is not appropriate for heating at the plasma center.

Figure 2 shows the dependence of $|\mathbf{E}_L(0)|/E_{\max}$ vs $k_z a$ for the $m = -1$ mode for the temperatures $T = 200$, 300, 500 eV. On this figure the rf frequency is $\omega/\omega_{ci}(0) = 0.89$. One can see that the amount of left-handed component on axis is substantial (for heating purposes) for a range of wave numbers k_z. It is not strongly dependent on plasma temperature and can be considered as a property of the particular mode. A similar amount of left-handed component is obtained with a cold plasma dielectric tensor. Therefore if there is a left-handed component near the resonance in the cold plasma limit then this property does not change in the hot plasma when the resonance is broadened. For the heating on axis to be strong at this frequency, the resonance should be broad enough:

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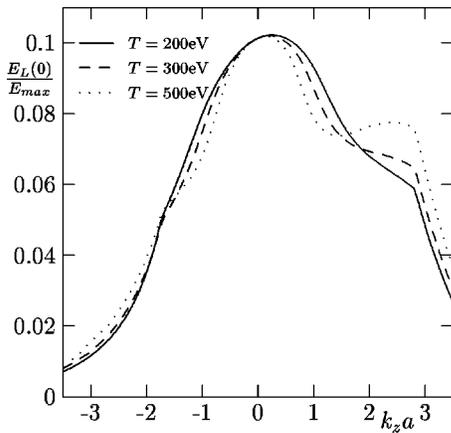Plasma heating in reversed field pinches . . .   1345

FIG. 2. Dependence of $E_L(0)/E_{\max}$ vs $k_z a$ for three temperatures. $m = -1$, $\omega/\omega_{ci}(0) = 0.89$.

$$|\zeta_{i-}| = \left| \frac{\omega - \omega_{ci}(0)}{k_z v_{Ti}} \right| \lesssim 1. \tag{8}$$

For $k_z a = 1.5$ and $T = 300$ eV, $|\zeta_{i-}| = 1.5$. This value is approximately in the range defined in Eq. (8). For these parameters the radial profile of the power absorbed per unit volume P (in arbitrary units) is shown in Fig. 3(a) and the
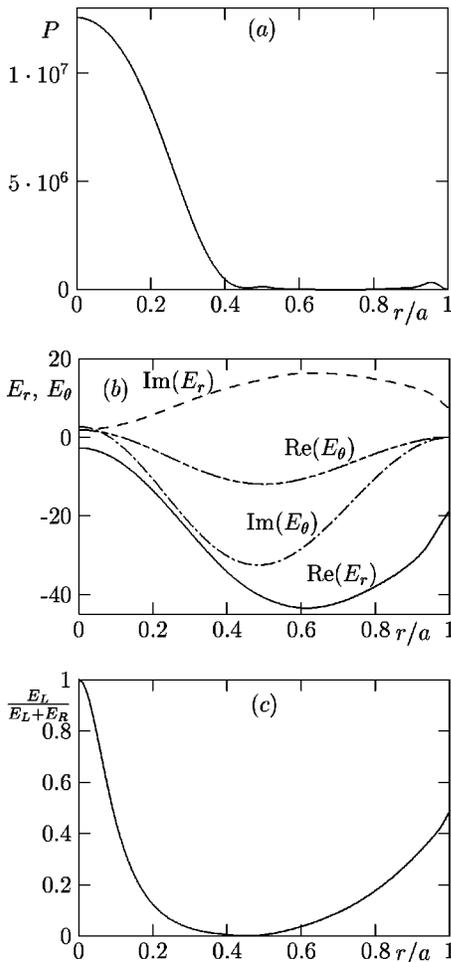

FIG. 3. Radial profiles of (a) P; (b) E_r, E_θ; (c) $E_L/(E_L+E_R)$. $m = -1$, $k_z a = 1.5$, $T = 300$ eV, $\omega/\omega_{ci}(0) = 0.89$.
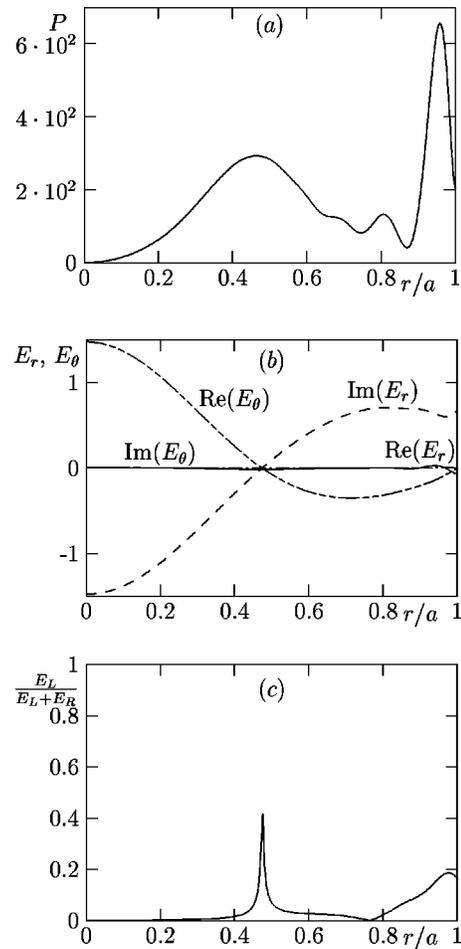

FIG. 4. Radial profiles of (a) P; (b) E_r, E_θ; (c) $E_L/(E_L+E_R)$. $m = 1$, $k_z a = 1.5$, $T = 300$ eV, $\omega/\omega_{ci}(0) = 0.89$.

profile of the components E_r, E_θ is presented in Fig. 3(b). The heating on axis is much stronger than it is near the plasma edge (even with the assumption of a hot plasma near the edge). At the center the power is absorbed by ions and the electron heating is negligible. Near the edge the power is absorbed mostly by electrons. From Fig. 3(b) one can see that there is a good penetration of the wave into the plasma with some finite wave amplitude near the plasma center. The wave amplitude on axis is relatively small; however, its polarization there is purely left-handed. Figure 3(c) shows the radial profile of the ratio $E_L/(E_L+E_R)$ ($E_{L,R}$ are the absolute values of the corresponding amplitudes). At the location of the cold plasma resonance ($\omega = \omega_{ci}$, $r/a \approx 0.4$) the wave is right-handed.

One should note that the use of Eq. (6) for the calculation of $k_\parallel$ instead of the corrected Eq. (7) would not result in significant change of the wave polarization profile but it would result in the suppression of the central heating in this particular example.

We compare the results of Figs. 3(a)–3(c) obtained for the $m = -1$ mode with the results for the $m = 1$ mode. Figures 4(a)–4(c) show the corresponding profiles for the $m = 1$ mode for the same parameters as on Figs. 3(a)–3(c). One can see that despite a good penetration of the wave into the plasma, the wave polarization is right-handed near the axis
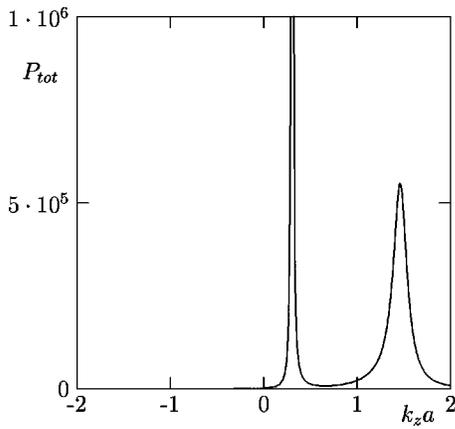
FIG. 5. P_{tot} vs $k_z a$, $m = -1$, $T = 300$ eV, $\omega/\omega_{ci}(0) = 0.89$.

FIG. 6. P vs r/a. $n_0 = 5 \times 10^{13}$ cm^{-3}, $k_z a = 3.5$, $a = 50$ cm, $B_0 = 1$ kG, $T = 300$ eV, $\omega/\omega_{ci} = 0.9$.

and there is no heating of the plasma center. The electron heating is now comparable with the ion heating. The power P in Fig. 4(a) is in the same scale as in Fig. 3(a). In Fig. 3(a) the absorbed power is much higher than that in Fig. 4(a). This is partially because the $m = -1$ harmonic is left-handed in the vicinity of the resonance and is partially because an eigenmode of the cylindrical RFP plasma is excited for the conditions of Figs. 3 [the wave amplitudes on Fig. 3(b) are significantly larger than those on Fig. 4(b)].

Figure 5 shows the dependence of the total power absorbed in the plasma P_{tot} (per unit length of the cylinder, in arbitrary units) vs $k_z a$ for the $m = -1$ mode and for $T = 300$ eV and $\omega/\omega_{ci}(0) = 0.89$. The two peaks at $k_z a \approx 0.3$ and $k_z a \approx 1.5$ correspond to the excitation of the eigenmodes of the cylindrical RFP plasma. $k_z a$ is relatively small at the first (narrow) peak, the condition of Eq. (8) is not satisfied near axis so that mostly the edge plasma (electron component) is heated for these parameters. The large absorption is due to a very large excited wave amplitude. Near the second (broad) maximum the condition of Eq. (8) is approximately satisfied and the plasma center is heated. Because the maximum is broad, it is feasible to excite such wave fields by an antenna. From Fig. 5 one can expect that for these plasma parameters the most efficient heating is for a directional antenna which excites the $m = -1$ mode with the wave numbers corresponding to this maximum. Analysis of different ranges of plasma parameters shows, however, that generally there is no favorable direction of wave excitation for effective heating. Excitation of the eigenmode of the RFP plasma is preferable from the antenna loading considerations. However, the plasma center is heated effectively also in cases when no eigenmode is excited.

For a given distribution of em fields we have estimated the power dissipated in aluminum walls and in a cold collisional edge plasma. Results indicate that for the $m = -1$ mode the power absorbed in the hot plasma is usually several orders of magnitude higher than the power absorbed because of the other mechanisms. In the case of the $m = 1$ mode both rates can be comparable.

Our analysis shows that the $m = -1$ mode launched with the wave number k_z satisfying the condition of Eq. (8) and for which the wave energy penetrates deeply into the plasma,
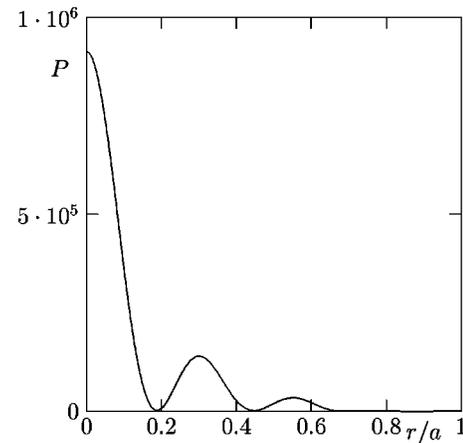
can be used for an effective heating of the central part of the plasma. This result is supported by calculations using the cold plasma dielectric tensor with the resonance resolved by artificially enhanced collision rates.

For the parameters chosen in the above analysis the rf field profiles correspond to the excitation of the lowest radial wave numbers. For higher plasma densities or larger radius of the vessel wave fields with higher radial wave numbers are excited. In this case the favorable property of the $m = -1$ mode remains. It is left-handed on the axis. Figure 6 shows the radial profile of the power absorbed per unit volume P (in arbitrary units) for higher density $n_0 = 5 \times 10^{13}$ cm^{-3} and $k_z a = 3.5$. On this figure $a = 50$ cm, $B_0 = 1$ kG, $T = 300$ eV, $\omega/\omega_{ci}(0) = 0.9$. Because of the excitation of a higher radial wave number the absorbed power has an oscillatory behavior. The most power is absorbed near the center. For higher density the value $k_z a$ is larger and Eq. (8) is better satisfied. These values of $k_z a$ are feasible for real antennas.

The value of magnetic field B_0 which we used to demonstrate effective heating of the plasma center was chosen to be relatively small in order to satisfy the condition of Eq. (8). For a fixed value of $\omega/\omega_{ci}(0)$ the mode dispersion relation and polarization profile is weakly dependent on the value of B_0 while with the increase of B_0 the absolute value of the difference $\omega - \omega_{ci}(0)$ in Eq. (8) increases. Thus for higher magnetic fields the amount of left-handed field component in the center is substantial but the condition of Eq. (8) is not satisfied and there is no central heating of the plasma. This conclusion is supported by our numerical analysis.

In Eq. (8) for a fixed $\omega/\omega_{ci}(0)$ (which approximately fixes the relative amount of the left-handed field component in the center) the difference $\omega - \omega_{ci}(0)$ is proportional to B_0, while k_z found from a dispersion relation is approximately proportional to $\sqrt{n_0}$. Thus the condition of Eq. (8) separates plasma parameters into regions where the heating is effective and ineffective. These two regions are presented approximately in Fig. 7 for $a = 50$ cm and $T = 200, 400, 800$ eV for a hydrogen plasma. For higher B_0 (region above separation line) the heating is ineffective and for higher densities
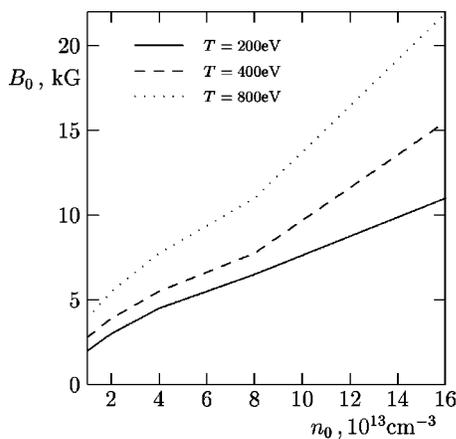
FIG. 7. Approximate separation of parameter ranges with effective and ineffective heating for three ion temperatures. $m = -1$, $a = 50$ cm.

n_0 (region below separation line) the heating is effective. This chart is found from a visual analysis of the power deposition and polarization profiles for different $\omega/\omega_{ci}(0)$ and k_z and should be considered as an approximate one. There is no sharp separation of the two regions. The separation lines on this figure approximately satisfy $B_0 \propto \sqrt{n_0}$ and $B_0 \propto \sqrt{T}$. This chart should not change significantly with the change of the vessel radius a. To ensure an effective plasma heating one should operate at some distance below the specified line.

The separation lines in Fig. 7 are in the range of parameters of modern RFP experiments. One can use this chart to determine whether this heating method is effective for particular operating conditions.

The possibility of plasma heating in the RFP near the fundamental ion cyclotron resonance is an important finding. This effect is due to the particular polarization of the $m = -1$ mode and also because the RFP operates at low magnetic field. It raises the question of whether this mode polarization is a unique property of the reversed field pinch in which the magnetic field direction varies strongly with radius or is a general property of the $m = -1$ mode in a magnetized plasma.

To address this issue we performed similar calculations for two additional magnetic configurations. In the first case we assumed that there is only a B_z component of magnetic field present which varies with the radius as the intensity of the RFP field shown in Fig. 1. Now an antenna excites an E_θ component on the surface of the vessel. In the second case we considered the field configuration similar to the previous one but assumed that B_z is uniform in cross section. These calculations show that in the first case the wave properties and heating efficiency is close to those in the RFP case. In the second case the $m = -1$ mode still has some finite left-handed polarization in the center (for $\omega \neq \omega_{ci}$) but the effect is weaker than in the case of the RFP. One can conclude that the considered properties of the $m = -1$ mode are the general to this mode which are exhibited in different magnetic field configurations. The accessibility of the wave with the

left-hand polarization to the plasma center is better, however, for the radial profile of the magnetic field intensity of an RFP type.

In tokamaks the magnetic field is stronger than in RFPs so that it is harder to satisfy the condition of Eq. (8). This, when combined with the smaller accessibility of the left-handed field component to the plasma center, makes this method of plasma heating ineffective for tokamaks.

IV. SUMMARY

The plasma heating efficiency in the reversed field pinch by excited rf waves with the frequency in the vicinity of the fundamental ion cyclotron resonance is examined. For this, appropriate treatment of the dielectric properties of the hot magnetized plasma is required.

Results showed that the effective central heating of ions is possible for some range of parameters when the $m = -1$ mode with the appropriate wave number k_z is excited. The $m = 1$ mode is not suitable for the heating. This effect is due to a thermal broadening of the ion cyclotron resonance in a hot plasma; it disappears in the cold plasma limit in which the resonance is narrow. Heating is due to a general property of the $m = -1$ mode which has a substantial amount of left-handed polarization on axis when the wave frequency is close to the ion cyclotron frequency at the center. The method is effective in a low magnetic field. Analysis indicates that the effect is substantial in an RFP configuration rather than in tokamaks.

Plasma heating in spherical tokamaks and in other low field configurations by this method can be effective also. A separate analysis for these configurations is encouraged by the results of our study.

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