Spectroscopic Observation of Fluctuation-Induced Dynamo in the Edge of the Reversed-Field Pinch

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The fluctuation-induced dynamo $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle$ has been investigated by direct measurement of $\tilde{\mathbf{v}}$ and $\tilde{\mathbf{b}}$ in the edge of a reversed-field pinch and is found to be significant in balancing Ohm's law. The velocity fluctuations producing the dynamo emf have poloidal mode number $m = 0$, consistent with MHD calculations and in contrast with the core $m = 1$ dynamo. The velocity fluctuations exhibit the parity relative to their resonant surface predicted by linear MHD theory.

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Self-generation of magnetic field in conducting and flowing media, the "dynamo effect," occurs in a broad range of settings—stars, planets, the interstellar medium, and laboratory plasmas [1]. It has long been known that in the toroidal laboratory plasma known as the reversed-field pinch (RFP), the magnetic field which confines the plasma is partly self-excited [2]. The equations of magnetohydrodynamics (MHD) applied to the RFP predict a dynamo arising from spatial fluctuations in the flow ($\tilde{\mathbf{v}}$) and magnetic field ($\tilde{\mathbf{b}}$). These fluctuating fields combine to form an equilibrium electromotive force ($\tilde{\mathbf{v}} \times \tilde{\mathbf{b}}$), which produces an equilibrium current and magnetic field [3]. This fluctuation-induced electromotive force is believed also to underlie many dynamos in naturally occurring plasmas, although the source of free energy for the fluctuations differs from the laboratory case, being primarily gravitational rather than from an externally imposed magnetic field.

In this Letter, we report localized measurement of the fluctuating flow velocity, and its spatial structure, in the outer region of the RFP to determine the dynamics responsible for the dynamo. Measurements were performed in the Madison Symmetric Torus (MST) RFP [4]. We find that the measured dynamo electromotive force accounts for the measured plasma current. Whereas past work detected only the $\tilde{\mathbf{E}} \times \mathbf{B}$ component of the flow, the present measurement directly detects the flow velocity, which can include many contributions beyond the $\tilde{\mathbf{E}} \times \mathbf{B}$ effect. We also measure that the flow responsible for the dynamo is locally resonant, i.e., the poloidal mode number is $m = 0$, corresponding to resonance at the reversal surface at which the safety factor $q = 0$. Combined with earlier measurements that the core dynamo is driven by $m = 1$ fluctuations, the picture emerges that the dynamo is a superposition of relatively localized magnetic reconnection events. The direct measurement of the fluctuating flow also displays the expected parity for a tearing mode: The radial component reverses phase across the resonant surface, while the toroidal component does not.

In prior work, the dynamo electromotive force has been measured in the extreme edge of the RFP [5]. The flow velocity was inferred from measurement of the fluctuating electric field by Langmuir probes, applying the assumption that the flow is an $\tilde{\mathbf{E}} \times \mathbf{B}$ drift. The measured dynamo was able to account for the current driven in the edge region. Similar measurements were undertaken in a spheromak plasma [6]. In the core of the RFP plasma, chordal Doppler spectroscopy was employed to measure the fluctuating ion flow velocity [7,8]. It was shown that the dynamo electromotive force again was consistent with the generation of the modeled current density profile, subject to considerations of spatial averaging of the diagnostic.

A simple form of the parallel mean-field Ohm's law which includes the fluctuation-induced dynamo effect can be written

$$E_{\parallel} + \langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_{\parallel} = \eta j_{\parallel},$$

where η is the parallel resistivity, $E_{\parallel}$ and $j_{\parallel}$ are the parallel components (approximately poloidal in the edge of MST) of the flux surface average of, respectively, the electric field $\mathbf{E}$ and the current $\mathbf{j}$, a tilde denotes fluctuations which average to zero over a flux surface, and $\langle \cdots \rangle$ denotes a flux-surface average. Note that products of two fluctuating quantities can generate mean-field effects, while terms which are linear in fluctuations do not.

To investigate the role of the fluctuation-induced dynamo, $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_{\parallel}$, in Ohm's law, independent, local measurements of $E_{\parallel}$, $\tilde{\mathbf{v}}$, $\tilde{\mathbf{b}}$, and $j_{\parallel}$ were made in the outer region of the plasma ($0.75 < r/a < 0.95$, where $a =$ 52 cm is the minor radius). The measurement of the dynamo term, $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle$, constitutes the primary novelty of the current work. In the experiment, fast, local measurements of ion velocity fluctuations $\tilde{\mathbf{v}}$ were measured using a unique insertable optical probe [9]. The probe collects light from two perpendicular lines of sight limited by view dumps to produce radial localization of the measurements to ± 2–3 cm. The collected light is transmitted fiber-optically to a high-throughput, fast-time-resolution spectrometer [10]. With the spectrometer tuned to an emission line from a plasma impurity species, centroid shifts of the Doppler profile of the line give ion velocity fluctuations. Hydrogen plasmas were doped with a small amount of helium and the He II 468.57 nm line was used for spectroscopy. It has been assumed that the velocity fluctuations

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of helium ions reflect those of the bulk; it has furthermore been assumed that the ion velocity accurately represents the single-fluid plasma velocity which appears in Ohm's law (i.e., that $m_e v_e \ll m_i v_i$).

The optical probe's two lines of sight provide simultaneous measurements of the radial and toroidal components of the plasma velocity, $\tilde{v}_r$ and $\tilde{v}_t$, the quantities which arise in the parallel component of $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle$ (i.e., $\langle \tilde{v}_r \tilde{b}_t \rangle - \langle \tilde{v}_t \tilde{b}_r \rangle$). To obtain the complete dynamo product, $\tilde{v}_r$ and $\tilde{v}_t$ were correlated with $\tilde{b}_t$ and $\tilde{b}_r$, respectively; the magnetic fields were obtained from a multicoil magnetic probe mounted at the same toroidal and radial locations as the optical probe, separated from it poloidally by 18°. Thus the velocity and magnetic fluctuation measurements are close together and sample approximately the same flux surface.

The induced electric field profile $E_\parallel(r)$ was obtained from the poloidal surface loop voltage V_L corrected for the flux change inside the torus but outside the radius of the electric field measurement; thus,

$$E_\parallel(r) = \frac{V_L}{2\pi r} - \frac{1}{r}\frac{\partial}{\partial t}\int_r^a B_t(r')r'\,dr'.$$

The $B_t(r)$ profile was determined with an additional insertable magnetic pickup coil, and circular, axially symmetric flux surfaces have been assumed.

The current profile $j_\parallel(r)$ was obtained using an insertable Rogowski coil [11]. The plasma resistivity η was not directly measured, since measurements of local effective ion charge state $Z_{\rm eff}$ were not available. However, at the low electron temperature (<50 eV as measured by Langmuir probes) of the edge plasma in MST, the probability of stripping impurity ions to high charge states is small; hence, a $Z_{\rm eff}$ as high as 2 is unlikely. For the present comparison, then, a $Z_{\rm eff}$ of 1.5 has been assumed and η is calculated from Spitzer's formula [12]. Since the contribution of the current term in Ohm's law is modest, the uncertainty in the estimated resistivity does not affect the conclusions which follow in this Letter.

Spontaneous generation of toroidal magnetic field in MST, as in some other RFP's, occurs in cyclical discrete dynamo events (sometimes referred to as sawtooth oscillations) [13,14]. Each cycle is usefully divided into three phases: first, a quiescent period with low levels of fluctuations and slow changes in the plasma magnetic fields, then a rising phase of the dynamo event during which fluctuations increase and the magnetic field rises rapidly, and finally a decay phase as the plasma returns to its original equilibrium (Fig. 1). All terms in the parallel Ohm's law are small in the quiescent phase but become significant as fluctuations increase during the dynamo event itself. These repeatable discrete events therefore provide convenient access to the study of the dynamics of the RFP dynamo. Ensembles of hundreds of similar dynamo events were selected at each radial insertion and controlled for plasma current, line-averaged density, and reversal parameter prior to the event. The plasmas studied were of relatively low current, $I_p \approx 210$ kA, with density $\bar{n} \approx 1.0 \times 10^{19}/\mathrm{m}^3$.

At large radii the measured dynamo product balances Ohm's law extremely well (Fig. 2). This measurement is qualitatively consistent with earlier dynamo measurements using Langmuir probes [5]. The electric field $E_\parallel$ is negligible during the quiescent phase, while $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_\parallel$ approximately accounts for $\eta j_\parallel$ and both are small. As the velocity and magnetic fluctuations increase during the rising phase, both $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_\parallel$ and $E_\parallel$ rise to about 10 V/m; the two terms cancel while $\eta j_\parallel$ remains small. In the decay phase the current magnitude rises modestly and compensates for a slight decrease in the magnitude of the fluctuation term to continue to balance $E_\parallel$. Finally all three terms return to their low, pre-dynamo-event level.

A plot of $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_\parallel$ overlaid with $\eta j_\parallel - E_\parallel$ at $r/a = 0.92$ [Fig. 2(b)] reveals two interesting characteristics of the dynamics of parallel Ohm's law. The first is the significant contribution of the fluctuation-induced dynamo in balancing the process. Indeed, the contribution of the parallel current term in Ohm's law is surprisingly small, while the dynamo balances a large induced $E_\parallel$, particularly during the rising phase of the dynamo event. The second observation is the remarkable agreement in time dependence between the two plots, particularly during the decay phase of the dynamo event cycle, when $\eta j_\parallel$ becomes significant. Comparing $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_\parallel$ with $-E_\parallel$ alone does not yield nearly such good agreement during this part of the cycle.

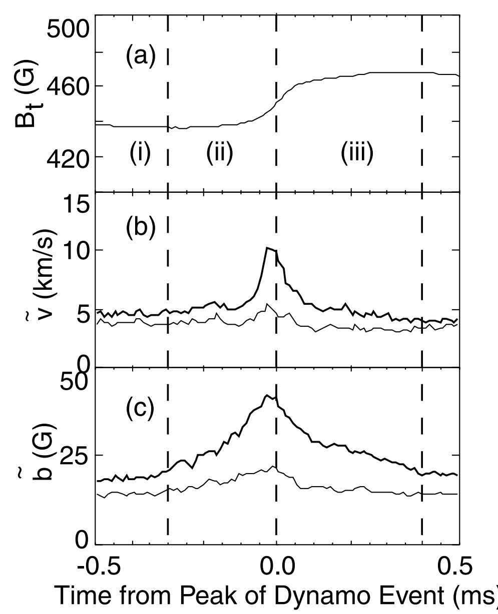

FIG. 1. The dynamo event cycle in MST. (i) Quiescent phase. (ii) Rising phase. (iii) Decay phase. Shown are (a) the toroidal magnetic field flux in the plasma B_t, (b) the amplitude of the toroidal (heavy line) and radial (light line) velocity fluctuations, and (c) the amplitude of the toroidal (heavy line) and radial (light line) magnetic field fluctuations.

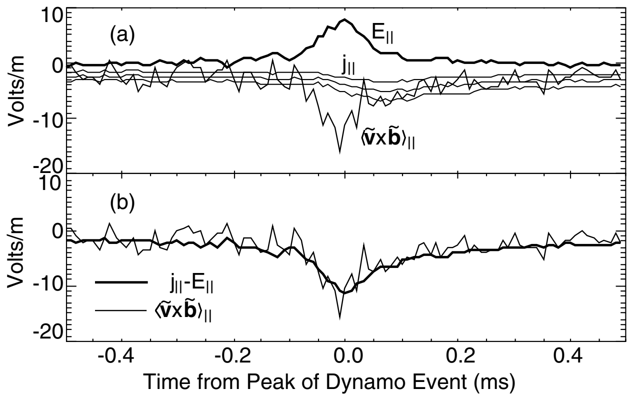

FIG. 2. Ohm's law during a dynamo event cycle. (a) The parallel electric field $E_{\|}$, fluctuation-induced dynamo $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_{\|}$, and $\eta j_{\|}$ with Z_{eff} of 1, 1.5, and 2. (b) $\eta j_{\|} - E_{\|}$ and $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_{\|}$ with $Z_{\mathrm{eff}} = 1.5$, showing good time-dependent agreement.

The assumption that η does not change much during the dynamo event cycle is validated by this observation.

At r/a of 0.87 to 0.90 the agreement of $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_{\|}$ with $\eta j_{\|} - E_{\|}$ continues to be good (Fig. 3), although slight discrepancies begin to appear just at the peak of the dynamo event, possibly indicating a slight increase of plasma resistivity or a contribution from other effects (such as a Hall effect, $\tilde{\mathbf{j}} \times \tilde{\mathbf{b}}$, not measured). However, at $r/a < 0.85$ (deeper than the reversal surface), the dynamo term virtually disappears. This is true despite the fact that the magnitudes of $\tilde{v}_r$, $\tilde{v}_t$, $\tilde{b}_r$, and $\tilde{b}_t$ individually remain large.

Separation of the dynamo product into its two constituent terms $\tilde{v}_r \tilde{b}_t$ and $\tilde{v}_t \tilde{b}_r$ reveals that at large radii both

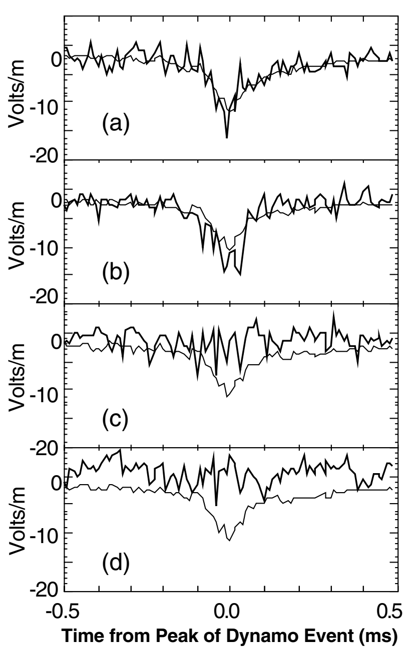

FIG. 3. Comparison of $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_{\|}$ (light line) with $\eta j_{\|} - E_{\|}$ (heavy line) during a dynamo event cycle at r/a of (a) 0.92, (b) 0.88, (c) 0.85, and (d) 0.81.

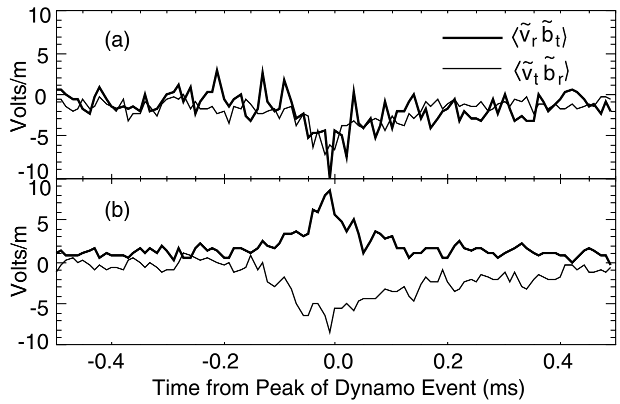

FIG. 4. Dynamo terms $\tilde{v}_r \tilde{b}_t$ (heavy line) and $\tilde{v}_t \tilde{b}_r$ (light line) at (a) $r/a = 0.92$ and (b) $r/a = 0.81$.

terms contribute to the dynamo. Deeper than the reversal surface, however, the $\tilde{v}_r \tilde{b}_t$ component changes sign and becomes an antidynamo (opposite in direction to the current), competing with $\tilde{v}_t \tilde{b}_r$ and effectively canceling the net dynamo effect (Fig. 4). Correlations of each fluctuating quantity with a reference signal from magnetic coils at the edge of MST show that $\tilde{v}_t$, $\tilde{b}_r$, and $\tilde{b}_t$ maintain their phase with radius, while $\tilde{v}_r$ flips phase by π radians at the reversal surface, producing the change in sign of the $\tilde{v}_r \tilde{b}_t$ contribution (Fig. 5).

The toroidal mode spectrum of the dynamo fluctuations was determined by correlating each relevant fluctuating quantity with each of the magnetic modes resolved by a toroidal array of 32 magnetic pickup coils distributed evenly around the inner wall of MST, yielding components of mode number $n = 1$ to 15. The mode spectrum of either component of the velocity fluctuations shows low correlations with the $n = 6$, 7, and 8 global modes, while the correlations with the $n = 1$ and $n = 2$ modes are significant (Fig. 6). The q profile known to exist in MST predicts

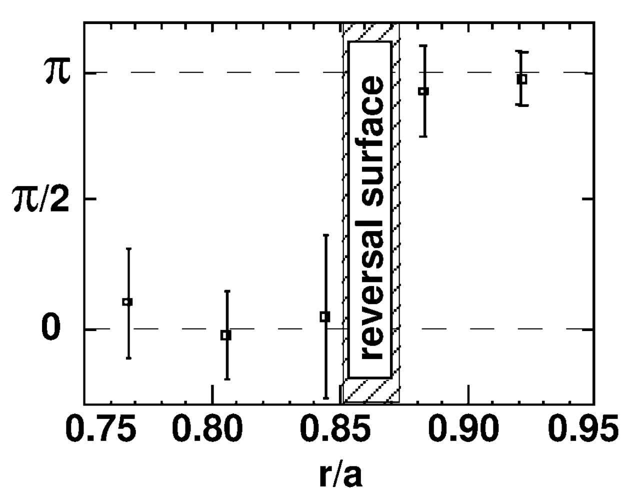

FIG. 5. Relative phase of $\tilde{v}_r$ with radius.

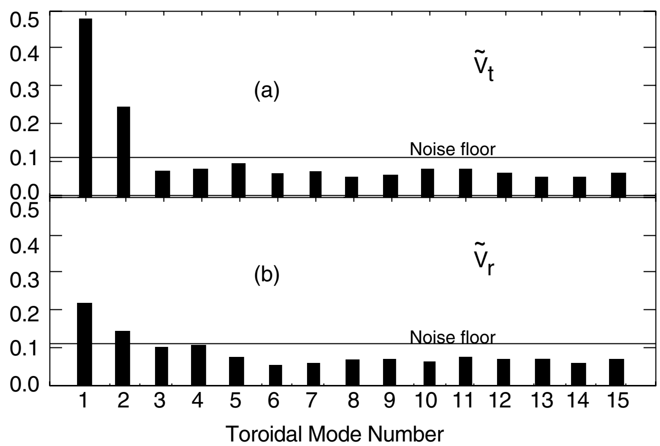

FIG. 6. Mode spectra of velocity fluctuations for (a) $\tilde{v}_t$ and (b) $\tilde{v}_r$ at $r/a = 0.92$. The numerical values represent the correlation (from 0 to 1) of the fluctuation with each mode resolved by the toroidal magnetic array.

that only the resonant low-n modes have poloidal mode number $m = 0$ and are resonant at the reversal surface, while modes with toroidal number $n = 6$, 7, and 8 are resonant in the core and have $m = 1$. The predominance of $n = 1$ and 2 components in the velocity fluctuation spectra therefore indicates that mainly $m = 0$ fluctuations are responsible for the dynamo at the edge. This is also consistent with nonlinear MHD computation, which predicts that the $m = 0$ component of the fluctuation-induced dynamo should be dominant (and in the direction of current generation) at the extreme edge of MST and that it changes sign near the reversal surface [15].

With the identification of the correlated velocity fluctuations as $m = 0$, the phase change of the radial velocity fluctuations mentioned above is significant. Linear MHD theory predicts a phase change in $\tilde{v}_r$ only across the resonant surface, while $\tilde{v}_t$ should have even parity there [16]. The measured flip in phase of $\tilde{v}_r$ at the reversal surface (the resonant surface for $m = 0$ fluctuations) therefore constitutes a confirmation of this aspect of the theory.

In summary, the spectroscopically measured flow fluctuations permit a direct observation of the MHD dynamo effect arising from the $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle$ term in Ohm's law. Throughout a cycle in which the dynamo effect varies in strength, the $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle$ term is measured to satisfy the simple mean-field Ohm's law. During the relatively quiescent phase of the cycle, the dynamo accounts for the parallel current; the electric field is small. In contrast, when the dynamo becomes very large the induced electric field also rises quickly, while the current is slower to respond. The dynamo in the edge is driven by fluctuations which resonate at the reversal surface ($m = 0$); combined with earlier measurements of the core dynamo in which $\tilde{\mathbf{v}}$ was found to be localized to resonant surfaces [7], we conclude that, although the dynamo is a global effect, it arises from a superposition of relatively localized reconnection events. Moreover, the flow velocity fluctuations are measured to have the parity expected for tearing modes resonant at the reversal surface. These observations are all consistent with the standard MHD model of the dynamo. However, at the smallest radii of our measurement, we observe that, while the separate terms in the dynamo remain large, the total fluctuation-induced dynamo vanishes; the identification of another current drive mechanism there (such as a fluctuation-induced Hall effect) awaits further research.

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