Measurement of Internal Magnetic Field Fluctuations in a Reversed-Field Pinch by Faraday Rotation

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Magnetic field fluctuations (and the associated current perturbation) have been measured in the core of a high-temperature reversed-field pinch using a newly developed fast-polarimetry system. Radial magnetic field fluctuation levels of ~1% are measured in standard-reversed-field pinch discharges which increase to ~4% during the sawtooth crash (enhanced dynamo). The fluctuation level is reduced fourfold for high-confinement plasmas where the core-resonant tearing modes are suppressed.

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In many laboratory and natural plasmas, the magnetic field fluctuates spontaneously in space and time. Magnetic fluctuations can have a large effect on the macroscopic behavior of plasmas. Two effects are of particular importance. First, magnetic fluctuations can cause magnetic field lines to wander stochastically. Energy and particle transport can then arise from particles streaming along the stochastic magnetic field lines. Second, fluctuations can generate a large-scale magnetic field through current driven by a dynamo effect.

Energy transport and dynamo effects from magnetic fluctuations are especially crucial to the reversed-field pinch (RFP) laboratory plasma configuration. It has long been considered that the transport arises from stochasticity induced by overlapping magnetic islands, and that the current density profile is partly determined by the dynamo [1]. In addition, improved confinement in the RFP, induced by control of the current density profile, has been conjectured to be related to the reduction of magnetic fluctuations within the plasma [2–4]. Understanding transport and dynamo physics requires measurement of the fluctuating magnetic field inside the high-temperature plasma. Magnetic probes have been used to measure the edge magnetic field fluctuations, as well as the internal fluctuation radial profiles in smaller, colder RFP plasmas [5,6]. Until now, however, direct nonperturbing measurement of the magnetic fluctuations in the hot plasma core has been lacking in the RFP, as well as in other toroidal configurations.

In this Letter, we report the first measurement of magnetic field fluctuations (and the related current density fluctuations) in the core of a high-temperature RFP. Their relation to transport and dynamo in the RFP is investigated. This is accomplished through measurement of the Faraday rotation of an injected far-infrared laser beam using a new high-speed polarimeter system.

We report three new results. First, we find that broadband internal magnetic fluctuations grow rapidly at the sawtooth crash, indicating their relation to the dynamo. The change in fluctuation amplitude correlates with the relaxation of the core current density profile and is consistent with the standard model of MHD relaxation and dynamo in the RFP. Second, fluctuating current associated with the dominant magnetic islands is measured and found to have a spatial extent ≤ 8 cm about the rational surface. The localization of the current, while the magnetic fluctuation is radially global, is consistent with the theoretical expectation for tearing modes. Third, we find that in plasmas with improved confinement the long-wavelength core magnetic fluctuation amplitude decreases fourfold. The decrease in core magnetic fluctuations is larger than has been previously inferred from measurements at the plasma surface.

The diagnostic approach employs two distinct but collinear far-infrared (FIR) laser beams to probe the plasma [7,8]. The nonperturbing probe beams are frequency offset and have counterrotating circular polarizations. Because of plasma birefringence, each beam experiences a different value of refractive index upon propagation through the plasma. The difference in refractive index is related to the Faraday rotation angle, Ψ, by the relation

$$\Psi = \frac{2\pi}{\lambda_0}\int \frac{(\mu_L - \mu_R)}{2}dz = \frac{\lambda_0^2 e^3}{8\pi^2 c^3 \varepsilon_0 m_e^2}\int n_e B_z dz, \quad (1)$$

where μ_R and μ_L are the indices of refraction for the right- and left-hand circularly polarized waves, B_z is the component of the magnetic field parallel to the vertically propagating FIR beams, n_e is the electron density, λ_0 is the laser wavelength, $z^2 = r^2 - x^2$, x is the impact parameter of probe beam ($x = 0$ corresponds to vacuum vessel center), and r is the minor radius, all in MKS units.

The FIR laser beams are at nominal wavelength 432 μm (694 GHz) and operated at a difference frequency of $\Delta\omega/2\pi \approx 750$ kHz. The 11 chord polarimetry and interferometry data (measured along the same sightlines) are routinely sampled at 1 MHz with the difference frequency being aliased down to 250 kHz. This results in a 4 μs time response (nonaliased time response $\sim$1 μs), which is sufficient to follow the dynamics of interest. A digital phase comparator technique is employed to extract the phase information for both the polarimeter and interferometer [9]. Typical polarimeter system rms noise levels are 1 mrad ($\approx 0.1°$) at 100 kHz bandwidth. Data can be ensemble averaged over the time window of the sawtooth cycle for further noise reduction.

Experimental work is carried out on the Madison Symmetric Torus (MST) RFP which is Ohmically heated with major radius $R_0 = 1.50$ m and limiter radius $a = 0.51$ m. Data presented are for plasmas with toroidal current $I_p = 400$ kA and nominal electron density $n_e = 1 \times 10^{19}$ m^{-3} using a deuterium working gas.

Typical MST fast-polarimetry data for a sawtoothing discharge are shown in Fig. 1. As expected, the Faraday rotation angle changes sign about the magnetic axis due to a change in the direction of the equilibrium poloidal magnetic field with respect to the polarimeter chords [see Fig. 1(b)]. Zero Faraday rotation angle indicates that the probing beam passes through the magnetic axis.

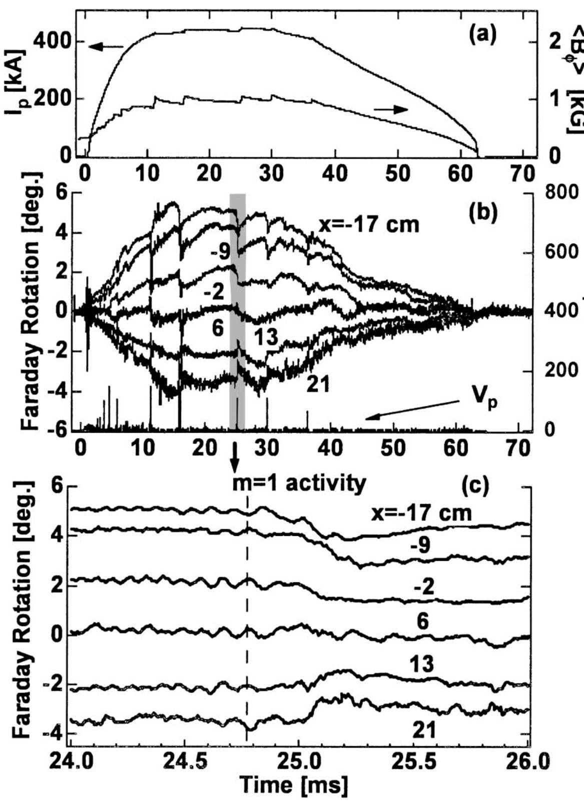

FIG. 1. (a) Plasma current and average toroidal magnetic field. (b) Faraday rotation angle for standard Ohmic MST plasma from six polarimeter chords. (c) Expanded time scale. Sawtooth crashes are denoted by the prompt generation of toroidal magnetic field and spikes in poloidal voltage, V_p. Faraday rotation data are low-pass filtered with 20 kHz cutoff.

Each chord shows distinct sawtooth cycles, corresponding to the slow changes in the equilibrium (axisymmetric) magnetic field. Sawtooth crashes, characterized by changes in toroidal flux, are denoted by prompt increases in the average toroidal field, $\langle B_\phi \rangle$ [see Fig. 1(a)].

To examine the fluctuations in the Faraday rotation signals, we expand the time axis and isolate an individual sawtooth crash ($t = 25$ ms) as shown in Fig. 1(c). The sawtooth crash or relaxation time scale is $\approx$ 100–200 μs. A clear coherent oscillation is observed on all chords prior to the sawtooth crash. The frequency ($\sim$ 12 kHz) of these fluctuations matches the dominant $m = 1$ tearing mode in MST (as measured by external magnetic coils). Finite coherence is observed for frequencies up to 100 kHz between polarimeter chords and an external magnetic sensor, indicating that the measured fluctuations are associated with magnetic activity.

Since the measured Faraday rotation angle depends on both the density and magnetic field, it is necessary to separate the two in order to isolate the fluctuating magnetic field [10]. By rewriting Eq. (1) in terms of the equilibrium and fluctuating quantities for each variable (e.g., $\Psi = \Psi_0 + \tilde{\Psi}$, $B_z = B_{0z} + \tilde{B}_z$, $n_e = n_0 + \tilde{n}$), the fluctuating part of the Faraday rotation signal becomes

$$\tilde{\Psi} = c_F\left(\int B_{0z}\tilde{n}dz + \int \tilde{B}_z n_0 dz\right), \tag{2}$$

where the second order term, $c_F \int \tilde{B}_z\tilde{n}dz$, is negligible because both $\tilde{n}$ and $\tilde{B}_z$ are small. From this equation, we see that the fluctuating part of the polarimetry signal is the sum of the fluctuating electron density weighted by equilibrium magnetic field, and the fluctuating magnetic field weighted by equilibrium density.

For all polarimeter chords shown in Fig. 1(c), the $\int B_{0z}\tilde{n}dz$ term of Eq. (2) is negligible and $\tilde{\Psi} \approx c_F \int \tilde{B}_z n_0 dz$. By using measured values for equilibrium poloidal magnetic field and electron density fluctuations, it can be shown that $\int B_{0z}\tilde{n}dz < 0.04°$, which is less than the polarimeter rms noise level (0.1°). Finite contributions from the toroidal magnetic field to the Faraday signal resulting from misalignment have also been considered and found to be negligible.

Confirmation that the measured fluctuations are magnetic is realized by examining the change in amplitude and phase of the polarimetry signals. Phase measurements indicate that fluctuations (both density and magnetic field) with frequencies $>$ 5 kHz have poloidal mode number $m = 1$ in MST. For an $m = 1$ density perturbation, we observe a minimum, $\int \tilde{n}dz \to 0$, for a chord going through the magnetic axis and a π-phase change for chords on opposite sides [11]. However, the measured $\tilde{\Psi}$ shows a maximum at the magnetic axis with 0-phase change. In addition, the π-phase change observed on $\tilde{\Psi}$ away from the magnetic axis occurs at the mode resonant surface. Both of these observations are consistent with expectations for $m = 1$ magnetic perturbation. For the central chord (i.e., $x = 6$ cm), the fluctuating Faraday rotation signal provides a chord-integrated measurement of radial magnetic field fluctuations.

The line-integrated magnetic fluctuation amplitude can be estimated from the relation $\bar{\tilde{B}} \cong \bar{n}_0(\Delta z)^{-1} \int n_0 \tilde{B}_z dz \cong (c_F \bar{n}_0 \Delta z)^{-1} \tilde{\Psi}$. Using the measured line-averaged density ($\bar{n}_0$), chord length (Δz), and Faraday rotation, we find the time-averaged rms amplitude of the magnetic field fluctuations, $\bar{\tilde{B}}_r \sim 33$ G or $\bar{\tilde{B}}_r/B_0 \sim 0.6\%$. The polarimeter rms noise level is $\sim$10 G. Since the density profile is centrally peaked [4], the line-averaged $\tilde{B}_r$ measurement is weighted to the plasma core. In addition, output from the nonlinear resistive MHD simulation [12] predicts that the eigenfunctions for the dominant tearing modes in MST (i.e., $m = 1$, $n = 5$–10, $f \sim 10$–20 kHz) peak in the plasma core. Both of these factors suggest that the measured $\bar{\tilde{B}}_r$ is primarily a measure of magnetic fluctuations in the plasma core.

Global magnetic field fluctuations are generated by current fluctuations in the vicinity of the resonant surface. The data shown in Fig. 1(c) are at a time when a single $m = 1$, $n = 6$ mode dominated all other core-resonant modes. Equilibrium magnetic field measurements and equilibrium reconstruction (not shown here) indicate the $q(= m/n) = 1/6$ rational surface is located between chords ($x = 13$ and 21 cm) and ($x = -9$ and -17 cm). The π phase change observed on the polarimeter chords across the $q = 1/6$ surface [see Fig. 1(c)] is consistent with expectations for a current channel associated with the magnetic island. The fluctuating toroidal current ($\tilde{I}_\varphi$) for this mode can be estimated using Ampere's law:

$$\oint_L \tilde{\vec{B}} \cdot d\vec{l} \approx \left[\int \tilde{B}_z dz \right]_{x1} - \left[\int \tilde{B}_z dz \right]_{x2} \approx \mu_0 \tilde{I}_\phi, \quad (3)$$

where L represents the closed loop formed between any two chords as shown in Fig. 2(a). Contributions from the portion of the loop along the plasma edge but between the chords are negligible due to both the short distance and small fluctuation amplitude. Evaluating Eq. (3) between outboard chords ($x_1 = 13$ and $x_2 = 21$ cm) gives $\tilde{I}_\phi \approx (2/\mu_0) \int \tilde{B}_z dz \approx (2\tilde{\Psi})/(\mu_0 c_F \bar{n}_0) \approx 5.4$ kA. Taking $\tilde{\Psi} = \int n_0 \tilde{B}_z dz \approx \bar{n}_0 \int \tilde{B}_z dz$ is a good approximation for MST plasmas where the electron density profile is flat in the core and the fluctuating magnetic field falls off rapidly towards the wall.

By taking the loop between adjacent pairs of chords, the spatial distribution of the fluctuating current can be obtained as shown in Fig. 2(b). For adjacent chords with out-of-phase oscillations, the two terms in Eq. (3) add. When applying Ampere's law to adjacent chords with in-phase oscillations (e.g., $x_1 = 6$ and $x_2 = 13$ cm), the toroidal current perturbation is small because the two terms of Eq. (3) essentially cancel. The fluctuating current shows an $m = 1$ nature as expected ($m/n = 1/6$), peaking sharply near the resonant surface with spatial extent ≤ 8 cm. Precise measurement of the current perturbation spatial extent is limited by chord spacing. Observation of a localized current fluctuation with a radially global magnetic fluctuation is consistent with theoretical expectations for tearing modes. The fluctuating current is predicted to be singular in the ideal MHD limit, but develops a finite width from nonideal effects. The average current density perturbation for this large ($m/n = 1/6$) mode is approximately $\tilde{j}_{m=1,n=6}/J_0 \sim 3\%$ before a sawtooth crash in MST. This value is comparable to predictions from a 3D, nonlinear, resistive MHD simulation using the DEBS code [12] which has been widely utilized to investigate RFP dynamics.

A strong correlation between internal magnetic field fluctuations and the current density dynamics is experimentally observed. The magnetic fluctuation amplitude for ensembled data is $\bar{\tilde{B}}_r/B_0 \sim 1\%$ (before the crash) but varies significantly during the sawtooth cycle as shown in Fig. 3. Turbulence levels are fairly constant until 100 μs before the crash when they jump over fourfold within $\sim$50 μs, comparable to edge measurements. The equilibrium current density, also measured with the fast polarimeter [13], gradually increases during the slow ramp phase of the cycle and promptly (100–200 μs) drops 20% at the sawtooth crash (see Fig. 3). The enhanced magnetic

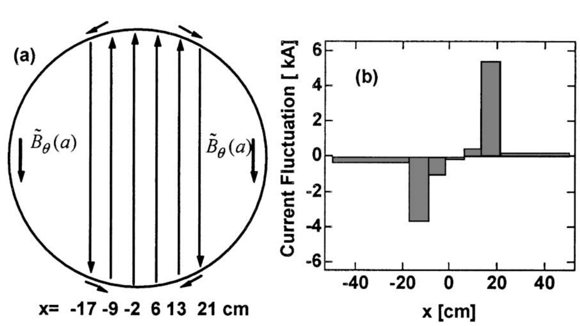

FIG. 2. (a) MST cross section showing chord positions and loops used to calculate current perturbation. Arrows indicate the direction of fluctuating magnetic field. (b) Spatial distribution of fluctuating toroidal current.

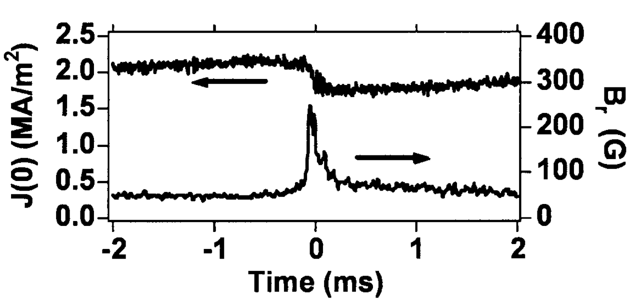

FIG. 3. Changes in core magnetic fluctuation amplitude and current density $J(0)$ during a sawtooth cycle (crash occurs at $t = 0$). Data ensemble averaged over 400 sawtooth events.

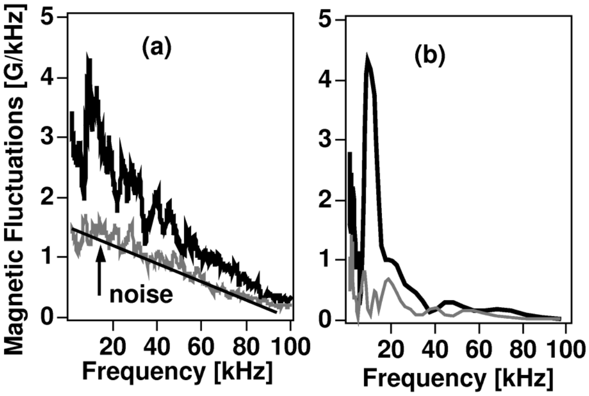

FIG. 4. (a) Radial magnetic field spectrum, and (b) coherence weighted spectrum for standard sawtoothing (bold line) and PPCD (grey line) plasmas in MST. System noise level is indicated by straight line.

fluctuations are predicted to generate a dynamo electric field which acts to reduce toroidal current density in the core [13–15].

Recently, it has been experimentally demonstrated [2–4] that appropriate programming of the electric field parallel to the edge magnetic field (known as pulsed parallel current drive, or PPCD), can greatly enhance the RFP confinement properties. Modeling has predicted and experiment suggests that improved confinement is largely attributed to suppression of $m = 1$ core-resonant tearing modes [1] and associated transport [2,3]. Energy and particle confinement times have been increased tenfold [3,4] while the magnetic fluctuations, as measured by external coils, are halved. New direct measurement of core magnetic fluctuations shows that the fluctuation reduction extends to, and is actually stronger in, the plasma core.

The core radial magnetic field fluctuation frequency spectrum for a high-confinement PPCD plasma ($\tau_E =$ 8.8 ms) is shown in Fig. 4(a) along with the broadband fluctuation spectra (using 5 ms time average) for a standard sawtoothing discharge ($\tau_E = 1.0$ ms). Here it is clearly seen that the magnetic field fluctuation amplitude is reduced across the entire spectrum (by a factor of 2 for the core modes, $f \sim$ 10–20 kHz). In fact, the fluctuation spectrum measured during PPCD is essentially the same as the polarimeter noise spectrum implying that we are limited by the instrumental resolution. In an effort to extract the coherent portion of the spectrum from these data, we plot the coherence-weighted frequency spectra in Fig. 4(b). The coherence-weighted spectra is the product of the fluctuation spectrum and computed coherence between the $x = 6$ cm polarimeter chord and an external magnetic coil. Here we see the coherent portion of the spectrum (10–20 kHz), which corresponds to the dominant core modes, is reduced fourfold during PPCD. This indicates that the core fluctuation amplitude is more strongly suppressed than at the edge. The core current density perturbation is reduced by a similar factor. A change in the radial structure of the fluctuations is implied. Reduction of core magnetic fluctuations strongly correlates with dynamo suppression and improved energy and particle confinement.

In summary, magnetic field fluctuations have been nonperturbatively measured, for the first time, in the core of a high-temperature RFP plasma using a newly developed fast Faraday rotation diagnostic. This diagnostic technique is also directly applicable to other toroidal confinement devices, in particular tokamaks. The magnetic fluctuation amplitude increases during the sawtooth crash consistent with expectations for the enhanced dynamo and magnetic relaxation. The frequency spectrum is broad with the fluctuations being observed up to 100 kHz. Current fluctuations associated with magnetic islands are measured to have $\tilde{j}_{m=1,m=6}/J_0 \sim 3\%$ and width ≤ 8 cm. For high-confinement MST plasmas, the core magnetic fluctuation amplitude is observed to decrease fourfold, larger than previously inferred from measurements at the plasma surface.

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