Two Fluid Dynamo and Edge-Resonant m=0 Tearing Instability in Reversed Field Pinch

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Abstract. Current-driven tearing instabilities dominate magnetic relaxation in self-organized high temperature plasmas such as the reversed field pinch (RFP) and spheromak. In the Madison Symmetric Torus (MST) RFP experiments, they are observed in the form of magnetic field, flow velocity, and current density fluctuations that follow a temporally cyclic sawtooth behavior. During a sawtooth crash, a surge occurs in the dynamo - a fluctuation-induced mean electromotive force in the generalized Ohm's law that combines the MHD $\mathbf{v} \times \mathbf{B}$ and $\mathbf{j} \times \mathbf{B}$ Hall dynamos. We report new results on the physics of two-fluid dynamos as well as on the problem of spontaneous (linear) instability of edge-resonant m=0 tearing modes. The key findings are: **(1)** two fluid effects are critical for dynamo through their influence on the phase between the fluctuations; in cylindrical RFP, the two fluid tearing instability becomes oscillatory due to significant curvature of the field lines; **(2)** the two fluid version of the NIMROD code confirms analytic results during the linear stage of the instability but exhibits significant broadening of the Hall dynamo profile on the longer time scales of nonlinear evolution; **(3)** improved modeling of force-free RFP equilibrium predicts a wide range of RFP parameters in which m=0 tearing mode is spontaneously unstable, a result that is consistent with recent MST experimental observations.

1. Introduction

Current-driven tearing instabilities are believed to dominate magnetic relaxation in self-organized high temperature plasmas such as the reversed field pinch (RFP) and spheromak. In the Madison Symmetric Torus (MST) RFP experiments, tearing instabilities are observed in the form of magnetic field, flow velocity and current density fluctuations that follow a temporally cyclic sawtooth behavior. During a sawtooth crash, a surge occurs in the dynamo - a fluctuation-induced mean electromotive force in the generalized Ohm's law that combines the MHD ($\mathbf{v}\times\mathbf{B}$) and Hall ($\mathbf{j} \times\mathbf{B}$) dynamos. The dynamo modifies parallel electric field and plasma current profiles. In particular, the mean current density at the magnetic axis drops significantly during plasma relaxation events (magnetic reconnection) accompanied by current increase in the edge. This ultimately leads to current flattening in the core and current sustainment in the plasma edge. The underlying physics of plasma relaxation is of great importance for both laboratory and astrophysical plasmas. Dynamo activity in the RFP has been intensively studied analytically [1,2], by 3D MHD computations [3] and experimentally [4,5]. We report here new analytic and numerical results on the physics of two-fluid dynamos as well as on the problem of spontaneous (linear) instability of the edge-resonant m=0 tearing mode.

The two fluid quasilinear Hall dynamo theory [6] that was originally derived for a sheared slab geometry is generalized to cylindrical geometry and illuminates the effects of current gradient and field line curvature on the dynamo. We found that (1) two fluid effects are critically important for the Hall dynamo through their influence on the phase between the fluctuations that yields a non-zero flux surface averaged Hall dynamo, absent in resistive MHD; (2) field line curvature results in an oscillatory character of two fluid tearing instability in RFP; (3) the quadrupole structure of the out-of-plane component is strongly deformed by the equilibrium current gradient in RFP; (4) the two fluid version of the NIMROD code

confirms analytic results during the linear stage of the instability but exhibits significant broadening of the Hall dynamo profile on the longer time scales of nonlinear evolution.
The second group of the results is related to linear stability of the edge resonant m=0 mode. The m=0 mode is of special importance for RFPs because of its impact on mode coupling, ion heating, momentum and energy transport. Robust linear stability of the m=0 mode was verified in the past by many calculations based on a single parameter model for plasma current. Recent MST experiments [7] have shown that in some regimes with improved plasma confinement the m=0 mode becomes unstable. This motivates our interest in revisiting the m=0 stability analysis including a broader range of current profiles. A four-parameter model is introduced which permits us to vary the position and the width of the current gradient independently. Improved modeling of force-free RFP equilibria predicts a wide range of RFP parameters in which m=0 tearing mode is spontaneously unstable, a result that is consistent with MST experimental observations.

2. Two fluid dynamo theory

Standard RFP discharges in the MST are characterized by the periodic sawtooth fluctuations in the plasma velocity $\mathbf{v}^{(1)}$, current density $\mathbf{j}^{(1)}$ and magnetic field $\mathbf{B}^{(1)}$. Corresponding MHD $<\mathbf{v}^{(1)}\times\mathbf{B}^{(1)}>$ and Hall $<\mathbf{j}^{(1)}\times\mathbf{B}^{(1)}>/ne$ dynamos generate mean electric field and mean current accordingly to the generalized Ohm's law $<\mathbf{E}>_{\|}$ - $\eta<\mathbf{j}>_{\|} = -(1/c) <\mathbf{v}^{(1)}\times\mathbf{B}^{(1)}>_{\|} + (1/en^{(0)}c) <\mathbf{j}^{(1)}\times\mathbf{B}^{(1)}>_{\|}$, where < > denotes the mean (flux surface averaged) value. We model the dynamo by two methods: analytically using two fluid quasilinear tearing mode theory and numerically with the use of the two fluid NIMROD code. Quasilinear theory deals with flux surface averaged quadratic combinations of the linear tearing eigenfunctions of $\mathbf{v}^{(1)}$, $\mathbf{j}^{(1)}$ and $\mathbf{B}^{(1)}$. Non-zero quasilinear Hall dynamo was first calculated in [6] for a sheared slab model, large values of the stability factor Δ' and the "kinetic Alfven" regime of instability [8]. Detailed analysis [9] showed that the case of small Δ' and the transient regime between "kinetic Alfven" and whistler mediated reconnection is more suitable to the MST (see, Fig.1).

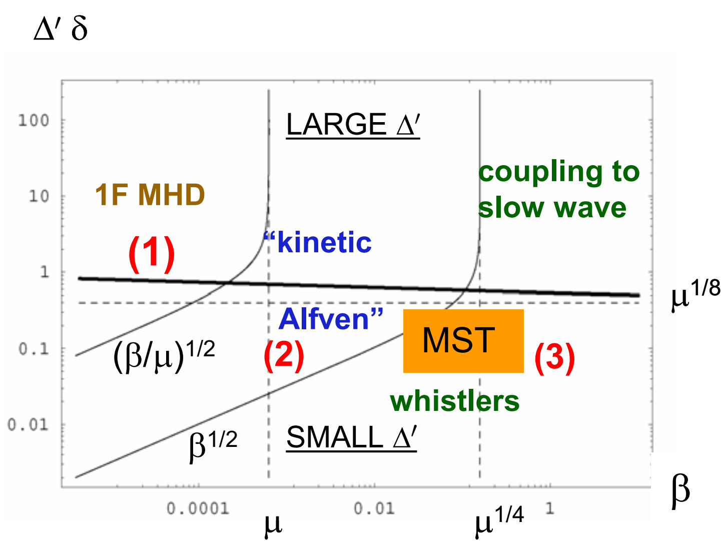


Fig.1. Classification of the different regimes of collisionless two-fluid tearing instability in terms of Δ' and plasma β ($\delta = c/\omega_{pe}$ is the electron skin depth, $\mu = m_e/m_i$). The thick line is the boundary between small and large Δ' cases, the thin lines separate single-fluid MHD regime (1) and the regimes (2),(3) where two-fluid effects dominate. The MST case, $\Delta'\delta \sim 0.2$, $\beta \sim$(5-10)% belongs to small Δ' and transitional regime between "kinetic Alfven"(2) and Whistler mediated (3) reconnection.

Focusing on this particular case, we consider two fluid tearing mode in force free cylindrical RFP with $p_{e,i}^{(0)} \neq 0$, $dp_{e,i}^{(0)} / dr = 0$, and, correspondingly, the effects of diamagnetic drifts $\omega^*_{e,i}$ are ignored. The reduced MHD approximation is not applicable for the RFP where perturbations of the guide field $B_{||}^{(1)}$ are comparable to the radial component $B_r^{(1)}$. Using helical harmonics expansion

$$B_{r,||}^{(1)} = \sum_{m,n} B_{r,||}^{(m,n)}(r)\exp(\gamma t + im\theta - ikz), \qquad k = n/R$$

and performing flux surface averaging (integration over z and θ) yields contributions from single tearing mode to the parallel components of the Hall and the MHD dynamos

$$\epsilon_{||}^{(H)} = \frac{1}{nec} < \mathbf{b}^{(0)}[\text{Re } \mathbf{j}^{(1)} \times \text{Re } \mathbf{B}^{(1)}] > \to \frac{\epsilon_0 d_i}{2r}\frac{\partial}{\partial r}\left(r|B_r||B_{||}|\cos\phi^{(H)}\right)$$

$$\epsilon_{||}^{(MHD)} = -\frac{1}{c} < \mathbf{b}^{(0)}[\text{Re } \mathbf{v}^{(1)} \times \text{Re } \mathbf{B}^{(1)}] > \;\to\; \frac{\epsilon_0}{2k_\perp r}\frac{\partial}{\partial r}\left(r|v_r||B_r|\sin\phi^{(MHD)}\right)$$

where $\phi^{(H)}$ and $\phi^{(MHD)}$ are, respectively, the phase shifts between complex B_r and $B_{||}$, v_r and B_r, $\mathbf{v} \to \mathbf{v}/v_A$, $\mathbf{B} \to \mathbf{B}/B_0$, $\mathbf{r} \to \mathbf{r}/a$, $\varepsilon_0 = v_A B_0/c$, indices (m,n) are omitted. Standard single fluid resistive MHD predicts a peak of the MHD dynamo localized to the rational surface and a small (but non-zero) $\varepsilon_{||}^{(MHD)}$ in the outer region. Using, for example, the ideal relation between v_r and B_r, yields a small (due to smallness of γ) MHD dynamo in the outer area

$$\gamma B_r = ik_{||}Bv_r \quad \to \quad \epsilon_{||}^{(MHD)} = \frac{\gamma\epsilon_0}{2k_\perp r}\frac{\partial}{\partial r}\left(\frac{r|B_r|^2}{k_{||}B}\right)$$

Formal application of single fluid resistive MHD to a Hall dynamo gives zero result due to the 90^0 phase shift between B_r and $B_{||}$. The situation changes radically with the transition to two fluid MHD where B_r and $B_{||}$ are in-phase yielding a large Hall dynamo that dominates in the vicinity of the rational surface. The "phase effect" is caused by electron-ion decoupling on short scales. In the two fluid case, the cross-field currents of decoupled electrons flow in The $\mathbf{E}_\perp\times\mathbf{B}$ direction while in single fluid MHD they are determined by polarization currents which are oriented along $\mathbf{E}_\perp$. This shifts the spatial profile of cross-field current vortexes with respect to the magnetic island and, correspondingly, changes the phase between $B_{||}$ and B_r. (see, Fig. 2.)

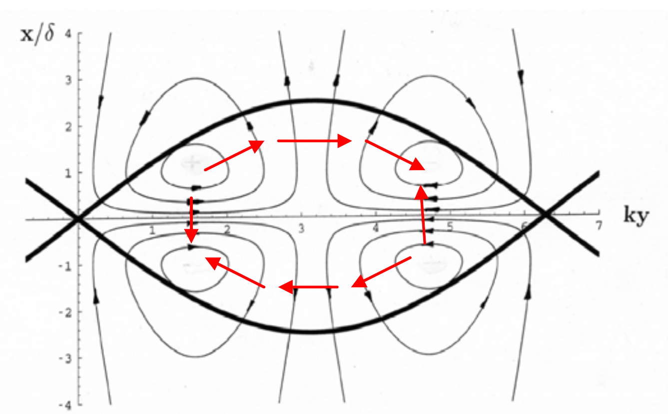


*Fig. 2. Qualitative illustration of the phase shift between B_r and $B_{||}$ caused by transition from single to two fluid MHD. The thick line represents position of the magnetic island, the thin lines and the red arrows are, respectively, the stream lines (equipotential surfaces) of **ExB** plasma flow and the polarization currents in single fluid MHD. In two-fluid MHD, the current vortexes are represented by the thin lines. They result from **ExB** flows of decoupled electrons while the ions are immovable.*

Quantitative treatment of $\varepsilon_{||}^{(H)}$ is based on linear eigenfunctions of the system. In tearing mode theory, they are calculated by matching of inner (resistive) and outer (ideal) solutions. The interrelation between outer (ideal) solutions for $B_{||}$ and B_r is as follows

$$B_{||}^{(outer)} = \frac{i}{k_0^2}\left(\frac{k_{||}}{r}\frac{\partial}{\partial r}(rB_r) - \lambda k_\perp B_r\right), \quad k_0^2 = m^2/r^2 + k^2, \qquad \lambda = \frac{4\pi a}{cB_0}\frac{j}{B}$$

This equation predicts a constant (not dependent on r) phase shift $\phi^{(H)} = \pi/2$ between $B_{||}$ and B_r and, therefore, $\varepsilon_{||}^{(H)} = 0$ in the outer region. Single-fluid resistive tearing layer equations [11] show that these functions are out of phase not only in the outer region but in the resistive layer too. Thus, within the scope of single fluid resistive MHD, $\varepsilon_{||}^{(H)} \equiv 0$ everywhere in a cylinder.

Transition from single to two fluid MHD does not change the plasma momentum equation but brings an additional $\nabla \times \mathbf{j} \times \mathbf{B}$ term into the induction equation. This term describes magnetic field frozen into the electron component. It modifies standard shear Alfven (SA), compressional Alfven (CA) and magnetoacoustic (MA) modes on short scales and gives rise to "kinetic Alfven" and whistler mediated regimes of tearing instability. At small values of Δ', the tearing instability is driven mainly by the electrons and, correspondingly, the ion motion can be ignored. Then, the components of the induction equation simplified in the vicinity of the resonant magnetic surface $k_{||} = 0$ are as follows

$$B_r - \delta^2 \frac{d^2 B_r}{dr^2} = \frac{d_i k_{||} k_0 B}{\gamma} \tilde{B}_{||}, \quad \tilde{B}_{||} = B_{||} + \frac{i\lambda}{k_0} B_r, \quad \beta = \frac{4\pi p^{(0)} \Gamma}{B_0^2}$$

$$\left[1 + B^2 \left(\frac{1}{\beta} + \frac{k_{||}^2}{\gamma^2}\right) + \frac{2 i d_i k_0}{\gamma} \frac{B_\theta^2}{rB}\right] \tilde{B}_{||} - \delta^2 \frac{d^2 \tilde{B}_{||}}{dr^2} = \frac{d_i B}{\gamma} \left(\frac{k_{||}}{k_0} \frac{\partial^2 B_r}{\partial r^2} - \frac{d\lambda}{dr} B_r\right)$$

where dimensionless γ is $\gamma\tau_A$, electron skin depth $\delta^2 = 1/(\gamma S)$, ion skin depth $d_i = c/(a\,\omega_{pi})$, $\Gamma = 5/3$, 1 is, respectively, the adiabatic or isothermal constant, the CA branch is decoupled by ignoring the inertia term in the $\mathbf{b} \times \mathbf{e}_r$ component of plasma momentum equation. The new function $\tilde{B}_{||}$ describes deviation of $B_{||}$ from its asymptotical outer solution $B_{||}^{(outer)}$. The term proportional to B_θ^2 in the second equation describes the effect of curvature that does not exist in the slab model. Ignoring this term, the solution for $\tilde{B}_{||}$ is in phase with B_r providing non-zero Hall dynamo in the tearing layer. Outside the layer, $\tilde{B}_{||}$ tends to zero changing the phase between $B_{||}$ and B_r to $\phi^{(H)} = \pi/2$. Thus, although the amplitude of $B_{||}$ is finite everywhere, the Hall dynamo vanishes in the outer area due to this phase effect. The curvature term and the term proportional to the gradient of the equilibrium current affect the growth rate and profile of $\varepsilon_{||}^{(H)}$ but do not change the general properties of the phase effect. We found that at small $\beta << \beta_c$, the curvature term can be ignored yielding the "kinetic Alfven" regime of instability with $\gamma \propto \beta^{1/3}$. At $\beta >> \beta_c$ this term becomes important leading to the saturation of γ as a function of β. The limiting value γ_c is a complex number with equal imaginary and real parts. It scales with Δ', d_i and S similar to the growth rate of the Whistler mediated instability in a slab geometry, $\gamma_c \propto \Delta' (d_i / S)^{1/2}$. The factor $\beta_c \propto (B/B_\theta)^3$ strongly depends on the curvature and is, therefore, significantly different for cylindrical RFP and tokamak-like equilibria ($\beta_c^{(RFP)} \sim 0.1\%$, $\beta_c^{(T)} \sim 15\%$), so that the instability is mostly growing at $B_\theta/B \sim 0.1$ and of an oscillatory type in the RFP ($B_\theta/B \sim 1$).

3. Numerical simulations

Numerical studies of Hall dynamo driven by two-fluid tearing instability are performed in a sheared slab geometry using the two-fluid version of NIMROD. In the linear stage of tearing instability, the Hall dynamo is localized on a short scale very near the resonance surface and is much larger than the MHD dynamo effect. On larger scales of order of ion gyroradius, the Hall dynamo diminishes and is comparable to the MHD dynamo. Nonlinear single mode computations are also performed in a wide parameter range. In the nonlinear regime, the

Hall dynamo broadens to the same scale as the MHD dynamo and contributions from the Hall and MHD terms to the dynamo are comparable (see, Fig. 3)

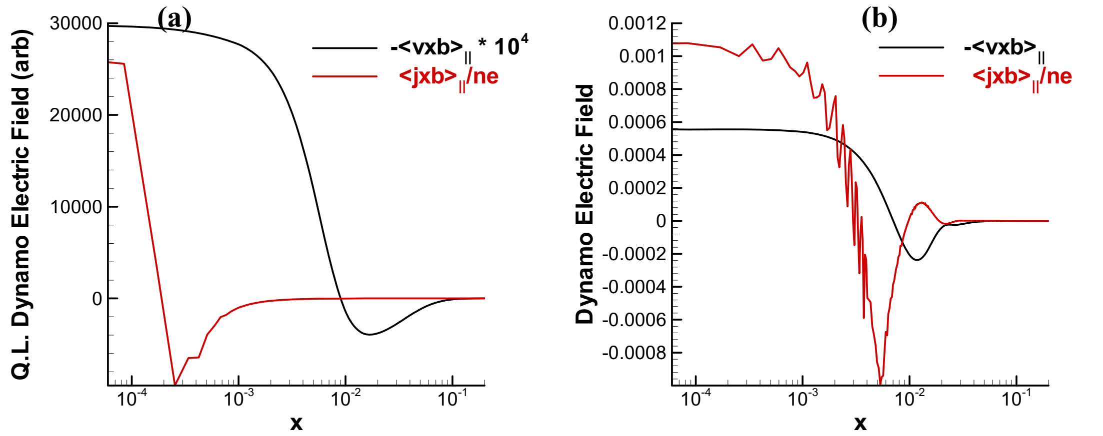


Fig.3. MHD and Hall dynamos vs. distance x from the resonant surface normalized to the equilibrium scale L during linear (a) and nonlinear (b) stages of tearing instability. Quasilinear computations (a) show strong and localized Hall dynamo, consistent with the analytical prediction. In nonlinear saturation (b), the Hall dynamo expands to the ion sound gyroradius scale ρ_s *, and the two dynamos become comparable in magnitude.*

Numerically computed growth rates are in good agreement with the analytical dispersion relation, provided that the tearing layer is sufficiently small with respect to the equilibrium scale. Viscous dissipation was anticipated for the nonlinear computations, and a parameter scan shows very little impact on linear growth rates with some broadening of the **V** profile of the eigenfunction. Examining the dynamo contributions locally in y (the direction in the reconnection plane orthogonal to x) shows that both Hall and MHD dynamos act in phase with respect to y in the quasilinear result; though, the x-scales and magnitudes are very different. In the nonlinear state, the local Hall electric field ($(\mathbf{j}^{(1)}\times\mathbf{B}^{(1)})_{\|}$ prior to surface-averaging) acts primarily near the separatrix (see, Fig. 4.) and remains large in magnitude relative to the MHD contribution while a broad profile is expected for current density redistribution caused by nonlinear MHD dynamo shown in Fig.5.

Fig.4. Contours of local fluctuation-induced Hall dynamo.

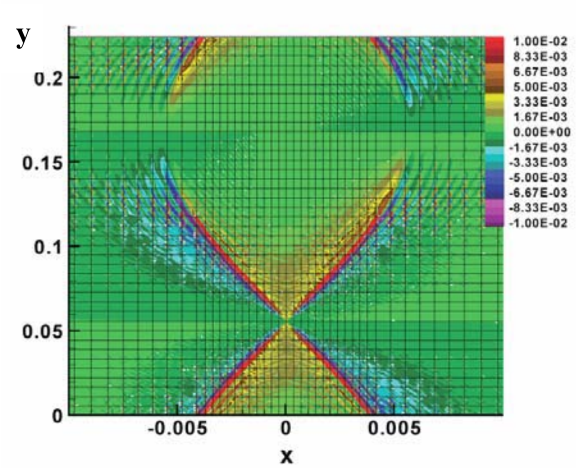
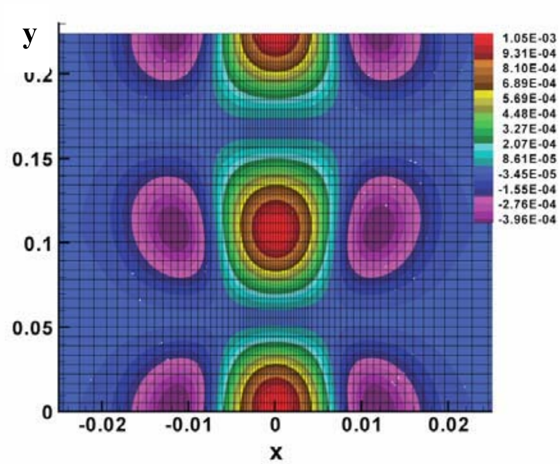


Fig. 5. Contours of local fluctuation-induced MHD dynamo.



4. Spontaneous tearing instability of m=0 modes

Typical MST regimes of operation exhibit cyclic sawtooth oscillations associated with core (m=1, n=6,7,…) and edge (m=0, n=1) resonant tearing modes. In typical discharges, it is believed that the core tearing modes are spontaneously unstable, while linearly stable m=0 modes are nonlinearly driven by coupling to core resonant modes. This scenario of forced m=0 magnetic reconnection is based in part on robust linear stability properties demonstrated in the past by various Δ' calculations which predict stability. Recent MST experiments [7] have shown that in some regimes with improved plasma confinement the m=0 mode becomes spontaneously unstable. Efforts to resolve this observation with theory inspired improved modeling of the equilibrium current profile. A four-parameter cylindrical model is introduced that allows independent variation in the radial position and width of the current gradient. These calculations determine a wide class of unstable equilibria showing a strong sensitivity of stability properties to small variations of current profiles. We report on ideal MHD Δ' analysis as well as the results obtained from a cylindrical resistive eigenvalue code.

Force free RFP equilibria are characterized by the parallel current profile $j = \lambda(r) B$ where $\lambda(r) = \mu_0 f(r)$ and

$$f(r,d,w,\alpha) = \frac{\left[\exp\left(\frac{r^\alpha - d^\alpha}{w}\right)+1\right]^{-1} - \left[\exp\left(\frac{1-d^\alpha}{w}\right)+1\right]^{-1}}{\left[\exp\left(-\frac{d^\alpha}{w}\right)+1\right]^{-1} - \left[\exp\left(\frac{1-d^\alpha}{w}\right)+1\right]^{-1}}$$

Four factors μ_0, w , d and α allow us to control independently the amplitude, width and position of the current gradient. At w >> 1, this model reduces to the standard α model

$$\lambda(r) \rightarrow \mu_0(1 - r^\alpha)$$

The values of the stability factor Δ' are found by solving the Newcomb equation

$$\frac{d^2 b_r}{dr^2} + \frac{f^*}{r}\frac{db_r}{dr} - g^* b_r = 0 \ ,$$

and matching left and right solutions at the resonant surface r_s ($q(r_s) = 0$) with the boundary conditions $b_r(0) = b_r(1)$. The results are presented in Fig. 6. for a few profiles $\lambda(r)$ and $\mu_0 = 3.35$, $\alpha = 3$.

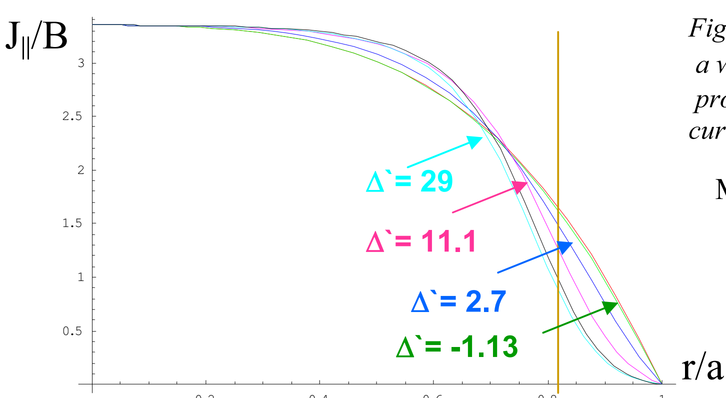


Fig. 6. Four parameter model describes a wide class of unstable ($\Delta' > 0$) current profiles while the α - model profile (green curve) is mostly stable.

Magnitude of Δ' as a function of RFP parameters F and Θ :

F=-0.06, Θ=1.6, Δ`= 29
F=- 0.24, Θ=1.7, Δ`= 11.1
F=- 0.23, Θ=1.6, Δ`= 2.7
F=- 0.22, Θ=1.6, Δ`= -1.13

Accordingly to the four parameter model there is a strong sensitivity of the stability properties to small variations of the current profile. The model reveals a wide class of unstable configurations with large positive Δ' whose $\lambda(r)$ profiles are just slightly different from the stable α-model curve. Calculations of the reversal F and pinch parameter Θ for these configurations show that the profiles with large Δ′ correspond to weakly reversed configurations. This is not consistent with the MST results where spontaneous m=0 instability is observed at F = – 0.5, while at F = -0.2 the mode is stable. In order to address this issue we analyzed the stability of these configurations not in terms of Δ′ , but by comparing the growth rates obtained from a resistive cylindrical eigenvalue code by V. Svidzinski. In Fig.7., the numerical results are compared with the analytic FKR expression [11] and more precise expression by G. Bertin [12] that takes into account the gradient of the equilibrium current.

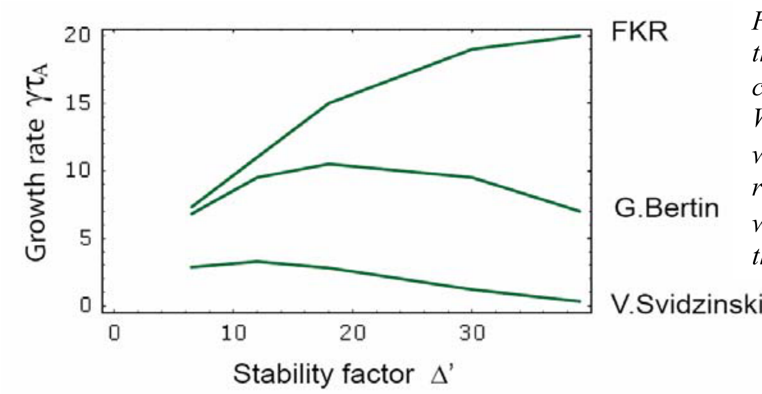


Fig. 7. Numerical calculations of the growth rate for different current profiles (β=0, $S=10^4$). Weakly reversed configurations with large Δ′ have smaller growth rates than deeply reversed states with small Δ′ that contradicts to the FKR analytic expression.

Strong suppression of the growth rates at large Δ′ may also result from finite β effects found in numerical calculations (see, Fig.8.)

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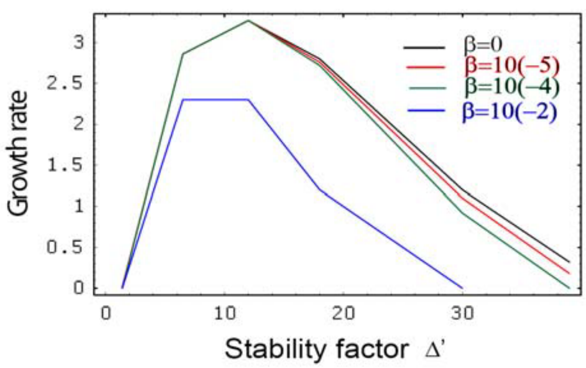

Fig. 8. Numerical calculations of the growth rate for different current profiles and values of β ($S=10^4$). The m=0 tearing instability driven by large Δ′ > 30 *are suppressed by finite β > 1% .*

These effects make m=0 tearing instability faster growing in deep reversal configurations with moderate Δ’ than in weak reversal configuration with large Δ’.

4. Summary

Three important effects for magnetic reconnection and Hall dynamo theory are obtained within the scope of the cylindrical RFP model:

(a) spatial variation of the phase between B_r and $B_{||}$ eliminates Hall dynamo in the ideal outer region, but makes this effect dominating in the tearing layer;

b) contribution from the field curvature can be ignored at small β; in the practical case of cylindrical RFP, the curvature leads to an oscillatory structure of Whistler mediated tearing instability, absent in tokamak-like configurations due to small curvature;

(c) despite the two fluid character of tearing instabilities in the RFP, the gradient of equilibrium current eliminates the quadrupole structure of the out-of plane component at typical values $\Delta' a < 5 - 6$; the quadrupole structure emerges at large $\Delta' a > 10$.

Two-fluid version of NIMROD accurately reproduces tearing mode theory in slab geometry with a large guide field. When the mode is at small amplitude, the computations reproduce the quasilinear prediction for Hall and MHD dynamo effects. With nonlinear saturation, the net Hall dynamo effect broadens and decreases in magnitude, becoming comparable to the net MHD dynamo effect; though, locally the amplitude of the Hall electric field remains greater. Multi-helicity two-fluid computations are needed to understand relaxation through nonlinear interactions among fluctuations.

Two-fluid Hall dynamo formalism is equivalent to the Maxwell stress calculations in the momentum transport theory. Thus, the results obtained are applicable to the problem of generation of parallel flows, momentum transport driven by tearing instabilities and magnetic reconnection in space plasmas.

Examining the wide class of unstable current profiles shows strong sensitivity of Δ' to small variation of the current profile. Dependence of the FKR growth rates on the reversal parameter F contradicts MST observations. An improved analytic model with $dj/dr \neq 0$ shows better agreement between the theory and the experiment. The effect of finite β makes m=0 tearing instability faster growing in deep reversal configurations with small Δ' than in weak reversal configuration with large Δ'.

Work supported by the U.S. Department of Energy and National Science Foundation.

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