Two-Fluid Hall Effect on Plasma Relaxation in a High-Temperature Plasma

W.X. Ding 1,3), V. Mirnov 2,3), A. F. Almagri 2,3), D.L. Brower 1,3), D. Craig 2,3), B.H. Deng 1,3), D. J. Den Hartog 2,3) , G. Fiksel 2,3), C.C. Hegna 2,3), S.C. Prager 2,3), J.S. Sarff 2,3)

1) University of California, Los Angeles, California 90095 USA
2) University of Wisconsin-Madison, Madison, Wisconsin 53706 USA
3) The Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas

e-mail contact of main author: wding@wisc.edu

Abstract. We report on experimental observations that the two-fluid Hall dynamo plays an important role in plasma relaxation. Measurements of the Hall effect during relaxation are made in the plasma core and edge. The Hall dynamo is found to be small at the edge ($r/a \geq 0.85$) but increases substantially towards the interior (r/a=0.82) as measured by probes. In the high-temperature plasma core (r/a=0.35), using a fast laser Faraday rotation diagnostic, the Hall electromotive force is found to be significant (~40 V/m), suppressing (flattening) the equilibrium core current near the resonant surface. Quasilinear two-fluid dynamo theory based on tearing modes predicts that the Hall dynamo can exceed the MHD dynamo under certain conditions. A one dimensional temporal model is used to determine how the effect of the Hall and MHD dynamos is divided between local current density production and local electric field production. Theoretical expectations are qualitatively consistent with experimental observations, implying that effects beyond single-fluid MHD are important.

1. Introduction

Plasma relaxation in the Reversed Field Pinch (RFP) has been described by Taylor theory where global helicity is conserved. However, while Taylor's theory successfully captures many RFP properties [1,2], it does not provide information on the processes of relaxation. In the RFP, the mean current (at the magnetic axis) drops significantly during a sawtooth crash accompanied by a toroidal flux increase. This relaxation occurs on a timescale of ~100 μs, much less than the resistive time [3]. A fundamental difficulty in understanding fast plasma relaxation in high-temperature plasmas is that the inductive electric field (E) is not completely balanced by the collision force (ηJ). Fluctuation-induced electromotive forces may be necessary to balance E. Multiple dynamo mechanisms have been proposed to explain plasma relaxation such as MHD, Hall dynamo, etc. The two-fluid physics of dynamo (electromotive force induced by plasma magnetic fluctuations) and reconnection are captured by generalized Ohm's law [4],

$$-\frac{m_e}{e^2 n_e}\frac{\partial \vec{J}}{\partial t}+\vec{E}+\vec{v}\times\vec{B}-\frac{1}{n_e e}\vec{J}\times\vec{B}+\frac{\nabla P_e}{n_e e}=\eta\vec{J} \quad , (1)$$

where n_e is the electron density, e is electron charge and P_e is electron pressure. *J,E,v,B,* η are the plasma current density, electric field, ion velocity, magnetic field and resistivity. By decomposing each quantity into mean and fluctuating (denoted by δ) parts, ensemble averaging (denoted by < …>) and taking magnetic field parallel component, the parallel mean-field Ohm's law can be written as

$$< E >_{//} - \eta_{//} < J >_{//} = - < \delta\vec{v}\times\delta\vec{B} >_{//} + < \delta\vec{J}\times\delta\vec{B} >_{//} / n_e e \quad , (2)$$

where inertial term $(\partial \vec{J}/\partial t)$ is neglected and all other quadratic terms driven by density and electron pressure fluctuations are assumed to vanish after the ensemble average. It is noted in Eq.(2) that along with the electric field, mean current can also be driven by magnetic field fluctuations coupled with either velocity (MHD dynamo) or current density fluctuations (Hall dynamo). Magnetic field fluctuations can play a crucial role in redistribution of mean current through dynamo formation. Probe measurements in the low-temperature plasma edge [5,6] reveal that current is indeed driven by the $<\delta\vec{v}\times\delta\vec{B}>$ (MHD) dynamo, which is in good agreement with nonlinear MHD computation [2]. It is interesting to note that the MHD dynamo in these experiments [5] disappears as the probes are moved towards the plasma interior (r/a=0.85). In the high temperature plasma core (r/a<0.5), limited measurements of the MHD dynamo have been made by Doppler spectrometry which show a finite amplitude without providing detailed spatial distribution [7]. These measurements imply that an additional dynamo mechanism could exist in the core region.

In this paper, we present three results on two-fluid Hall dynamo effect [2nd term RHS in Eq.(2)]. (1) The Hall dynamo, measured by magnetic probes in the plasma edge, is found to be small for $r/a \geq 0.85$ but substantially increases at the inner most point of r/a=0.82. (2) In the high-temperature plasma core (r/a=0.35), the fluctuation-induced Hall dynamo, $<\delta\tilde{J}\times\delta\vec{B}>_{//}/n_e e$, is also directly measured. This is accomplished by measuring both magnetic and current density fluctuations using a non-perturbing, high-speed, laser Faraday rotation diagnostic. It is found that the Hall electromotive force is significant, suppressing (flattening) the equilibrium core current near the resonant surface. (3) Quasi-linear calculations show that the Hall dynamo can exceed the MHD dynamo under certain conditions, consistent with experimental observation. These results imply that effects beyond single-fluid MHD are important.

2. Experimental measurement of Hall dynamo

In the MST edge region, measurements of the Hall dynamo, driven by correlated action of fluctuations in the plasma current density and magnetic field, have been made by magnetic probes. The Hall dynamo emf is small near the plasma boundary at the normalized plasma radius r/a=0.9 but sharply increases to ~20 V/m at the reversal surface (r/a=0.82), as shown in Fig. 1.

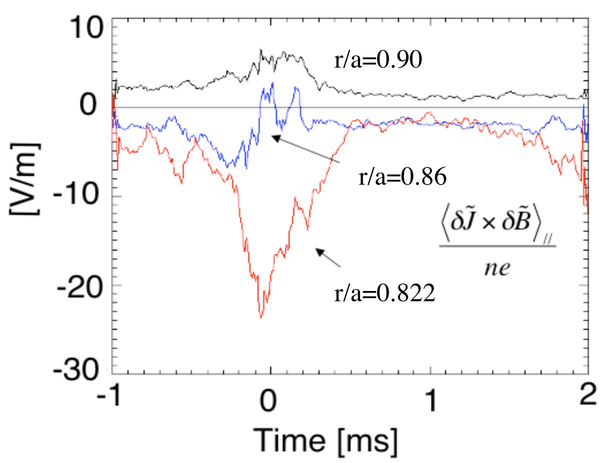

Fig. 1. Hall dynamo measured in plasma edge region by magnetic probes for different radial positions. Time t=0 denotes sawtooth-crash. Hall dynamo increases when probe is positioned deeper into plasma, at reversal surface.

This value is sufficient to account for the local mean parallel current density. In contrast, the previously measured MHD dynamo [5], driven by correlation of fluctuations in the plasma flow velocity and magnetic field, has the opposite trend with radius. The MHD dynamo is ~20 V/m near the plasma boundary but small at the reversal surface (r/a=0.82). Hence, the combined Hall and MHD dynamos more completely account for the mean current density in the edge region (probably the m=0 resonant surface) of MST plasmas.

In the high-temperature (T~ 300 eV) plasma core which is inaccessible to probes, the Hall dynamo effect, current density profile dynamics and inductive electric field E are measured directly by using a non-perturbing, laser-based, Faraday rotation system with time response ~1 μs and spatial resolution of a few cm [3]. Measurements of internal current density fluctuations and associated Hall electromotive force are accomplished by taking a set of parallel polarimetry chords which measure line-integrated magnetic field fluctuations [8]. This line-integrated magnetic field fluctuation measurement can be inverted using a fluctuation fitting procedure to obtain the magnetic perturbation spatial distribution [9]. The resulting magnetic field fluctuation and current density fluctuation spatial profiles for the dominant, core resonant, m/n=1/6 (f~ 20 kHz) mode are shown in Fig. 2. The radial magnetic field fluctuations slightly increase near the resonant surface and then fall off rapidly toward the wall, going to zero at the boundary. Radial magnetic field fluctuations are observed to extend continuously through the rational surface indicating their resistive nature. Poloidal magnetic fluctuations change sign across the resonant surface due to the localized current sheet, which is in agreement with MHD computation. Current density fluctuations ($\delta J / J_0 \sim 4.5\%$) are radially localized at the mode resonant surface ($r_s/a \sim 0.35$) with width $\sim 8 \pm 3$ cm. This width is greater than MHD predictions and approximately equal to the ion skin depth (c/ω_{pi}).

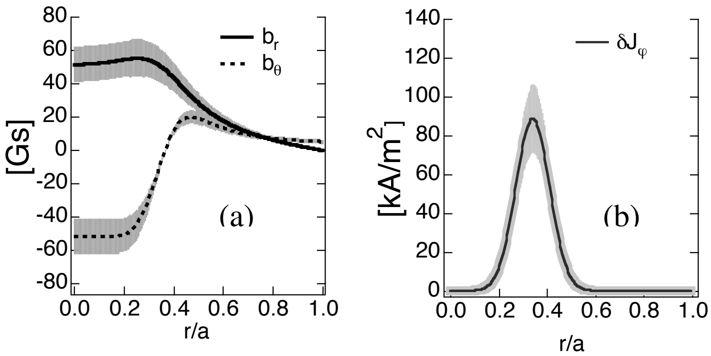

Fig. 2. (a) Radial and poloidal magnetic field fluctuation radial profile and corresponding (b) current density fluctuation spatial profile for m/n=1/6 (f~ 20 kHz) mode.

For the MST magnetic configuration, the parallel Hall dynamo near the mode resonant surface can be rewritten in a simple form [10],

$$\frac{<\delta\vec{J}\times\delta\vec{B}>_{//}}{n_e e} \approx \frac{A}{n_e e} < (\frac{1}{r}\frac{\partial}{\partial r} r\delta b_\theta)\delta b_r) >= A\frac{<\delta j_\phi \delta b_r>}{n_e e} \quad , \qquad (3)$$

where $A = B_\theta\left(1+\left(B_\phi / B_\theta\right)^2\right)/B$, and B_φ, B_θ and B are the known equilibrium toroidal, poloidal and total magnetic field. In MST, rotation of the low-n magnetic modes transfers their spatial structure in the plasma frame into a temporal evolution in the lab frame. Since the magnetic modes are global, for convenience we correlate δj_ϕ to a specific helical magnetic mode which is spatially Fourier decomposed from 32 wall-mounted magnetic coils. After averaging over an ensemble of similar events, we can determine the phase between $\delta j_{\phi}(r_s)$ and $\delta b_r(a)$ for the specified mode [10]. For tearing modes, the perpendicular magnetic perturbation has a constant phase over minor radius, which has been verified by probe measurements in lower temperature plasmas. This allows us to directly evaluate the Hall effect in the region of the mode resonant surface.

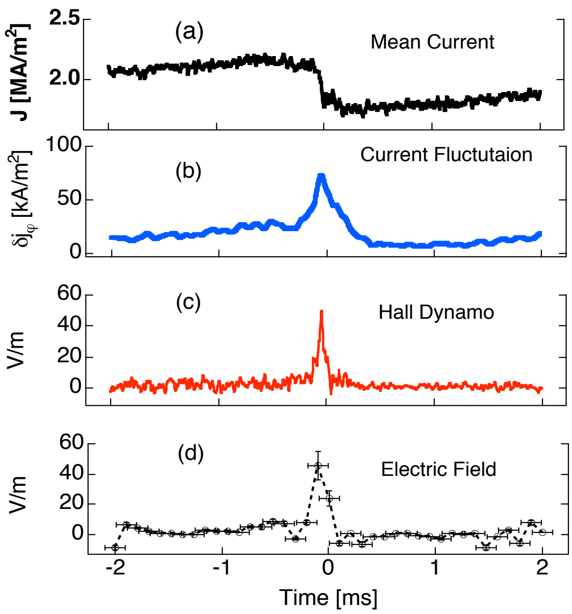

Fig. 3. Ensemble averaged plasma dynamics over sawtooth crash (t=0) measured by Faraday rotation at r/a~0.35.

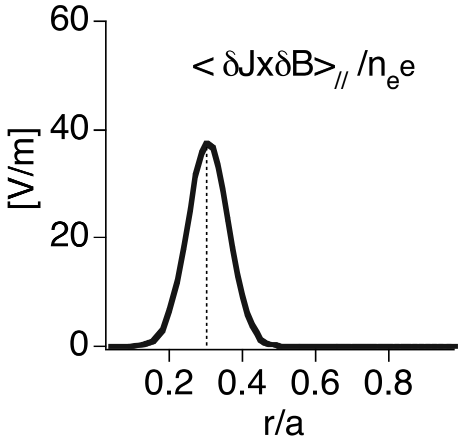

Fig. 4. Hall dynamo spatial profile. Hall dynamo peaks at resonant surface with finite width about 8 cm. Vertical dashed line denotes resonant surface location.

Figure 3 shows the measured core plasma dynamics averaged over many similar sawtooth events. The current density on axis increases before the sawtooth crash as shown in Fig. 3(a). This provides the free energy required to drive current density fluctuations [see Fig. 3(b)] at the resonant surface for the dominant (m/n=1/6) resistive tearing mode (r/a~0.35). The measured Hall dynamo increases dramatically at the sawtooth crash, reaching ~ 50±10 V/m as shown in Fig. 3(c). This occurs because the phase difference between the fluctuating current and magnetic field deviates from 90 degrees (by ~10 degrees) while their amplitudes increase. Away from the sawtooth crash, the Hall electric field (averaged over time window –2 ms to –1 ms) is relatively small (~1.75 ± 0.5 V/m). At this time, the current density fluctuation has a near 90 degree phase difference with the magnetic fluctuations, although both δj_{ϕ} and δb_r are non-zero.

Enhanced current density fluctuations coupled to magnetic field fluctuations generate a mean Hall electromotive force in the opposite direction of mean current and reducing the plasma current. The measured inductively-driven electric field (see Fig. 3(d)) is comparable to the Hall dynamo (see Fig.3(c)) which acts to suppress equilibrium current during plasma relaxation. The resistive force, ηJ, is small compared to both the induced electric field and the Hall dynamo during the sawtooth crash. This direct measurement of a substantial Hall dynamo in a high-temperature laboratory plasma indicates that two fluid effects are necessary to understand plasma relaxation.

The fluctuating current peaks at the resonant surface and hence the Hall dynamo has a maximum there as well, falling off rapidly away from $r=r_s$ as shown in Fig. 4. This implies that the Hall dynamo is spatially localized to resonant surfaces (width ~8 cm), and an additional dynamo mechanism is required to balance the induced electric field elsewhere.

3. Theoretical model

In experiment, Hall dynamo displays two important characteristics; (1) Hall dynamo is spatially localized to region of resonant surface (see Fig. 4), and (2) Hall dynamo is balanced by inductive electric field at sawtooth crash (see Fig. 3). Recently, two-fluid quasilinear dynamo calculations were performed to determine the spatial structure of the fluctuation induced two-fluid dynamo [11]. Both the MHD dynamo $<\delta\vec{v}\times\delta\vec{B}>$ and the

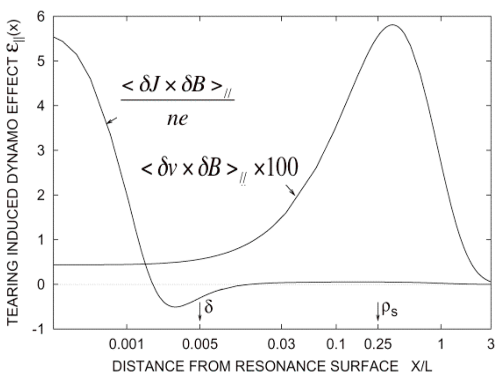

Fig. 5. Hall and MHD dynamo effect. (all functions are even in x, δ is the combined collisionless and collisional electron skin-depth $\delta^2 = c^2/\omega_{pe}^2 + \eta c^2/4\pi\gamma$, where γ is the growth rate of tearing instability, and $\rho_s = c_s/\omega_{ci}$ is the ion-sound gyroradius)

Hall dynamo $<\delta\vec{J}\times\delta\vec{B}>_{//}/n_e e$ contribute to the parallel component of generalized Ohm's law (see Eq.(2)) which can be rewritten $<E>_{//} - \eta <J>_{//} = \varepsilon_{//}(x,t)$, where $\varepsilon_{//}(x,t)$ denotes the total electromotive force driven by the MHD and the Hall dynamo terms, and $x = r - r_s$ is the distance from the mode-resonant surface. Theory predicts that the contribution to $\varepsilon_{//}(x,t)$ from the Hall term is locally much larger than the contribution from the MHD dynamo but the strong enhancement takes place in a narrow electron skin layer (δ). The MHD term has a smaller amplitude, but it is broadened in its spatial width to the order of an ion-sound gyroradius ($\rho_s=c_s/\omega_{ci}$). The fact that the Hall dynamo dominates in the vicinity of the resonant surface is in agreement with the experiments. However, the measured Hall dynamo radial width exceeds quasilinear prediction, indicating a need for more complete nonlinear dynamo theory.

As far as dynamo dynamics are concerned, the amplitudes of fluctuations in magnetic field (δB), flow velocity (δv), and current density (δJ) follow a periodic sawtooth cycle in their time dependence as shown in Fig.3. During the sawtooth crash, a surge occurs in the dynamo. As a response to the dynamo, a large change occurs in the mean plasma current density profile along with an even larger in the mean electric field profile. Even if spatial and temporal dependences of $\varepsilon_{//}(x,t)$ are known, the electron momentum equation does not determine $<E>_{//}$ and $<J>_{//}$ separately. In order to understand how the total dynamo effect is divided between local current density production and local electric field production, we combine two-fluid tearing mode theory with a one-dimensional temporal model based on Faraday's law. This represents a new element of the theoretical treatment and provides a general approach for interpretation of the MST dynamo experiments. The model is described by the diffusion equation for $<E>_{//}$ including a source term. In a plasma slab geometry (that is applicable for narrow tearing layer near plasma edge), it has a form

$$\left(\frac{\partial}{\partial t} - \frac{\eta c^2}{4\pi}\frac{\partial^2}{\partial x^2} - \frac{c^2}{\omega_{pe}^2}\frac{\partial}{\partial x^2 \partial t}\right) < E >_{//} (x,t) = \frac{\partial \varepsilon_{//}}{\partial t} \qquad (4)$$

The source term is presented by the partial derivative $\partial\varepsilon_{//}/\partial t$ and is localized in the vicinity of the resonant surface. A variety of solutions is possible depending on the relationship between the width of localization of $\varepsilon_{//}(x,t)$ and the characteristic diffusion scale. The latter can be expressed in terms of combined resistive and collisionless electron skin depth δ (*where* $\delta^2 = c^2/\omega_{pe}^2 + c^2\eta/4\pi\gamma$, *and* γ *is the growth rate of the tearing instability)*, calculated with respect to the characteristic temporal scale of $\varepsilon_{//}(x,t)$.

We apply this model to two different cases: (a) $\varepsilon_{//}(x,t)$ is driven by an exponentially growing tearing mode with two spatial scales corresponding to narrow Hall and broad MHD dynamo profiles (see Fig. 5), (b) $\varepsilon_{//}(x,t)$ is a periodic function of time with experimentally observed sawtooth temporal profile and single spatial scale l (see Fig. 6). In case (b), we ignore the two spatial scales of $\varepsilon_{//}(x,t)$ and focus attention on the effects caused by the two temporal scales of sawtooth time dependence. This dependence is characterized by a relatively long time T between the crashes and large amplitude narrow peaks of the width $\Delta T << T$.

In case (a), the Hall dynamo term in $\varepsilon_{//}(x,t)$ generates mainly mean parallel current in the vicinity of the resonant surface while the broad MHD dynamo drives mainly mean electric field on scales larger than the electron skin-depth. In case (b), there are three different limiting regimes which allow us to explain the general situation. Let us decompose $\varepsilon_{//}(x, t)$ into its mean value (time average) and oscillating part, so that $\varepsilon_{//}(x,t) = \overline{\varepsilon(x,t)} + \delta\varepsilon_{//}(x, t)$ where $\overline{\delta\varepsilon_{//}(x,t)} = 0$. (1) If the spatial scale of $\varepsilon_{//}(x,t)$ is broad, $l >> \delta_T$, where δ_T is the electron skin depth calculated with respect to long time between the crashes (by substituting $1/\gamma = T$ into the relation for δ), then the oscillating part of dynamo generates most strongly oscillating electric field while its mean value contributes to smooth dc current $\eta<J>_{//} = -\overline{\varepsilon(x,t)}$.

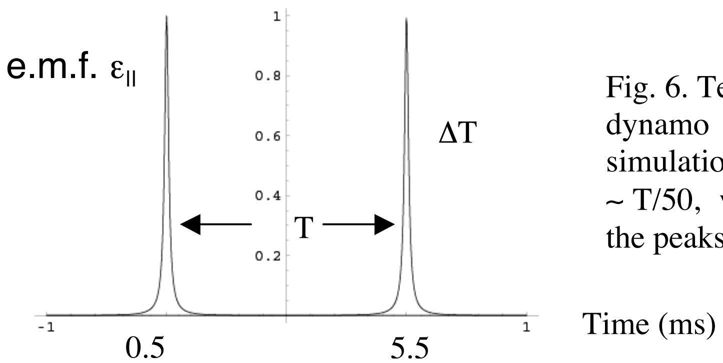

Fig. 6. Temporal profile of periodic dynamo e.m.f. used for numerical simulation (peak width $\Delta T \sim 100$ μsec $\sim T/50$, where time interval between the peaks $T \sim 5$ msec).

(2) In the limiting case of narrow spatial profile, $l << \delta_{\Delta T}$, where $\delta_{\Delta T}$ is the electron skin depth calculated with respect to short time of crash ($1/\gamma = \Delta T$), the dynamo contributes mostly to the generation of strongly oscillating current $\eta\langle J\rangle_{//} = -\varepsilon_{//}(x,t)$ while the amplitude of electric field is negligibly small.

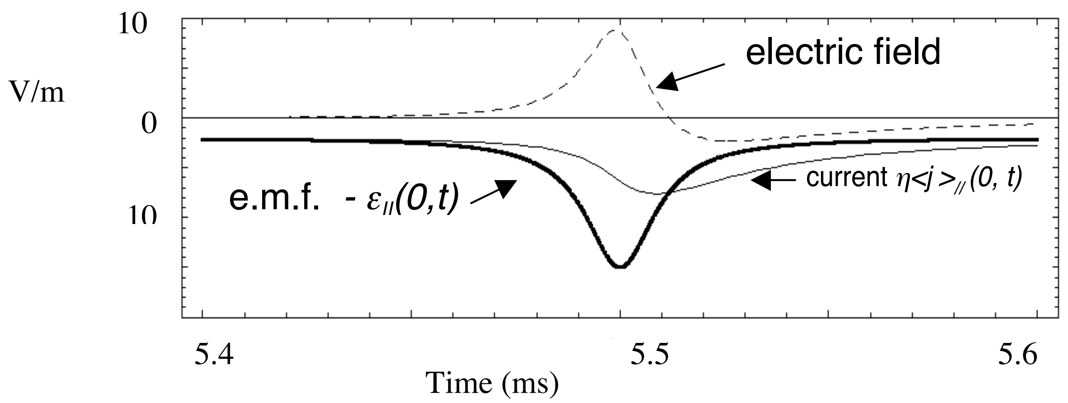

Fig. 7. Temporal profile of periodic dynamo e.m.f. used for numerical simulation (peak width $\Delta T \sim 100$ μsec $\sim T/50$, where time interval between the peaks $T \sim 5$ msec).

(3) In the intermediate situation, $\delta_{\Delta T} << l << \delta_T$, which is of primary interest for MST operation. The dynamo electromotive force is balanced in generalized Ohm's law by the $\eta \langle J\rangle_{//}$ term during the long time between the crashes while $\varepsilon_{//}(x, t)$ contributes mostly to the generation of the electric field during the short time of crashes. Numerical integration of Eq.(4) (see Fig. 7) shows good quantitative agreement with the edge dynamo measurements and reasonably good qualitative agreement with the core measurements. For core dynamo, a more accurate cylindrical model is needed instead of the slab approximation. In general, the self-inductance of the tearing layer prevents fast changes to the mean parallel current, thereby providing "stiffness" of the profiles with respect to perturbations.

4. Summary

Hall dynamo effects during plasma relaxation have been directly measured by magnetic probes in the edge and by laser Faraday rotation in the core. It is found that Hall dynamo is small in the edge region (r/a>0.85), becoming significant in the core (r/a=0.35) at the resonant surface of the dominant global mode. The Hall dynamo peaks in the same region where the current density fluctuation has a maximum. In the plasma core, Hall dynamo driven current is in the opposite direction of the mean current density and is comparable to the inductive electric field. In the edge, Hall dynamo actually drives mean current. These experimental measurements are qualitatively consistent with quasi-linear theory which suggests that the Hall dynamo is localized near the resonant surface. A one dimensional electro-dynamical model explains the temporal behavior of plasma response. The dynamo drives mostly mean current between crashes while it generates mostly mean electric field at the crash. These results show that two-fluid effects in high temperature plasmas play an important role in plasma relaxation (or sawtooth crash).

Acknowledgements

This work is supported by the U.S. Department of Energy and the National Science Foundation.

References

[1] TAYLOR, J. B., Phys. Rev. Lett. **33**,1139(1974).
[2] ORTOLANI, S. and SCHNACK, D. D., *Magnetohydrodynamics of Plasma Relaxation* (World Scientific, Singapore,1993).
[3] BROWER, D. L., DING, W.X., TERRY, S. D., et al., Phys. Rev. Lett. 90, 035002(2003); TERRY, S. D., BROWER, D. L., DING, W.X., *et al.*, Phys. Plasmas **11**,1079(2004).
[4] SPITZER, L., Jr. *Physics of Fully Ionized Gases* (2nd Revised Edition) Interscience Publishers, New York, 1962), p.28
[5] FONTANA, P. W., DEN HARTOG, D.J., FIKSEL, G., PRAGER, S. C., Phys. Rev. Lett. **85**, 566(2000).
[6] JI, H., *et al.* Phys. Rev. Lett. **69**, 616(1992); JI, H., ALMAGRI, A. F., PRAGER, S C., SARFF, J.S., Phys. Rev. Lett. **73**, 668(1994).
[7] DEN HARTOG, D. J., CHAPMAN, J.T., CRAIG, D., FIKSEL, G., FONTANA, P.W., PRAGER, S.C., SARFF, J.S., Physics of Plasmas, **6**, 1813(1999).
[8] DING, W.X., BROWER, D.L., TERRY, S. D., CRAIG, D., PRAGER, S. C., SARFF, J. S., WRIGHT, J., Phys. Rev. Lett. **90**, 035002(2003).
[9] DING, W.X., BROWER, DENG, B.H., CRAIG, D., PRAGER, S.C., SVIDZINSKI, V., Review Scientific Instruments. **75 ,** 3387(2004).
[10] DING, W.X., BROWER, CRAIG, D., DENG, B.H., FIKSEL, G., MIRNOV, V., PRAGER, S.C., SARFF, J.S., SVIDZINSKI, V., Phys. Rev. Letts. **93**, 04502(2004).
[11] MIRNOV, V.V., HEGNA, C.C., PRAGER, S.C., Plasma Physics Report **29**,612(2003).