Determination of Z_{eff} in the Madison Symmetric Torus through Integrated Data Analysis

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Future fusion devices will require new data analysis methods

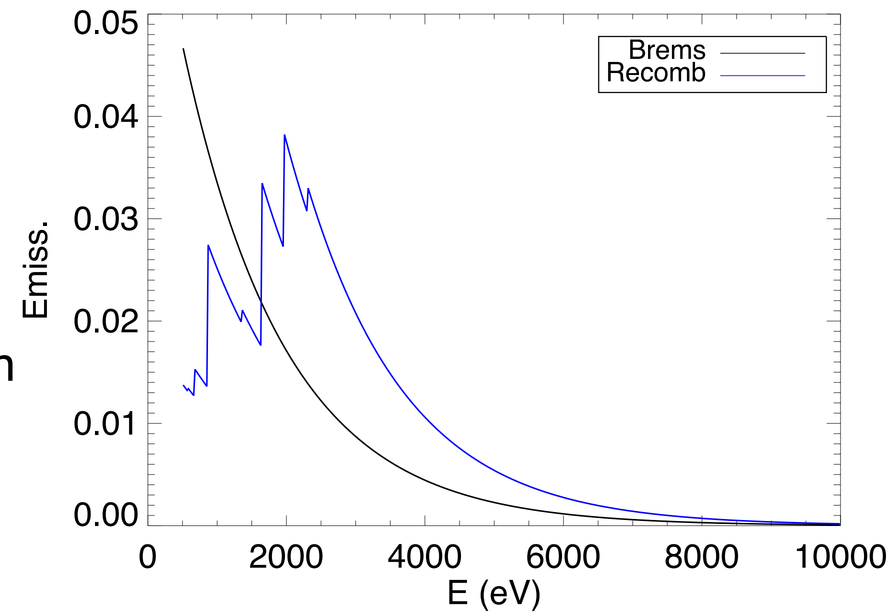
- Limited diagnostic access will make data more difficult to obtain
  - Available data will be more valuable
- Need techniques to maximize information gained from data
  - Requires scientists trained in techniques
- Integrated data analysis presents method for maximizing knowledge gain
  - Applied at W7-AS, ASDEX, JET

One problem amenable to treatment through IDA is the determination of Z_{eff} in MST PPCD plasmas

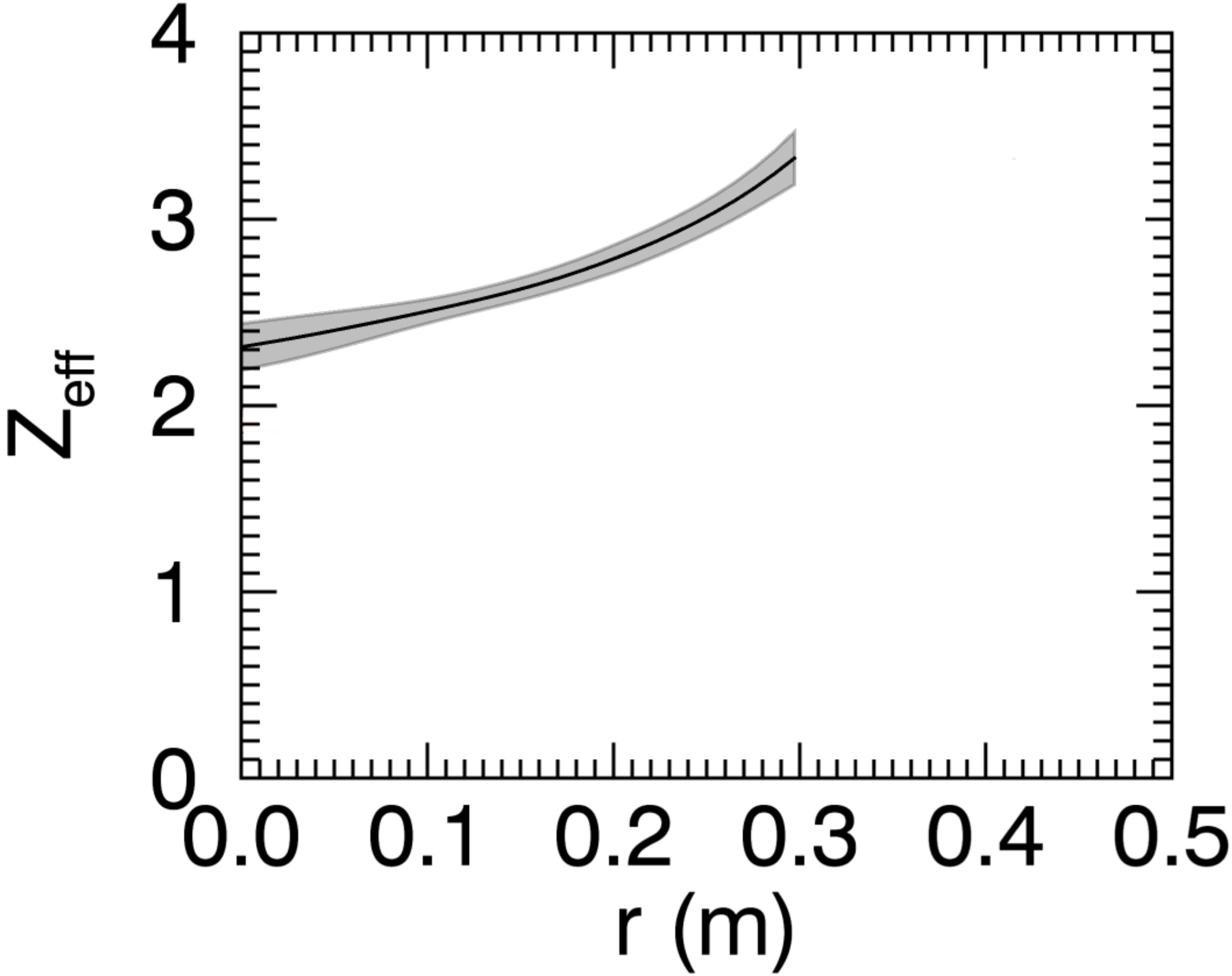
- Visible and NIR bremsstrahlung measurements were deemed infeasible
  - Large edge neutral density
  - Signal contaminated by molecular emission and electron-neutral bremsstrahlung

- CHERS measurements of highly-stripped impurity densities available
  - Missing impurities? (underestimate Z_{eff})
  - Prior estimates range between 2-6

- Soft X-ray (SXR) emission is a combination of both bremsstrahlung and impurity recombination
  - Prior estimates neglected recombination and produced unreasonably high Z_{eff}
  - Which impurities contribute?
  - Need model for recombination edges
  - Missing impurities? (overestimate Z_{eff})



By combining diagnostics with redundant dependence on Z_{eff} we can establish an estimate with higher confidence



Outline

- Brief introduction to integrated data analysis (IDA)
- How we use IDA to determine Z_{eff}
- Summary and future work

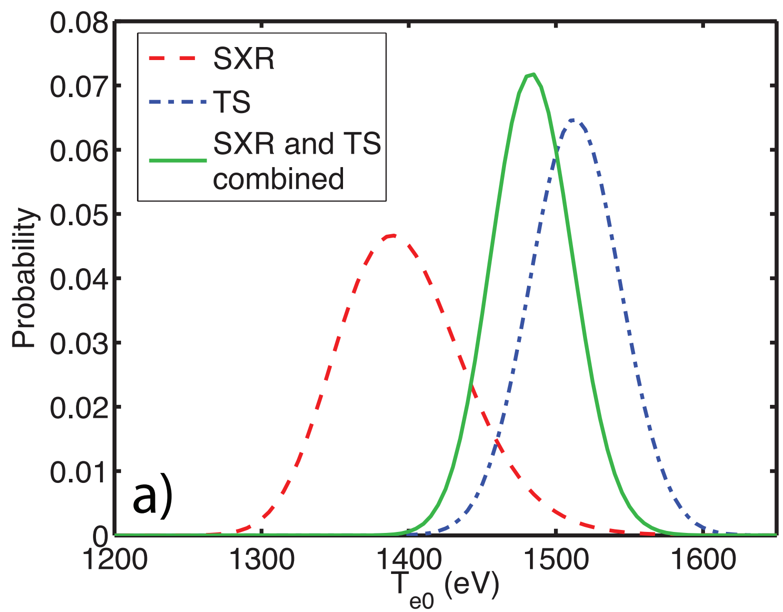
Brief introduction to Integrated Data Analysis

- "The concept of integrated data analysis (IDA) offers a unified way of combining data from various experiments with modeling information to obtain improved results." [1]

- Combine information from multiple, unique diagnostics to
  - Provide a parameter estimate that is more precise than any individual diagnostic
  - Allow for consistent error analysis

- Utilize diagnostic redundancy to determine parameters that cannot be measured by individual diagnostics

- Technique applied to T_e measurements from Thomson scattering and SXR has improved temperature profile estimate uncertainties by x4



[1]R. Fischer *et. al.* Fus. Sci. and Tech. (2010)

Bayesian probability theory provides a framework for IDA

- Create a forward model for diagnostic signals
  - Combine all relevant information including prior knowledge
  - Predict diagnostic signals given parameterized profile
  - Incorporate all known uncertainties, both systematic and statistical

- Result is most probable value for given quantity of interest
  - $\underbrace{p\left(n_{Z_i}|d\right)}_{\text{posterior}} \propto \underbrace{p\left(d|n_{Z_i}\right)}_{\text{likelihood}} \underbrace{p\left(n_{Z_i}\right)}_{\text{prior}}$

- Highly modular
  - Easy inclusion of many diagnostics by multiplying likelihoods
  - $p\left(n_{Z_i}|d\right) \propto p\left(d_{\mathrm{CHERS}}|n_{Z_i}\right) p\left(d_{\mathrm{SXR}}|n_{Z_i}\right) \ldots$

- While we are starting with uncorrelated, standard statistical likelihoods for the diagnostic set, the framework is general enough to handle correlated parameters and non-Gaussian PDFs.

Standard analysis techniques often rely on inversion and curve fitting techniques and produce separate results

FIR interferometry

line-integrated data
high time resolution

density inversion
uncertainty analysis

Thomson Scattering

local data
low time resolution

uncertainty analysis
spline curve fit

CHERS impurity density

local data
high time resolution

uncertainty analysis
spline curve fit

$n_e(r)$ $T_e(r)$ $n_Z(r)$

SXR emission

line-integrated data
high time resolution

emissivity inversion
uncertainty analysis

Is the forward model comparable to data?

The goal of Integrated Data Analysis is to create one self-consistent comprehensive model to explain diagnostic data

Parameterized model for $n_e(r,t)$, $T_e(r,t)$, $n_z(r,t)$

Forward model for interferometer	Forward model for Thomson Scattering	Forward model for SXR	Forward model for CHERS
Comparison with data	Comparison with data	Comparison with data	Comparison with data

Parameter optimization and uncertainty analysis

This work begins by integrating the impurity density dependence of CHERS and SXR emission

Forward model
for interferometer

density inversion
uncertainty analysis

Forward model
for Thomson Scattering

uncertainty analysis
spline curve fit

Profiles of $n_e(r)$, $T_e(r)$

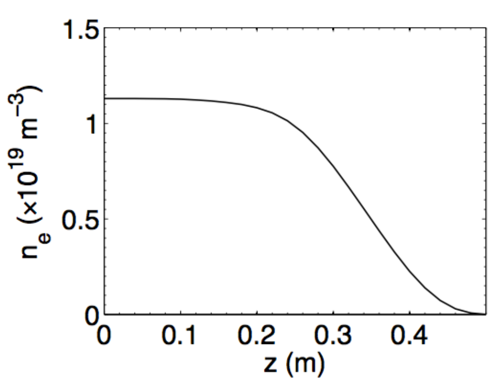
Parameterized model for $n_z(r)$

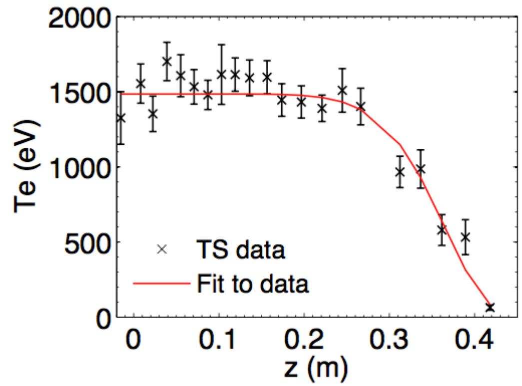
Forward model
for SXR

Forward model
for CHERS

Parameter optimization
and uncertainty analysis

Profiles of density and temperature from interferometry and Thomson scattering are assumed for the rest of this work





Profiles of $n_e(r)$, $T_e(r)$

Parameterized model for n_z (r)

Forward model for SXR

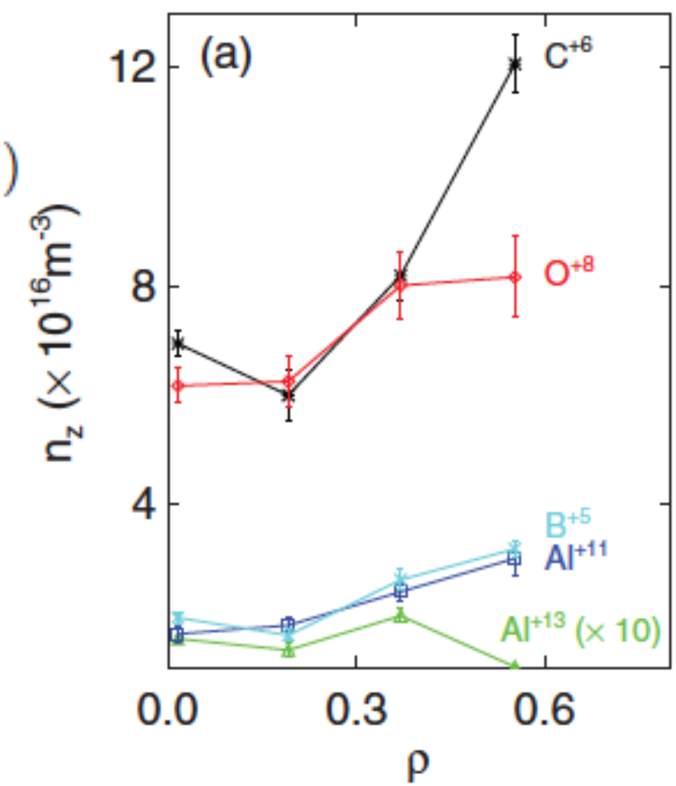
Forward model for CHERS

Parameter optimization and uncertainty analysis

Prior information: Impurity density measurements using Charge Exchange Recombination Spectroscopy

- Relative distribution of C, O, N, B impurities known from previous experiments
- Measurements of Al^{+10}, Al^{+11}, and Al^{+13}
- Infer Al^{+12} from ionization balance using ADAS
- Impurity density structure (profiles) known for certain species
  - Hollow in high current PPCD
  - Adequate parameterized profile:

$$n(r) = n_0(1-(r/a)^\alpha)^\beta + \Delta n \exp(-(\delta_r - r/a)^2/2\Delta r^2)$$

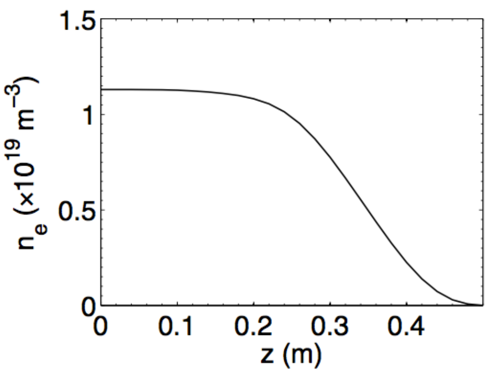


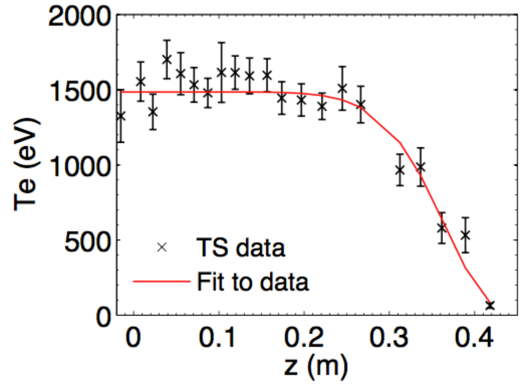
- Treat aluminum density separately from low-charge-state ions
  - O, N, B densities treated as percentage of carbon density

- Construct a prior distribution using measured values of n_{C+6} and n_{Al+11} with gaussian profiles for PDF based on diagnostic uncertainty

S. Kumar *et. al.* Phys. Plasmas (2012)

We are now left with determining the likelihood of the SXR emissivity data





Profiles of $n_e(r)$, $T_e(r)$

Parameterized model for n_z (r)

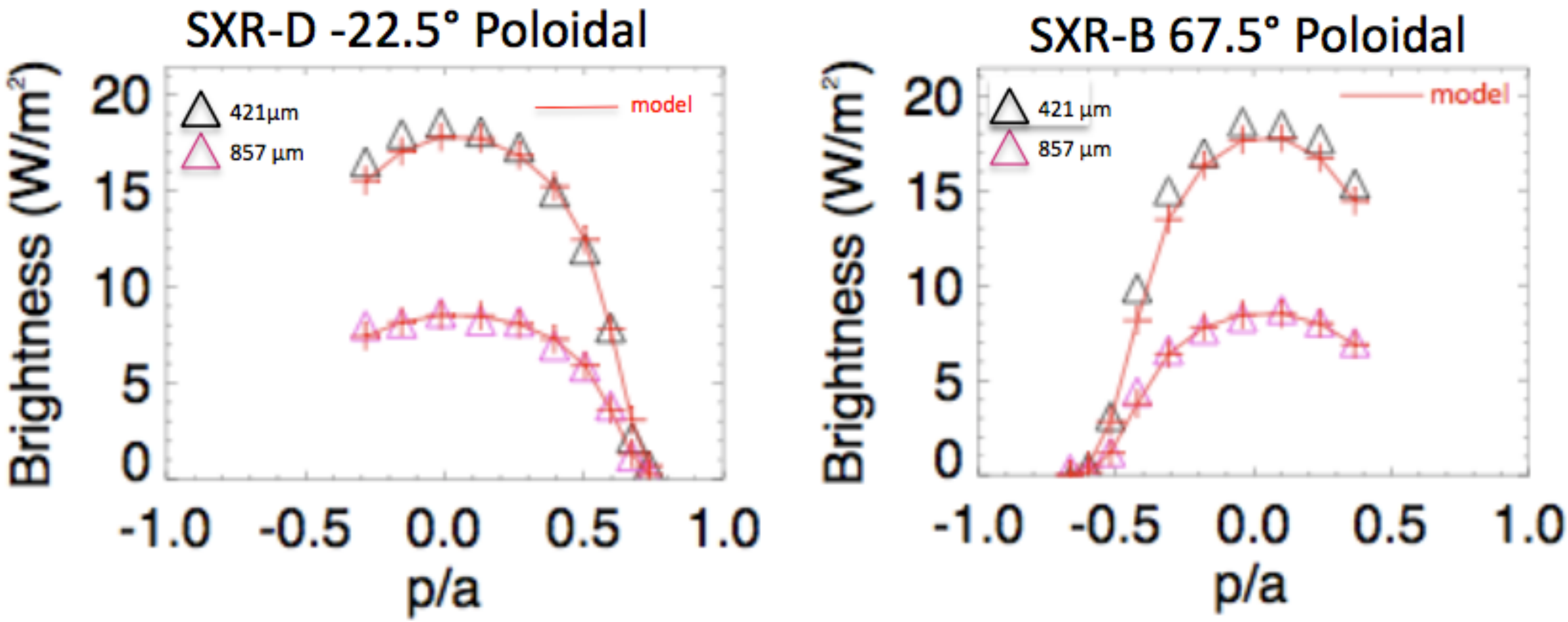
$$n(r) = n_0(1-(r/a)^\alpha)^\beta + \Delta n \exp(-(\delta_r - r/a)^2/2\Delta r^2)$$

Forward model for SXR

$$p\left(d_{\mathrm{CHERS}} | n_{Z_i}\right)$$

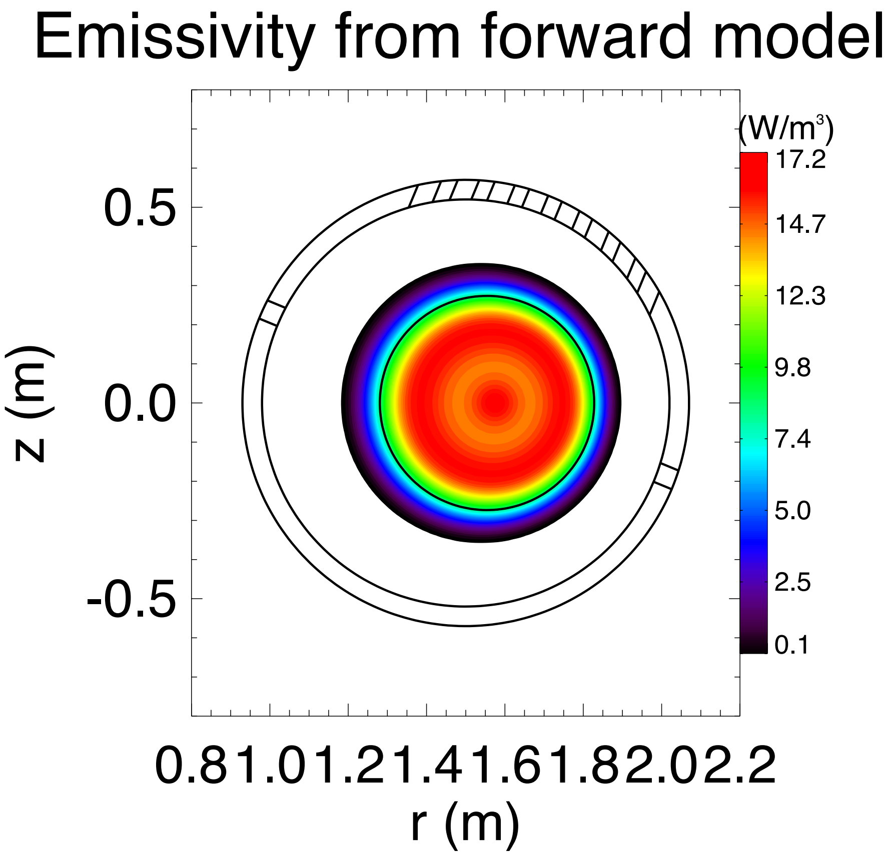
Parameter optimization and uncertainty analysis

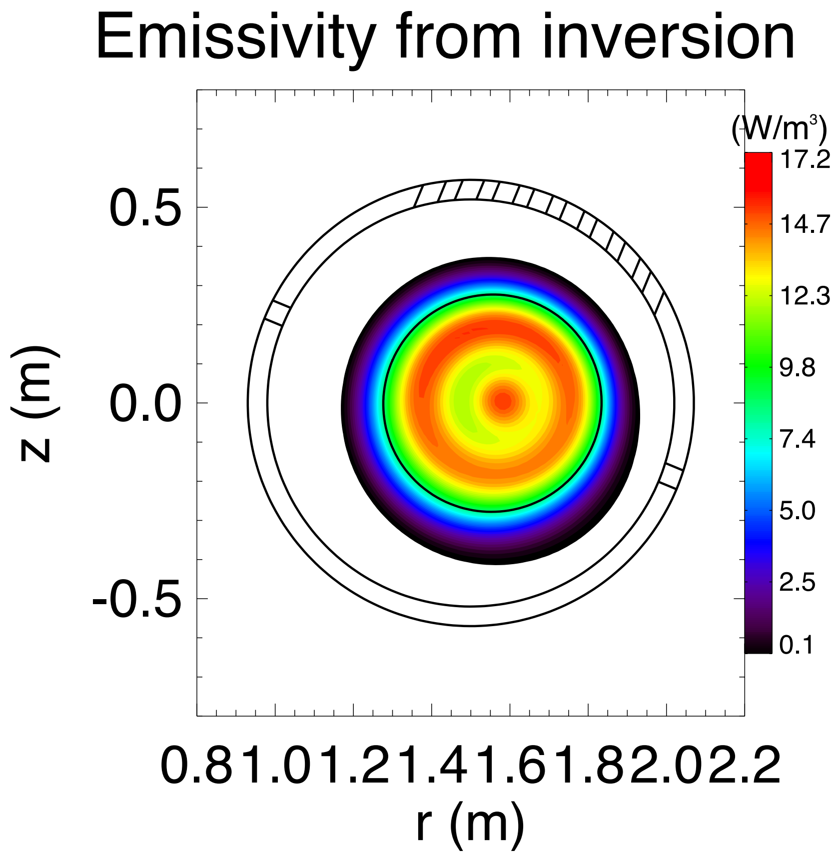
Forward model for SXR emissivity predicts bremsstrahlung and recombination emission



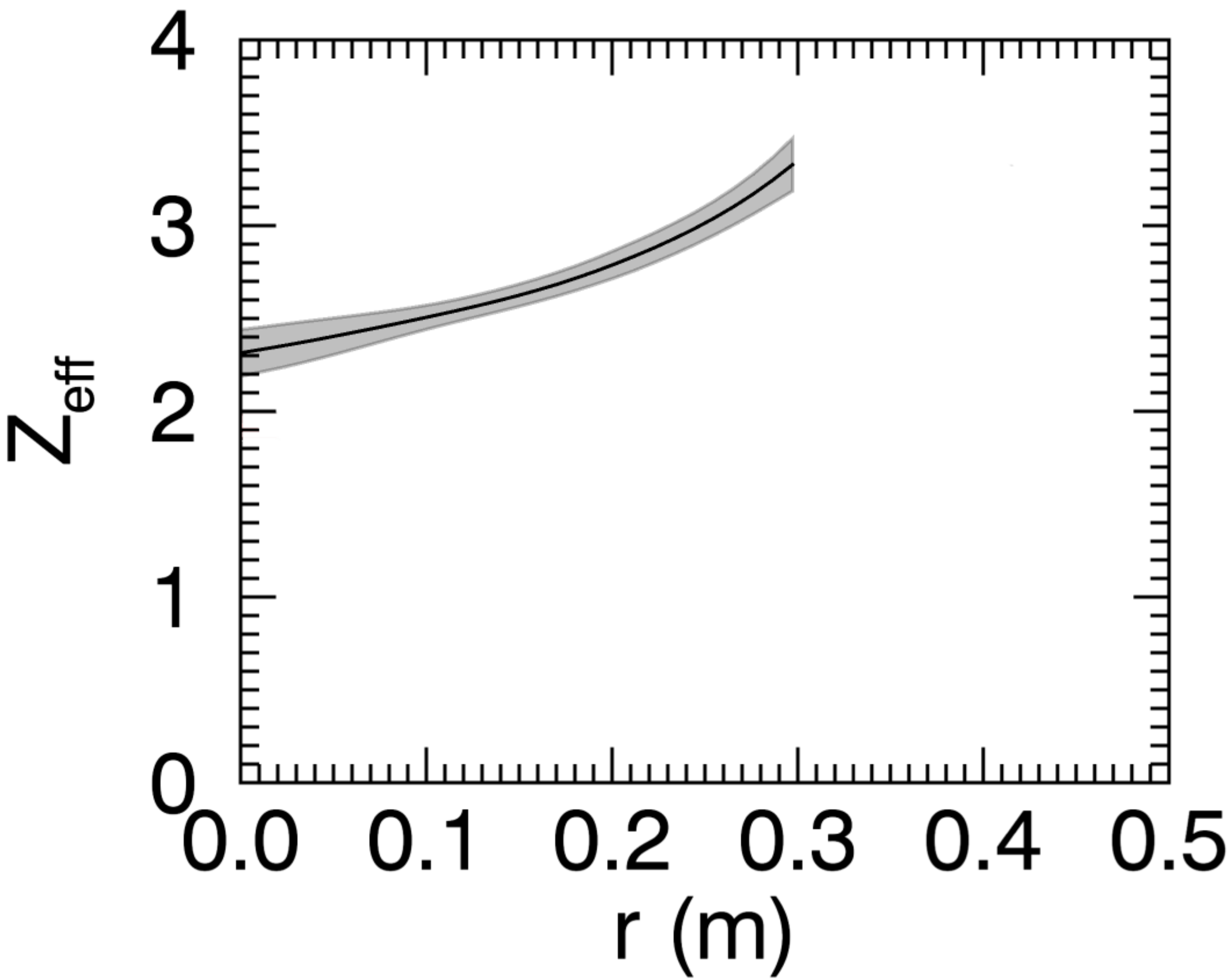
- Parameter search using Markov Chain Monte Carlo
  - Allows many model parameters with reasonable processing time
  - Provides flexibility for further modifications to the model emissivity

Emissivity from forward model is similar to emissivity from inversion technique

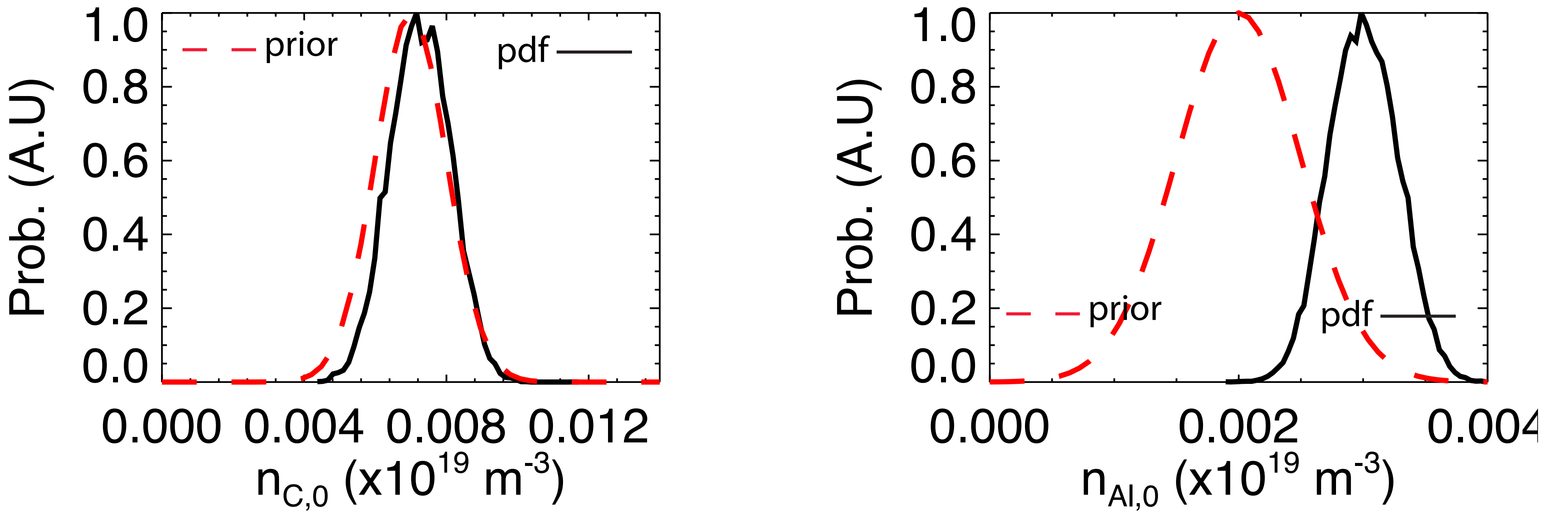




Z_{eff} in MST, as determined through IDA, is much better constrained to the level calculated from only CHERS data



However, histograms of the core impurity densities suggest we may have under-predicted the Aluminum concentration



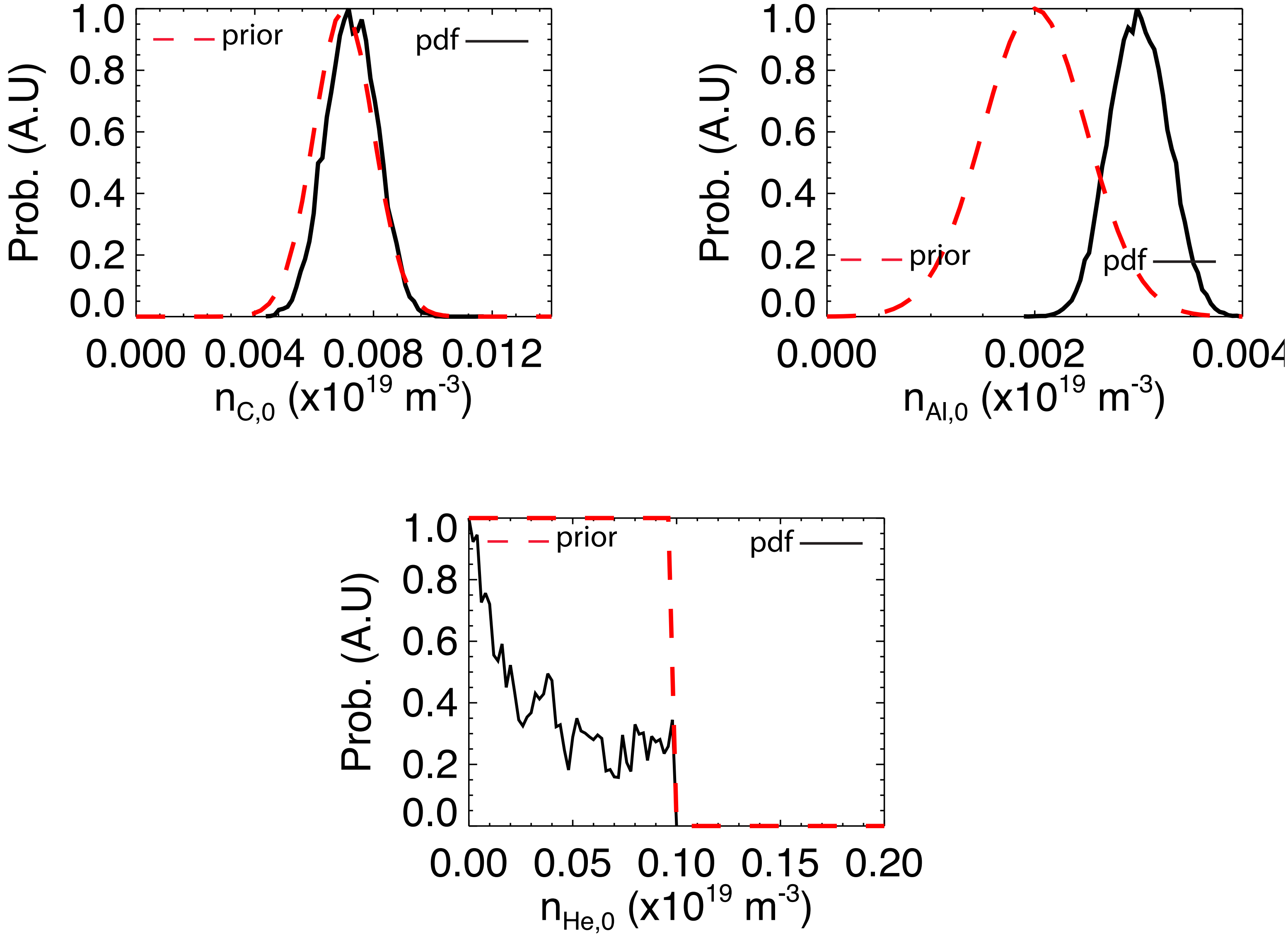
Hypothesis testing: What if we are missing an impurity?

- What if we eventually learn that there are other impurities in the plasma that are not part of the SXR emissivity model?

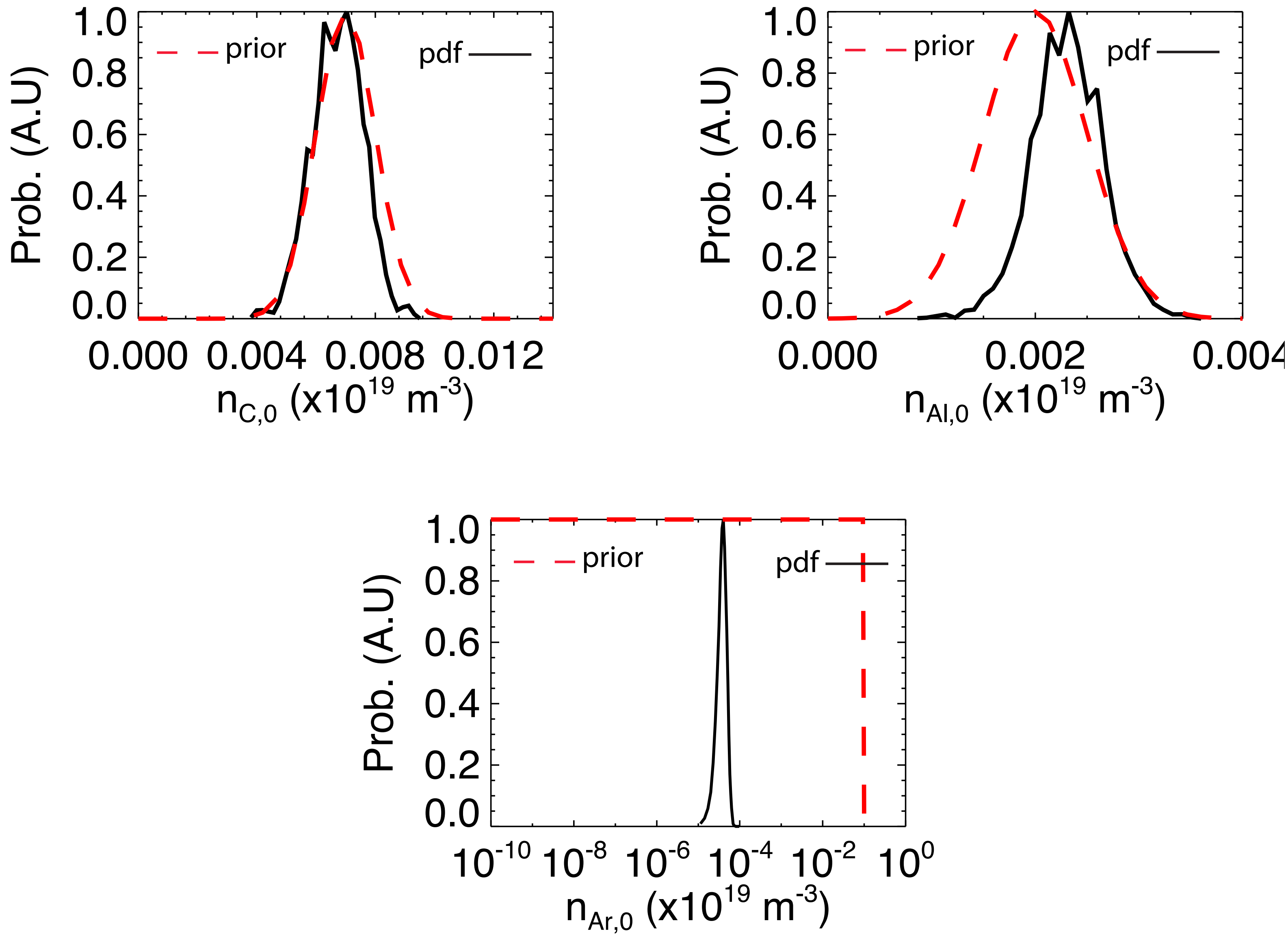
- Let's construct a new emissivity parameterization to test this hypotheses
  - Assume that there is He^{+2} as a proxy for low-Z impurities
    - Low-Z impurities could be present in large density without contributing much SXR emission
    - They also wouldn't contribute significantly to Z_{eff}
  - Assume that there is Ar^{+18} as a proxy for mid-Z impurities
    - Mid-Z impurities, if present, could contribute significantly to Z_{eff}
    - They should also produce significant SXR emission
    - How much unexplained SXR emission is there?

- No CHERS data for either case, so assume uniform priors up to 10% of density

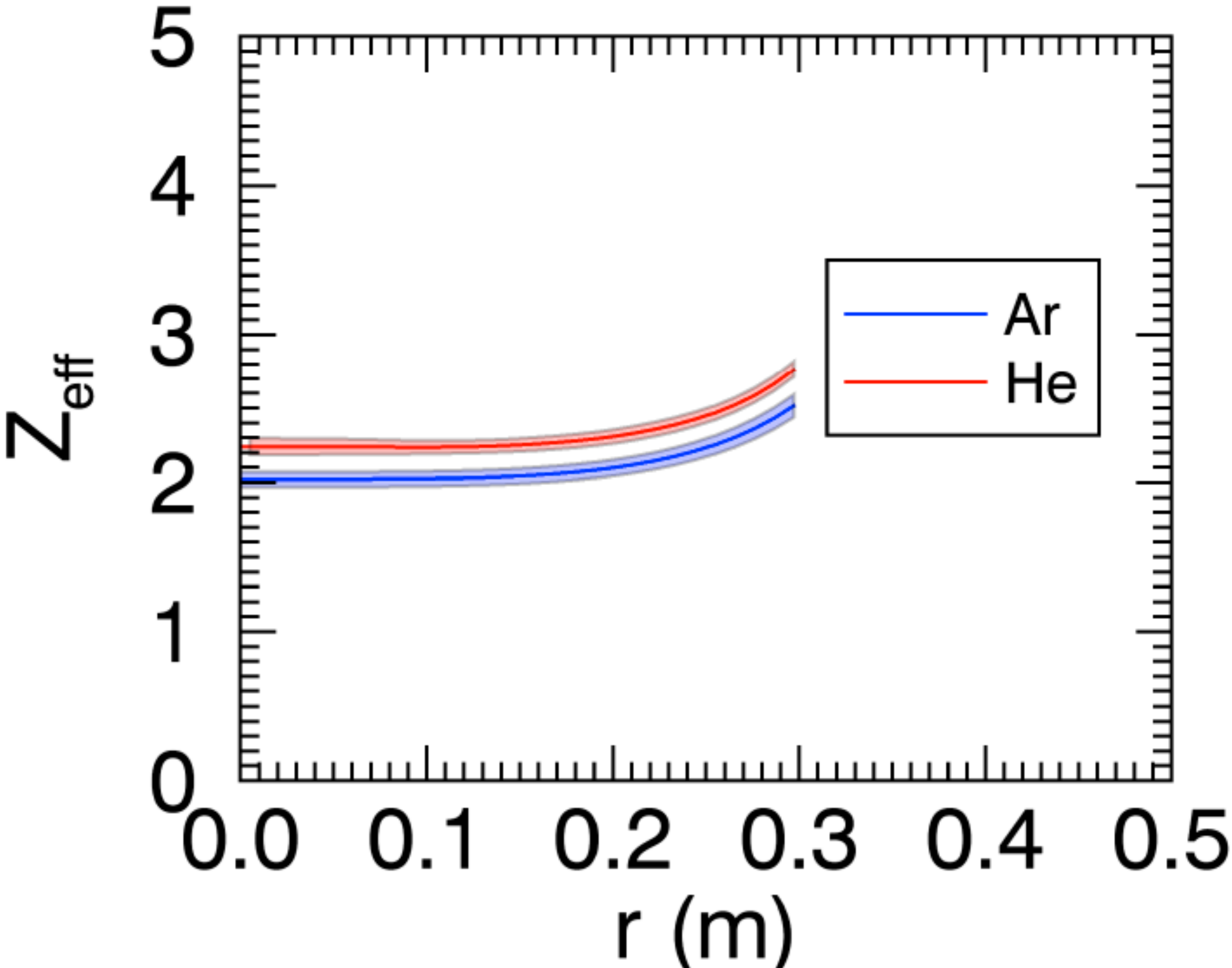
The added density of He is not well constrained by SXR emission



The density of the added Ar can be constrained by SXR and reduces Z_{eff} by diluting the core Aluminum concentration



Missing impurities can dilute the core Aluminum concentration thereby reducing Z_{eff}



Summary & future work

- IDA framework to determine Z_{eff} being developed at MST
  - Applied to SXR tomography system and existing CHERS data

- SXR impurity recombination emission forward model developed
  - Planning development of forward models for other diagnostics

- $Z_{eff} \sim 2$ in core of high current improved confinement discharges
  - Determine Z_{eff} in standard discharges

- Hypothesis testing suggests that mid-Z impurities may be missing from the model
  - Check for evidence of highly ionized mid-Z impurities like Argon in plasma

<u>References</u>

R. Fischer *et. al.*PPCF (2002)
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