Cause of Sudden Magnetic Reconnection in a Laboratory Plasma

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The cause for sudden reconnection in reversed field pinch plasmas is determined experimentally for two cases: large reconnection events (the sawtooth crash) and small reconnection events during improved confinement. We measure the term in the MHD equations which represents the driving (or damping) of edge tearing modes due to the axisymmetric magnetic field. The term is negative for large reconnection events (the modes are stable, implying that reconnection may be driven by nonlinear coupling to other modes) and positive for small reconnection events (modes are unstable, reconnection is spontaneous).

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Magnetic reconnection [1] occurs in a wide range of laboratory and astrophysical plasmas. Often reconnection occurs suddenly, following a longer period in which the magnetic field changes slowly. This occurs in tokamak and reversed field pinch (RFP) experiments through the well-studied sawtooth oscillation [2–4]. Magnetic field evolution during the long phase of the sawtooth cycle is followed by a "crash" phase in which reconnection rapidly alters the field. In natural plasmas like the solar corona and the earth's magnetosphere, impulsive reconnection is also observed to punctuate a slower evolution of the field [1,5]. The origin of impulsive reconnection is a nonlinear problem, often treated through nonlinear magnetohydrodynamics (MHD).

In this Letter, we determine experimentally the cause for sudden reconnection in two different situations in RFP plasmas: large reconnection events associated with the sawtooth crash and small reconnection events which appear during improved confinement. In both cases (illustrated in Fig. 1), impulsive reconnection is manifest as a burst of magnetic fluctuations (nonaxisymmetric components of the field) with poloidal mode number $m = 0$. The $m = 0$ modes cause reconnection in the plasma edge and subsequent growth of an $m = 0$ magnetic island at the radius where the axisymmetric toroidal magnetic field vanishes and the modes are resonant (i.e., the wave number parallel to the axisymmetric magnetic field vanishes) [6]. We have taken the rather new approach of measuring the term in the MHD equations which represents the driving (or damping) of $m = 0$ modes due to the axisymmetric magnetic field. We find that the $m = 0$ modes are stable (energy flows from the modes to the axisymmetric field) during the large reconnection events and that they are strongly coupled to $m = 1$ modes. This agrees with the standard picture for the RFP sawtooth developed through MHD computation [7,8]. In the small reconnection events, we find that the $m = 0$ modes are unstable (energy flows from the axisymmetric field to the modes), likely the result of changes in the edge equilibrium profiles during improved confinement. Hence, in one case the sudden reconnection is driven by nonlinear coupling to other unstable modes and in the other it is spontaneous, resulting from linear instability. It is interesting to note that in each case the onset of reconnection is sudden. Thus, the sudden onset might be independent of whether the reconnection is driven or spontaneous, but rather set by the geometry of the reconnection or the reconnection layer dynamics. The origin of these events is important to understand not only for the study of impulsive reconnection, but also because they can be confinement limiting.

The experiments were performed in the Madison Symmetric Torus (MST) [9] with major (minor) radius 1.5 m (0.52 m). For these studies, the plasma current was 200 kA. Standard MST plasmas exhibit regular, large reconnection events (sawtooth crashes) which have been well studied in experiment [10] and MHD computation [7,8]. The cycle is believed to be driven by the evolution of the current profile which causes a slow growth of a few marginally unstable $m = 1$ modes. The slow growth is followed by a sudden large reconnection event composed of a burst of $m = 0$ (and other) modes. Figure 2 shows the

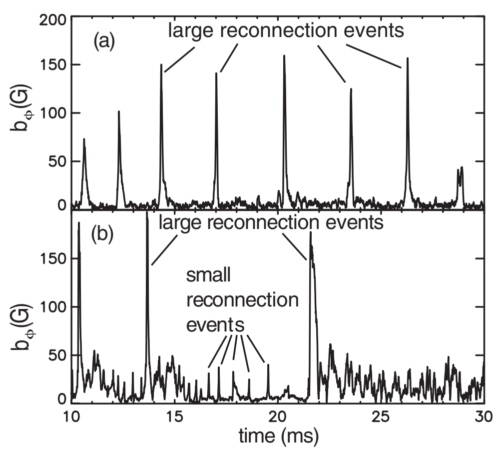

FIG. 1. The $(m, n) = (0, 1)$ toroidal magnetic field fluctuation amplitude vs time, measured at the plasma boundary during (a) a standard plasma with large reconnection events (sawtooth crashes) and (b) a plasma with both large and small reconnection events.

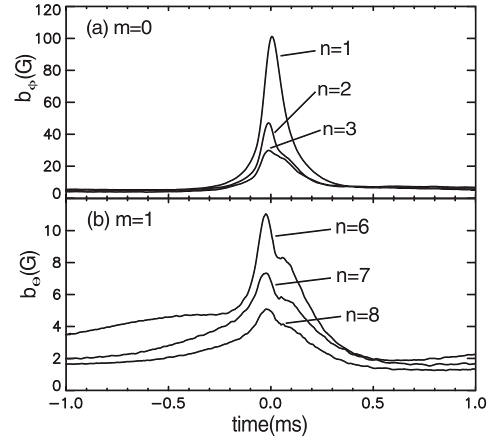

FIG. 2. (a) The toroidal magnetic field fluctuation amplitude for $m = 0$ and $n = 1, 2, 3$ and (b) the poloidal magnetic field fluctuation amplitude for $m = 1$ and $n = 6, 7, 8$ measured by an array of coils at the plasma boundary during a large reconnection event. Data represent the average of 1000 similar events.

evolution of several $m = 0$ and $m = 1$ modes averaged over a set of large reconnection events with reversal parameter (toroidal magnetic field at the plasma boundary/volume average toroidal field) $F \cong -0.2$. The small reconnection events are less common and have a different mode spectrum (Fig. 3). Instead of a prior slow growth of $m = 1$ modes, the $m = 1$ modes rise slightly after the $m = 0$ burst. These less-studied events have yet to be observed in MHD computation and appear during improved confinement which can be actively produced by modifying the driven current profile [11] or spontaneously obtained in a restricted operational space [12]. We have used spontaneous periods in low density, deeply reversed ($F \cong -0.5$) plasmas.

To determine the exchange of energy between the modes and the axisymmetric field, we examine the term in MHD

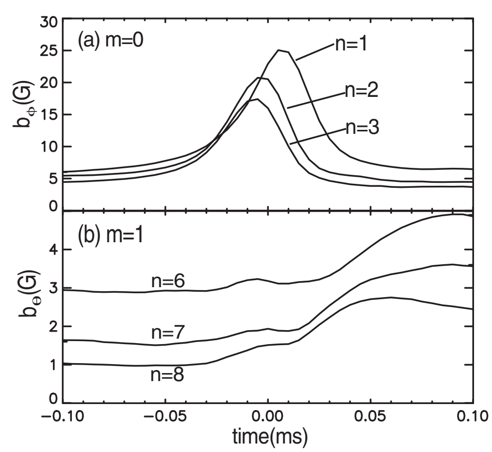

FIG. 3. (a) The toroidal magnetic field fluctuation amplitude for $m = 0$ and $n = 1, 2, 3$ and (b) the poloidal magnetic field fluctuation amplitude for $m = 1$ and $n = 6, 7, 8$ measured by an array of coils at the plasma boundary during a small reconnection event. Data represent the average of 350 similar events.

responsible for this exchange. From Maxwell's equations and Ohm's law,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\eta}{\mu_0} \nabla^2 \mathbf{B}. \quad (1)$$

[Departures from the simple Ohm's law occur in MST in some regions [13]. However, in the region of interest the simple form applies [14].] Inserting the spatial Fourier transform $\mathbf{B}(r, \vartheta, \varphi, t) = \sum_{m,n} \mathbf{b}_{mn}(r, t) e^{i(m\vartheta + n\varphi)}$ into Eq. (1) yields

$$\frac{\partial \mathbf{b}_{mn}}{\partial t} = \nabla \times (\mathbf{v}_{mn} \times \mathbf{b}_{00}) + \nabla \times (\mathbf{v}_{00} \times \mathbf{b}_{mn}) + \sum_{\substack{m=i+k \\ n=j+l}} \nabla \times (\mathbf{v}_{ij} \times \mathbf{b}_{kl}) + \frac{\eta}{\mu} \nabla^2 \mathbf{b}_{mn}, \quad (2)$$

where the axisymmetric $(m, n) = (0, 0)$ terms are removed from the sum. Multiplying Eq. (2) by $b^*_{\phi,mn}$ yields the equation for the evolution of energy in the toroidal magnetic field (the largest component for $m = 0$ modes)

$$\frac{\partial |b_{\varphi,mn}|^2}{\partial t} = \nabla \times (v_{mn} \times b_{00}) \cdot b^*_{\varphi,mn} + \nabla \times (v_{00} \times b_{mn}) \cdot b^*_{\varphi,mn} + \sum_{\substack{m=i+k \\ n=j+l}} \nabla \times (v_{ij} \times b_{kl}) \cdot b^*_{\varphi,mn} + \frac{\eta}{\mu} \nabla^2 b_{mn} \cdot b^*_{\varphi,mn} + \text{c.c.} \quad (3)$$

The first two terms on the right-hand side represent energy exchange between the mode and axisymmetric fields. If these terms are positive, then the mode gains energy from the axisymmetric fields and is unstable. If negative, the mode damps on the axisymmetric fields and is stable. The summation represents nonlinear interactions between modes. The second and last terms are measured with probes to be small relative to the first term.

We measure the first term on the right-hand side with probes in the MST edge. A multicoil probe detects the magnetic field vector. Velocity fluctuations are calculated from measured electrostatic potential fluctuations assuming $\mathbf{v}_{mn} = \frac{-\nabla\Phi_{mn} \times \mathbf{B}_{00}}{B_{00}^2}$, a good approximation for these low frequency modes and in agreement with spectroscopically measured velocities [14]. The potential is measured assuming $\Phi = \Phi_f + 2.2T_e$, where Φ_f is the floating potential and T_e the electron temperature, both measured with a triple Langmuir probe.

Measurements are made at a single toroidal location. To extract a particular mode number, the measurements are correlated with individual Fourier harmonics obtained from a toroidal array of 32 coils at the plasma boundary. The array resolves the toroidal mode number n, but not m. Fortunately, in MST the modes with $n = 1$–4 are dominantly $m = 0$, and are the modes considered here. An ensemble of 1000 (350) similar large (small) reconnection

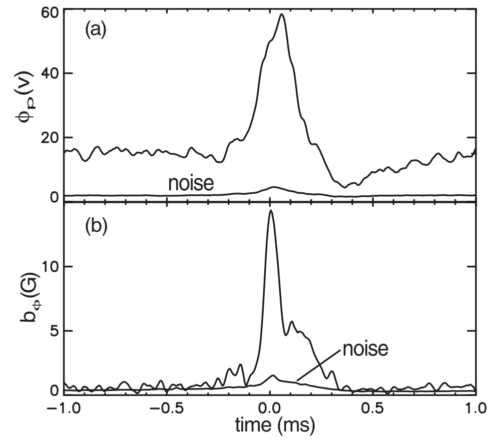

FIG. 4. The magnitude of (a) the electrostatic potential fluctuation and (b) the toroidal magnetic field fluctuation for $(m, n) = (0, 1)$ measured at $r = 46$ cm during the large reconnection event. The statistical noise level for the ensemble is also shown.

events is used to ensure good sampling of many different mode phases. As an example, the part of the magnetic probe signal due to mode n is obtained from $\langle b_{\text{probe}} \cdot b_n \rangle / |b_n|$ where b_{probe} is the magnetic probe signal, b_n is the complex Fourier coefficient for mode n of the array, and the average is done over the set of similar events. The correlations yield the complex v_{mn} and b_{mn} used in Eq. (3).

Measured electrostatic potential and magnetic fluctuations for $(m, n) = (0, 1)$ are shown in Fig. 4 for the large reconnection event along with noise levels based on finite ensemble size. Both fluctuations increase strongly at the event ($t = 0$) and are well above the noise near $t = 0$. Derivatives were evaluated from measurements at $r = 45.5$ cm and 46.5 cm. (The reversal surface, where $b_{\varphi,00} = 0$ and the $m = 0$ modes are resonant was at $r \cong 44$ cm.) A similar result is obtained for small reconnection events.

Combining the measurements, we evaluate the first term on the right-hand side of Eq. (3)—the energy exchange between the mode and the axisymmetric magnetic field, shown in Fig. 5(a) for the large reconnection event. The term is negative, indicating that the mode is stable; energy flows from the mode to the axisymmetric field. The uncertainty arises from the error in the amplitudes and phases of the correlated quantities. Figure 5(b) shows the left-hand side of Eq. (3), illustrating the $m = 0$ burst. The energy exchange between the mode and the axisymmetric field is such as to damp the mode during this entire time, consistent with the picture that the mode is not excited by the equilibrium profiles but instead by nonlinear coupling to other modes.

In principle, the sum over all modes in Eq. (3) could be measured to determine the nonlinear energy transfer between the $m = 0$ and all other modes. In practice, the number of terms in the sum and the $m = 1$ noise level in the edge is too large to obtain a meaningful result. However, nonlinear mode coupling is inferred from the

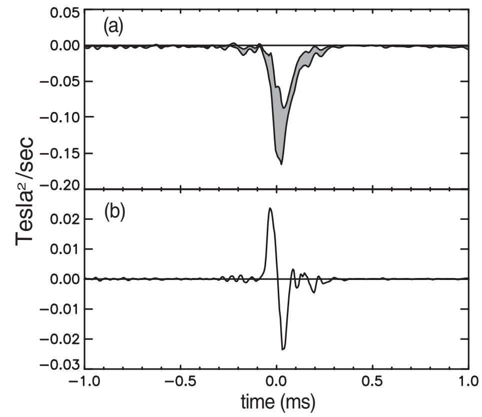

FIG. 5. The measured value of (a) the first term on the right-hand side of Eq. (3) and (b) the left-hand side of Eq. (3) at $r = 46$ cm during the large reconnection event. The two lines in (a) represent the upper and lower bound of the measurement.

bicoherence [15]

$$\beta_{ijk} = \sqrt{\frac{|\langle b_i b_j b_k \rangle|^2}{\langle |b_i b_j|^2 \rangle \langle |b_k|^2 \rangle}},$$

where i, j, k represent coupled modes and $\langle\rangle$ denotes an ensemble average. As Fig. 6 shows, the bicoherence between two marginally stable or slightly unstable $m = 1$ modes and the stable $m = 0$ mode increases substantially during the reconnection event.

The result that the $m = 0$ modes are stable with the implication that they are excited by nonlinear coupling to other modes is consistent with MHD computation. Figure 7 shows the same analysis of Eq. (3) from the 3D, nonlinear, resistive MHD code DEBS [16]. As in experiment, the left-hand side shows mode growth and decay during the sawtooth crash. Also as in experiment (Fig. 5), the first term on the right-hand side is negative throughout the entire event.

The conventional picture of nonlinearly driven $m = 0$ modes does not apply to small reconnection events. Figure 8(a) shows the measured energy exchange between the $(m, n) = (0, 2)$ mode and the axisymmetric magnetic field. The term is positive, indicating the mode receives

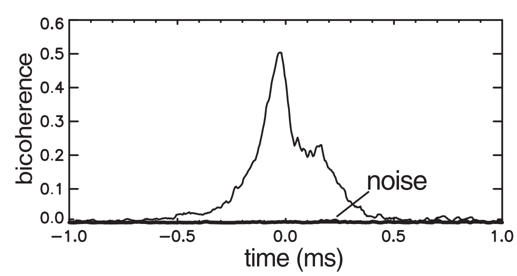

FIG. 6. The bicoherence between the modes $(0, 1)$, $(1, 6)$, and $(1, 7)$ measured at the plasma boundary during the large reconnection event. The statistical noise level obtained by adding a random phase shift to each mode is also shown for the 1000 event ensemble.

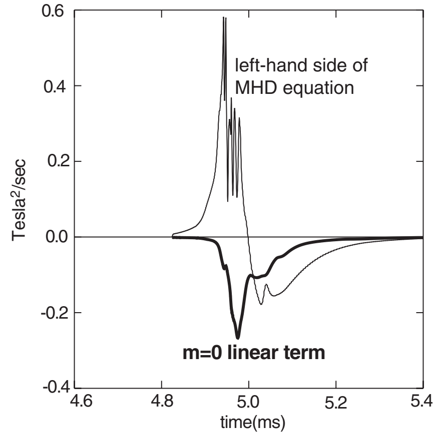

FIG. 7. The first term on right-hand side of Eq. (3) and left-hand side vs time for a single large reconnection event evaluated from MHD computation.

energy from the axisymmetric fields; i.e., it is unstable. Figure 8(b) shows the left-hand side, again indicating that the mode first grows, then decays. The drive of the $m = 0$ mode by the axisymmetric fields remains positive during this entire period, perhaps indicating that nonlinear coupling is transferring energy out of the $m = 0$ modes. Hence the picture is the opposite of the large reconnection case in which nonlinear coupling transfers energy into the $m = 0$ modes. The small reconnection has not been observed in MHD computation. However, $m = 0$ modes are expected to be unstable for steep edge current density or pressure profiles, for which there is some evidence from past measurements [12].

In summary, we have determined the cause for sudden reconnection in a laboratory plasma by measuring the term in MHD which accounts for the energy exchange between a magnetic fluctuation and the axisymmetric field. The measurement was made for two types of reconnection events in the RFP: the large reconnection present during the sawtooth crash and a smaller reconnection event during improved confinement. With large reconnection, the term is negative for modes with $m = 0$, indicating that energy flows from the mode to the axisymmetric fields. A significant measured bicoherence between the $m = 0$ and $m = 1$ modes indicates that nonlinear coupling is present. This supports the standard picture that $m = 0$ modes are stable, but $m = 0$ reconnection is driven by nonlinear coupling to other modes. In the small reconnection events, an opposite picture holds: $m = 0$ modes derive energy from the axisymmetric fields indicating they are unstable, corresponding to spontaneous reconnection. Such a picture awaits theoretical study and more complete equilibrium profile measurements. These results support the possibility that nonlinear mode coupling could be important in sudden reconnection in other laboratory and astrophysical venues. Indeed, the case with nonlinear coupling produces larger macroscopic changes in the plasma. However, it appears that the suddenness of the reconnection occurs whether the reconnection is spontaneous or driven. Hence, the suddenness may be an intrinsic feature of the reconnection layer dynamics or geometry. These techniques might also be useful elsewhere to examine the cause for sudden reconnection.

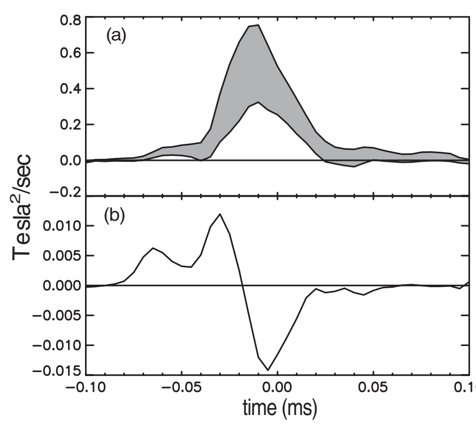

FIG. 8. The measured value of (a) the first term on the right-hand side of Eq. (3) and (b) the left-hand side of Eq. (3) at $r = 47$ cm during the small reconnection event. The two lines in (a) represent the upper and lower bound of the measurement.

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[1] See, e.g., D. Biskamp, *Magnetic Reconnection in Plasmas* (Cambridge University Press, Cambridge, England, 2000).
[2] S. von Goeler, W. Stodiek, and N. Sauthoff, Phys. Rev. Lett. **33**, 1201 (1974).
[3] R. G. Watt and R. A. Nebel, Phys. Fluids **26**, 1168 (1983).
[4] R. J. Hastie, Astrophys. Space Sci. **256**, 177 (1997).
[5] See, e.g., A. Bhattacharjee, Annu. Rev. Astron. Astrophys. **42**, 365 (2004).
[6] N. A. Crocker, G. Fiksel, S. C. Prager, and J. S. Sarff, Phys. Rev. Lett. **90**, 035003 (2003).
[7] K. Kusano and T. Sato, Nucl. Fusion **30**, 2075 (1990).
[8] Y.L. Ho and G. G. Craddock, Phys. Fluids B **3**, 721 (1991).
[9] R. N. Dexter, D. W. Kerst, T. W. Lovell, S. C. Prager, and J. C. Sprott, Fusion Technol. **19**, 131 (1991).
[10] D. L. Brower *et al.*, Phys. Rev. Lett. **88**, 185005 (2002).
[11] J. S. Sarff, S. A. Hokin, H. Ji, S. C. Prager, and C. R. Sovinec, Phys. Rev. Lett. **72**, 3670 (1994).
[12] B. E. Chapman, C.-S. Chiang, S. C. Prager, J. S. Sarff, and M. R. Stoneking, Phys. Rev. Lett. **80**, 2137 (1998).
[13] W. X. Ding *et al.*, Phys. Rev. Lett. **93**, 045002 (2004).
[14] P. W. Fontana, D. J. Den Hartog, G. Fiksel, and S. C. Prager, Phys. Rev. Lett. **85**, 566 (2000).
[15] Y. Kim and E. Powers, IEEE Trans. Plasma Sci. **PS-7**, 120 (1979).
[16] D. D. Schnack, D. C. Barnes, Z. Mikic, D. S. Harned, and E. J. Caramana, J. Comput. Phys. **70**, 330 (1987).