Direct measurements of the 3D plasma velocity in Single-Helical-Axis RFP plasmas

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The first local velocity measurements of helical equilibrium plasmas in the Reversed Field Pinch (RFP) Single Helical Axis (SHAx) state using a Charge Exchange Recombination Spectroscopy (CHERS) diagnostic are presented. Measurements show strong axisymmetric and non-axisymmetric flow, with n = 5 components of flow related to the (m,n) = (1,5) dominant magnetic mode on the order of the axisymmetric flow in certain regions of the plasma, as well as significant n > 5 flow. Flow measurements are compared with NIMROD simulations of visco-resistive, single-fluid MHD in toroidal and cylindrical geometries with limited axial periodicity. Both measurements and the simulation with toroidal geometry show stronger inbound flows relative to the outboard flows, which is attributed to the toroidal geometry of the device. In the experiment, the n = 5 component of flow is phase shifted from the reconnection-like flow pattern observed in the single-fluid simulations, possibly due to decoupling of the ion and electron fluids over much of the plasma. Finally, the strength of the helical angular flow shear relative to the critical shear necessary to disrupt nonlinear coupling between tearing modes is calculated around the helical magnetic axis. The shear in the measured flow is on the order of the theoretical critical threshold needed to nonlinearly decouple modes, but the measurement uncertainty in the gradient of the flow is large.

I. INTRODUCTION

Self-organized, non-axisymmetric plasma dynamics that have potentially useful features for fusion concepts can be present in toroidal plasmas. In the boundary plasma, resonant magnetic perturbations (RMPs) are used to deform the boundary in order to mitigate Edge Localized Modes (ELMs)1, 2. Plasma flow has been shown to stabilize resistive wall modes3. Islands produced by external magnetic perturbation coils in LHD can induce spontaneous flow and flow shear in the plasma4. The same device has shown that magnetic stochasticization of the plasma causes flow damping5. Plasma discharges with a long-lived non-axisymmetric core have been created in DIII-D6, ASDEX6, MAST7, TCV8, and are planned for ITER9, 10, 11. In the reversed field pinch (RFP), the plasma is sustained through a dynamo process of interacting magnetic and flow perturbations. A long-lived, self-organized helical equilibrium can arise in an RFP plasma with improved confinement properties12, 13. Magnetic and flow fields interacting to produce self-organization via nonlinear MHD processes underlies many of these features. Understanding the details of non-axisymmetric plasma self-organization relies on characterizing many aspects of the plasma. This work focuses on the first characterization of the local ion flows in a self-organized helical RFP equilibrium.

The RFP has a safety factor profile q < 1 that peaks on or near the magnetic axis and passes through zero at or near the plasma edge, causing it to contain many poloidal m = 1 and toroidal n tearing mode resonant surfaces at different radii13. Typically only one or two of the innermost tearing modes are unstable, but a wide spectrum of modes is observed due to nonlinear interactions between the modes14, 15. For standard RFP discharges, the tearing modes have comparable amplitude. The resulting island widths overlap leading to a high degree of magnetic stochasticity, which degrades energy and particle confinement in the plasma. Under certain conditions the innermost tearing mode can grow to large amplitude such that the magnetic energy in the mode is much greater than the energy in all other tearing modes combined. This situation is referred to as Quasi-Single Helicity (QSH)16, and we will refer to the kink-tearing mode that is larger than all the rest as the dominant mode. Within the QSH condition, if the dominant mode is sufficiently strong, the core region attains a helical magnetic axis, analogous to stellarator configurations. The change in magnetic topology results in a reduction in magnetic stochasticity leading to regions of plasma with better energy and particle confinement16. The resulting plasma is referred to as the Single Helical Axis (SHAx) state17, 18.

Measurements of ion velocities are critical to understand the nonlinear processes that generate the SHAx state and for fully characterizing the plasma in general. This paper presents the first measurements of local ion velocities in the saturated SHAx state. Previous measurements of flows in the QSH state have been made, but they either use passive, line-integrated measurements and infer local velocities using simulation results19-23, or they measure a growing QSH mode that did not saturate24, 25. Based on the local measurements reported here for plasmas in the Madison Symmetric Torus (MST) device, commentary is made regarding the effect of the toroidal geometry, two-fluid physics, and flow shear on the plasma.
The remainder of this paper is organized as follows: in section II, we discuss further background and details of the SHAx state. In section III, we describe the methods for obtaining the novel localized flow measurements in a self-organized, non-axisymmetric state. In section IV we present the flow data and make observations by analyzing a parameterized profile of the flow data. In section V, we present NIMROD simulations of a SHAx state and compare them to the experimental data. In section VI, we comment on the effects of the toroidal geometry of the device and two-fluid physics on the flow profiles, as well as provide an analysis of the helical angular shear flow measured in the plasma. Finally, in section VII we summarize the results and comment on the impact of the work.

II. THE SATURATED SHAX STATE

It is important to distinguish between QSH, SHAx, and a regime we refer to as the saturated SHAx state. As mentioned earlier, QSH describes the relative distribution of magnetic mode energy in the plasma, whereas SHAx indicates that the average flux surfaces that best describe the plasma are helical as opposed to axisymmetric. The QSH state in an RFP is often characterized using the spectral index of the plasma, defined as

$$N_e = \left[ \sum \left( \frac{E_n}{\sum E_n} \right)^2 \right]^{-1},$$

(1)

where $b_n$ is the perturbed magnetic field amplitude of a given tearing mode with toroidal mode number $n$. The summation in MST ranges from $n = 5$ to $n = 15$, where $n = 5$ is the lowest order tearing mode, and the mode energy for $n > 15$ modes is small. The plasma is considered to be in the QSH state when $N_e < 2$. Since the SHAx condition is a statement of the internal magnetic topology of the plasma, it is difficult to precisely identify when the transition to SHAx occurs. The plasma is estimated to enter the SHAx state when the dominant mode amplitude is $|b_n|/|b_m| = 2 - 4\%$, where $|b_n|/|b_m|$ is the perturbed magnetic field matching the helicity of the dominant mode at the wall, and $b_m$ is the axisymmetric field measured at the wall. In most cases, the dominant mode continues to grow beyond this level. In MST, the RFP used for this work, the dominant mode will not saturate until it reaches 6-9% of the edge magnetic field, well above the threshold for entering the SHAx state. MST plasmas in this regime have previously been characterized as having helical density profiles, temperature profiles, soft x-ray emissivity, and hard x-rays from a confined fast electron population, all indicating the plasma is in a SHAx state. The helical equilibrium is maintained at this level in a relatively steady-state condition. Figure 1 shows the time series of magnetic toroidal mode amplitudes measured at the device wall during a plasma discharge that reaches a saturated SHAx state at 26 ms.

Before entering the QSH state and subsequent saturated SHAx state, the plasma is in a multi-helicity (MH) state, where all the tearing modes are of comparable amplitude and $N_e > 2$. The transition between an MH plasma and a saturated

SHAx plasma is typically quick relative to the total plasma discharge time. The two states can be thought of as two poles in a chaotic attractor system, and the amount of time spent at either pole is influenced by the plasma Lundquist number,

$$S = \frac{\mu_0 J_V}{\eta} \sim \frac{T_a^3}{\sqrt{n_i}},$$

(2)

where $L$ is the characteristic system length, $V_A$ is the Alfven speed, $\eta$ is the resistivity, $T_e$ is the electron temperature, $I_p$ is the plasma current, and $n_i$ is the ion density. Empirically, higher Lundquist number non-reversed plasmas that have vanishing toroidal field at the plasma surface $B_T(a) = 0$, are more likely to enter, and stay in, the SHAx state. Plasma discharges for this dataset are non-reversed, operated at relatively high Lundquist number ($S \sim 10^4$), and spend much more time in the saturated SHAx state than the multi-helicity or transition states, as indicated by the colorscale in Figure 2. In these conditions, the plasma spent 4.5 times longer in the saturated SHAx state than in the MH state between the interval from 5 ms to 45 ms after the start of the discharge. Plasma discharges run at a lower plasma current would spend more time in the MH state.

III. DIAGNOSTICS AND DATA ANALYSIS

METHODOLOGY

As mentioned, the data for this work was obtained on the Madison Symmetric Torus (MST), which produces RFP plasmas with a minor radius of $a = 0.50$ m and a major radius of $R = 1.5$ m. MST has a 5 cm thick aluminum shell that provides a conducting boundary for the plasma as well as a poloidal and toroidal ring of carbon limiters to mini-
One possible reason is that the resonant surface shows representative flux surfaces that...
The new helical magnetic axis revolves about this axisymmetric axis as it is traced out in toroidal angle. There is one large viewing port for the toroidal view which allows data to be collected anywhere between 16 cm inboard and 26 cm outboard along the neutral beam. The radial locations of the poloidal and toroidal measurements used in this paper are displayed in Figure 4.

For most QSH plasmas, and all 500 kA SHAx plasmas that saturate, the plasma locks early in the discharge, well before saturation occurs. The plasma stability is not appreciably affected by locking, but it complicates the ability to obtain velocity measurements with toroidal spatial resolution, which is typically done by measuring the velocity in time as the plasma rotates through the neutral beam. While standard RFP plasmas naturally spin up\(^{35}\), the locking in SHAx states is caused by a braking torque localized to the tearing mode’s resonant surface that scales with the strength of the tearing mode perturbation\(^{35}\). The locking orientation of the helical equilibrium can be controlled, however, using a resonant magnetic perturbation (RMP) system that consists of 38 coils which apply a radial magnetic field at a single toroidal location in MST\(^{33}\). The plasma still remains locked for the duration of a discharge, but toroidal resolution is obtained by changing the locking orientation every shot.

RMP is used for the majority of this dataset and has an estimated \(n = 5\) mode strength of 1-2\% of the edge magnetic field, which has been shown to have no substantial effect on QSH confinement\(^{38}\). Even so, the RMP is only applied during QSH onset and is turned off before the plasma reaches the saturated SHAx state.

For each shot, the location of the flow measurement relative to the helical equilibrium is inferred from the phase of the dominant mode measured by the toroidal array of magnetic diagnostic coils at the edge of plasma. The phase measurement maps 1:1 to a toroidal angle of the helical equilibrium, \(\phi_{SHAx}\), used later in the text. By using the phase measurement, an assumption is made that the helical equilibrium has a toroidal \(n = 5\) periodic symmetry, consistent with the presence of a large, saturated \((m,n) = (1,5)\) mode. Furthermore, this assumption has successfully been used in V3FIT reconstructions of MST\(^{38}\) which yielded synthetic diagnostic results that closely matched actual measurements.

The average flow in the saturated SHAx state is measured by aggregating all flow measurements in time and over multiple discharges made at the same \(n = 5\) locking phase. Since the measurements are averaged over a time much longer than the carbon-deuterium collision time, the flow measurements are considered to be representative of the bulk deuterium velocity as well. The presence of an ion specific momentum drive or loss is not considered here. There is good shot-to-shot agreement in the average flow for any one measurement location, where the variance in the data is on the order of the variance due to photon statistics. The inboard flow measurements have higher variance than the outboard measurements due to attenuation of the neutral beam as it propagates from the outboard side to inboard side of the plasma.

The absolute wavelength calibration of the spectrometer performed each day allows us to know the absolute velocity to within \(\pm 3\) km/s. The calibration method, described in more detail in Craig et al.\(^{39}\), leverages our ability to measure the velocity in the plasma at a location where the velocity is negligible. Such a scenario occurs when the velocity is measured in the poloidal plane at the magnetic axis of an axisymmetric plasma where poloidal current drive is used to reduce tearing mode activity\(^{40,41}\). The velocity measured at this location is used as the zero velocity baseline to calibrate the spectrometer, and this calibration was performed at the beginning and end of each day flow measurements were taken.

IV. VELOCITY PROFILES IN SHAX

In this section we present the aggregate data collected in the saturated SHAx state of 1445 SHAx discharges. Figure 5 shows the contours of the averaged poloidal and toroidal velocity on the toroidal plane sampled by the neutral beam as the plasma is rotated through it. Negative radial values on the horizontal axis of the figure indicate the inboard side of the machine, where \(r/a = 0\) is measured from the point the neutral beam crosses the mid-plane. The vertical axis in the plots is the toroidal angle of the helical equilibrium, \(\phi_{SHAx}\). This toroidal angle is not measured relative to the machine but to...
the helical equilibrium that develops in the plasma for each shot. We chose as a convention in the paper to report features in terms of 5 times the toroidal angle, \(-\pi < 5\phi_{\text{SHA}} < \pi\) instead of \(-\pi/5 < \phi_{\text{SHA}} < \pi/5\). The zero point of \(\phi_{\text{SHA}}\) is chosen to be where the helical magnetic axis is in-line with the neutral beam on the outboard side, as shown by the black lines in Figure 5. The plane shown in Figure 5 is the one sampled by the plasma rotating through the neutral beam and should not be thought of as a helical flux surface.

There are many clear non-axisymmetric features in Figure 5. At \(|r/a| > 0.5\) the poloidal flow varies by more than 10 km/s along the toroidal angle. The same is true for the toroidal flow at \(r/a = 0.4\) and \(r/a = 0.1\), where the flow nearly goes to zero at \(5\phi_{\text{SHA}} > -3\) rad and \(5\phi_{\text{SHA}} > 0.5\) rad respectively. It is easier to parse the axisymmetric and non-axisymmetric contributions to the flow by fitting the data to a parameterized profile consisting of a Fourier series along the toroidal angle. Since the toroidal angle enforces a 5x periodicity assumption, the modes of the Fourier decomposition are harmonics of a base \(n = 5\) mode. These are harmonics correlated with the \(n = 5\) helical equilibrium and are not necessarily correlated with higher order tearing mode perturbations in the plasma.

The amplitudes of the modes are given in Figure 6 for the \(n = 0\) mode and five harmonics of \(n = 5\). Increasing the number of modes in the Fourier decomposition did not appreciably change the results for the \(n < 25\) modes, and the amplitudes of \(n > 25\) harmonics were small with uncertainty that included 0 km/s. The error bars in the reported amplitude values come from the standard deviation of fitting ten equally sized and randomly selected subsets of the total dataset.

As can be seen from Figure 6 a), the axisymmetric toroidal flow profile is a relatively uniform -9 km/s for \(|r/a| < 0.5\). The toroidal flow is in the same direction as the plasma rotation before mode locking occurred, which is opposite to the direction of \(B_r\) and in the direction of \(J_r\). The axisymmetric poloidal flow profile has a minimum in the plasma core and solid body rotation on the inboard side (\(-0.7 < r/a < 0\)). On the outboard side (\(r/a > 0\)), flow deviates from solid body rotation, peaking at \(r/a = 0.2\), and reversing direction at the outboard edge, \(r/a > 0.6\). This poloidal flow reversal has been observed in previous measurements of MST plasma flows. The source of asymmetry in the poloidal flow may partly be due to the toroidal geometry of the device, which will be discussed in section VI A.

Both the \(n = 5\) and the \(n > 5\) components of the flow shown in Figure 6 b) and c) are non-trivial in the plasma. The \(n = 5\) poloidal flow is comparable to the axisymmetric flow at \(r/a = -0.1\) and 0.7. The \(n > 5\) poloidal flow exceeds the \(n = 5\) flow at \(r/a = -0.7\) and 0.5. The poloidal flow harmonic amplitudes are also indicative of solid-body rotation, with a minimum in flow at the center that increases with increasing radius. It should be noted that the poloidal flow measurement at \(r/a = 0\) can be considered a radial flow according to the coordinate system presented, but it is only normal to the helical flux surfaces if \(\phi_{\text{SHA}} = \pi/2\) rad and \(-\pi/2\) rad. The lack of strong flow near \(r/a = 0\) is similar to previous reconstructed flow profiles from line-integrated velocity measurements using simulations with a resistivity similar to the experiment, but different than those relying on simulations run at reduced resistivity and/or viscosity. The toroidal \(n = 5\) harmonic is also strong, though it never exceeds the toroidal axisymmetric flow. The toroidal \(n > 5\) modes are less pronounced compared to the \(n = 5\) mode, but they are still comparable to the \(n > 5\) modes in the poloidal flow.

The \(n = 5\) poloidal flow is broader in extent compared to previous CHERS measurements in MH and growing QSH plasmas, which were made in \(I_p = 400\) kA reversed RFP conditions. Previous measurements of flow associated with tearing modes in MH quiescent phase is localized near the tearing mode’s rational surface. The mode velocity related to the dominant tearing mode in the growth phase of the QSH plasmas, is also broader than the MH quiescent phase, but still doesn’t extend as far as the SHAx plasmas, where \(n = 5\) flow is still measured at \(|r/a| > 0.5\). The peak mode velocities for the SHAx plasmas are also three times that of the velocities measured in the MH quiescent phase and the dominant mode of the growing QSH phase.

V. NIMROD SIMULATION OF A QSH STATE

To better understand what physical effects are influencing the flow profiles in the SHAx state, simulations were performed to compare to experimental data. There have been several past analyses of theoretical Single Helicity (SH) states and QSH states created in simulations. One method uses scalar viscosity and/or resistivity values much larger than the perpendicular Braginskii viscosity or Spitzer resistivity calculated from experiment to encourage the formation of a helical state by heavily dissipating the energy transferred from dominant to sub-dominant modes. A shift from MH to SH in computations is observed to depend on the plasma Hartmann number:

\[
H_a = \frac{B L}{\sqrt{\eta \mu}} = \frac{S}{\sqrt{\eta \mu}}
\]

where \(B\) and \(L\) are the characteristic magnetic field and length scale, \(\eta\) is the plasma resistivity, \(\mu\) is the viscosity, \(P_m\) is the Prandtl number, and \(S\) is the Lundquist number previously defined in Equation 2. The transition from MH to SH is observed to occur when viscosity and/or resistivity increase such that the Hartmann number decreases below a threshold Hartmann number of around 2000. In experimental plasmas, the Hartmann number obtained using Spitzer resistivity and perpendicular Braginskii viscosity is on the order of \(H_a = 10^9\). However, the effective dissipation relevant for tearing mode interactions may be different than that implied by these estimates. For example, viscosity is enhanced above perpendicular Braginskii viscosity by stochastic fields in the nonlinear limit, an effect implied by electrode biasing experiments and qualitatively captured in resistive-MHD simulations. It is also conceivable that parallel viscosity, which is much larger and scales differently than perpendicular viscosity, might be important in the stochastic-field limit of nonlinear MHD, further increasing the effective viscosity on the tearing modes. The role of
FIG. 5. a) Poloidal and b) toroidal velocity contours of averaged CHERS velocity measurements. The black lines are representative flux surfaces obtained from V3FIT. The horizontal axis is the radius measured along the neutral beam, where zero is the location the neutral beam crosses the mid-plane, $r/a < 0$ is inboard, and $r/a > 0$ is outboard. The vertical axis is the toroidal angle of the helical equilibrium, $\phi_{SHAx}$, where angles are reported from $-\pi < \phi_{SHAx} < \pi$. This figure is the same plane cut out by the plasma being rotated through the neutral beam as is shown in Figure 3.

The highly anisotropic viscosity tensor on nonlinear plasma evolution merits further investigation, though it is outside the scope of this paper.

Nonetheless, the high dissipation SH simulations also behave differently than experimental QSH plasmas. They lack significant sub-dominant modes, and the SH state persists with few of the reversions to MH that are typically observed experimentally. Furthermore, the probability of entering a QSH state is empirically observed to increase with increasing Hartmann number (based on perpendicular Braginskii viscosity), opposite the trend observed in simulations. These differences suggest the high dissipation SH simulations are in a different regime and are influenced by different mechanisms than experimental plasmas. The low Hartmann number simulations are not necessarily meant to assign a causal link between high dissipation and the formation of QSH states in experiment, but the simulations have been used to infer magnetic and flow profiles for analysis in experimental QSH plasmas. These flow profiles include substantial radial flow through the axis and poloidal flow near the axis, which are not present in the measurements made in this paper.

Another method of obtaining a QSH state is to apply an edge perturbation at the plasma boundary, which has also been successfully used in RFX-mod. The process uses RFX-mod’s set of magnetic perturbation (MP) coils to apply a radial magnetic perturbation of a specific poloidal and toroidal mode number on the order of 1-2% of the edge magnetic field. The simulations and experiments have shown helical regions of improved confinement matching the MP helicity. The simulations have also shown the same trend of increased QSH persistence with increasing Hartmann number that is observed experimentally. QSH states with a dominant mode that is different than the innermost tearing mode can even be created using this method. This is an interesting approach to achieving a SHAx state; however, the plasma self-organization mechanisms responsible for creating a SHAx state by continually applying an edge perturbation is likely different than the SHAx states under consideration in this paper. In contrast to applying edge perturbations, the thickness aluminum shell in MST suppresses radial edge perturbations on the plasma, keeping them well below 1%. The strength of the $n = 5$ component of the RMP in MST used to encourage entry into the SHAx state and to control the locking phase of the helical equilibrium is on the order of 1-2% of the edge magnetic field, but it is only applied temporarily and is terminated before the saturation of the dominant mode occurs.

The simulations run for this work use neither high dissipation nor perturbed boundary conditions to achieve a QSH state. Instead, a QSH state was obtained with a Hartmann number above the MH to QSH transition by only simulating toroidal modes that are harmonics of $n = 5$. This is equivalent to only simulating 1/5 of the torus. The poloidal spectrum is not limited other than by computational constraints. We emphasize the simulations are fully three-dimensional, only limited in toroidal harmonic content. Achieving a QSH state is attributed to the artificial elimination of tearing modes that are not harmonics of $n = 5$, such that energy is transferred away from the dominant mode less effectively and the dominant mode ampli-
FIG. 6. a) Axisymmetric ($n=0$) poloidal (blue) and toroidal (green) flow profiles. b) Poloidal and c) toroidal harmonic flow profiles. Error bars represent the standard deviation of the velocity amplitudes from fitting ten random subsets of the total dataset.

despite this limitation, the flow profiles are expected to be similar to the experimental measurements since lower resistivity and viscosity values are able to be used.

NIMROD$^{12}$ simulations are used in this paper to obtain a steady-state QSH plasma. NIMROD is an initial-value extended-MHD code that uses a 2D finite element mesh and Fourier components in the third direction. For these simulations the mesh is in $R$ and $Z$ and the Fourier direction is in $\phi$. Two simulations are used, one run using cylindrical geometry and one run using toroidal geometry, both using 22 modes (up to $n=105$) in a non-reversed equilibrium with an ideal conducting boundary, no slip flow boundary, and the parameters given in Table I. The Hartmann number of these simulations is 8944, well above the transition from QSH to MH observed in previous simulations. Despite this, steady-state QSH states are obtained, as shown in Figure 7, where the dominant $n=5$ magnetic mode energy remains persistently and significantly larger than the other higher $n$ tearing modes in time, after an initial relaxation event occurs. In the plot $W_0$ is the total magnetic energy per mode, and $W_B$ is the total axisymmetric magnetic energy in the plasma. Identification of axes in the NIMROD simulation is nontrivial, and it cannot be said definitively if the simulated plasma is in a SHAx state or not. However, a helical magnetic axis can be identified from the off-axis region of healed flux surface seen in Poincaré plots, as shown in Figure 8. Even with limited periodicity of the simulations, there is significant enough mode overlap and mode interaction to cause substantial stochasticity of the plasma in both the cylindrical and toroidal geometry simulations.

In order to provide as direct comparisons as possible to
Experimental measurements, NIMROD ion velocities are reported at locations and lines of sight that match those obtained from the CHERS diagnostic in the experiment. However, the poloidal axis in NIMROD is in a different location than the experiment due to different amounts of Shafranov shift. To properly match the experimental and simulated velocity measurements, the line of velocity measurements along an imagined neutral beam in NIMROD is shifted to intersect the poloidal axis for each simulation. For the cylindrical simulation, this means that the “neutral beam” intersects the mid-plane at the axis for each simulation. For the poloidal simulation Poincaré plot at $t/t_0 = 18069$ showing the region of healed flux surfaces.

![Poloidal cross-section of the toroidal geometry NIMROD simulation Poincaré plot at $t_f = 1.5$ rad and time point $t/t_0 = 18069$ showing the region of healed flux surfaces.](image)

**FIG. 8.** Poloidal cross-section of the toroidal geometry NIMROD simulation Poincaré plot at $t_f = 1.5$ rad and time point $t/t_0 = 18069$ showing the region of healed flux surfaces.

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**TABLE I.** Relevant NIMROD simulation run parameters.

where $V_{SIM}$ is the normalized velocity reported by NIMROD, $V_{AMST}$ is the Alfvén velocity of the experimental MST plasmas ($10^6$ m/s), $N_{SIM}$ is the Lundquist number of the NIMROD simulation, and $N_{MST}$ is the Lundquist number of the experimental MST plasmas ($10^{16}$). This scaling is motivated by the resistive tearing mode growths that lie between Alfvénic and resistive timescales. However, it is only an estimate, and simulations run at the same parameters as the experiment would be necessary to more accurately compare flow magnitudes. Given the simplifications used in the computation, it is not expected that the velocities will match the experimentally measured values, but it is expected they should reach the appropriate order of magnitude.

Cross-sections of poloidal and toroidal flow matching the cross-section measured in the experiment are shown in Figure 9 for both the cylindrical and toroidal geometry simulations. The mechanisms that drive net axisymmetric plasma flow in MST plasmas are not adequately simulated in resistive-MHD computations, so visual comparison between the two simulation and experimental figures is limited. What can be seen from the figures in Figure 9 is that the strength of the perturbations is larger for $r/a > 0.5$ than $r/a < 0.5$, as in the experimental measurements. The typical flow pattern is further characterized in Figure 10, showing the full flow profile for a poloidal cross-section for the toroidal geometry simulation at $t/t_0 = 18069$ and $\delta_{SHA} = 1.5$ rad, the same toroidal location as the Poincaré plot in Figure 8. Since experimental data is only taken at one poloidal location, the poloidal cross-section of flow cannot be easily compared to the flow in the experiment. Such a comparison would require relying on a model for reconstructing flow in the poloidal plane which makes assumptions about the poloidal mode content of the flow and would not be well constrained by the experimental data.

The flow profiles between the experimental data and the simulation data is most easily compared by considering the mode amplitudes of the flow. The poloidal flow profiles for both the cylindrical and toroidal geometry simulations are qualitatively similar to the experimental measurements, as can be seen in Figure 11 a and b). Specifically, there is an overall rotational profile with a maximum at $r/a = 0$ and amplitudes that increase with increasing radius as well as significant contributions from $n > 5$ modes. The amplitude of the $n > 25$ modes are small compared to the $n \leq 25$ modes and are not shown. The profiles shown in Figure 11 are averaged from the time points in the simulation indicated in dotted black lines in Figure 7, and the error bars represent the standard deviation of the data. The poloidal flow shows a strong $n = 10$ flow in both simulations, especially at larger radii. This is attributed to a strong $(m,n) = (1,10)$ tearing mode. The strength of the mode and the fact that it is present at larger radius also explains why the global kinetic energy in the plasma is dominated by the $n = 10$, as shown in Figure 7 b and c). The toroidal velocity profile is significantly different in the simulations compared to the experiment, however. Comparing to Figure 6 c), the NIMROD simulations have very weak non-axisymmetric toroidal flow at the center of the plasma, where the experimental measurements for toroidal flow show substantial non-

![Experimental and simulated poloidal flow profiles for the toroidal geometry simulation.](image)

**FIG. 9.** Experimental and simulated poloidal flow profiles for the toroidal geometry simulation. The experimental data is indicated by the solid black line, while the simulation data is indicated by the dotted black line. The poloidal flow profiles between the experimental data and the simulation data is most easily compared by considering the mode amplitudes of the flow. The poloidal flow profiles for both the cylindrical and toroidal geometry simulations are qualitatively similar to the experimental measurements, as can be seen in Figure 11 a and b). Specifically, there is an overall rotational profile with a maximum at $r/a = 0$ and amplitudes that increase with increasing radius as well as significant contributions from $n > 5$ modes. The amplitude of the $n > 25$ modes are small compared to the $n \leq 25$ modes and are not shown. The profiles shown in Figure 11 are averaged from the time points in the simulation indicated in dotted black lines in Figure 7, and the error bars represent the standard deviation of the data. The poloidal flow shows a strong $n = 10$ flow in both simulations, especially at larger radii. This is attributed to a strong $(m,n) = (1,10)$ tearing mode. The strength of the mode and the fact that it is present at larger radius also explains why the global kinetic energy in the plasma is dominated by the $n = 10$, as shown in Figure 7 b and c). The toroidal velocity profile is significantly different in the simulations compared to the experiment, however. Comparing to Figure 6 c), the NIMROD simulations have very weak non-axisymmetric toroidal flow at the center of the plasma, where the experimental measurements for toroidal flow show substantial non-

![Experimental and simulated poloidal flow profiles for the toroidal geometry simulation.](image)
FIG. 9. Perturbed velocity contours in NIMROD simulations at the same poloidal location as the experimental CHERS measurements in Fig. 5. Figures a) and b) are the poloidal and toroidal velocities in cylindrical geometry respectively, and figures c) and d) are the poloidal and toroidal velocity in toroidal geometry respectively. The data shown is for a single time point from the simulations, $t/t_A = 18069$ for the toroidal geometry simulation and $t/t_A = 8631$ for the cylindrical geometry simulation. The black ‘x’ in the figures represents the estimate of the helical axis location.

FIG. 10. Poloidal cross-section of the toroidal geometry NIMROD simulation showing velocity at $\phi_{SHAx} = 1.5 \, \text{rad}$ and time point $t/t_A = 18069$. The black arrows show the direction of velocity in the poloidal plane, with the length of the arrow representing the relative strength of the flow, and the color scale represents the toroidal velocity.

axisymmetric flow. The decrease in non-axisymmetric flow observed in the inboard side of the experimental toroidal flow is also not observed in the simulation. The NIMROD velocity amplitudes overall are about five times weaker than the measured values. This is not surprising given the missing physics (i.e. two-fluid, significant beta, diamagnetic flow, and lower viscosity). Since the $n = 0$ axisymmetric flow is not expected to match the experiment, it is not reported in Figure 11.

Simulations were also run with high dissipation by increasing the Prandtl number from 20 to 2000, giving a Hartmann number of $Ha = 894$. The simulations were run in toroidal geometry for a simulation of 1/5 of the torus and a simulation of the entire torus. Both simulations produced a QSH state, as shown in Figure 12. The simulation of only 1/5 of a torus produced a strong $n = 5$ magnetic mode that was consistently dominant, but oscillated strongly in time, never reaching a steady-state. The simulation of the entire torus produced a strong $n = 5$ dominant mode that gave way later in time to...


A simple analysis of the poloidal flow can help characterize the strength of the flow asymmetry. Since the measurement points are equidistant from the poloidal axis, the ratio of the total inboard poloidal flow and the total outboard poloidal flow can be used to get a basic quantification of the inboard/outboard flow asymmetry. The ratio is

$$R_{IN/OUT} = \frac{\sum_{R=IN}^{R=IN/OUT} V_n(R_{IN})}{\sum_{R=OUT}^{R=IN/OUT} V_n(R_{OUT})}$$

where $R_{CHERS,inboard}$ is the set of inboard CHERS locations, $R_{CHERS,outboard}$ is the set of outboard CHERS locations, and $V_n$ is the $n$ toroidal mode velocity. A device without toroidal geometry effects is expected to have a poloidally symmetric flow profile, and the ratio would be one. A deviation from unity is attributed to the toroidal geometry of the system. The ratio of flow for each harmonic of the parameterized profile of poloidal flow for the experimental data, the cylindrical NIMROD simulation, and the toroidal NIMROD simulation is shown in Figure 13. It is clear from Figure 13 that both the experiment and toroidal NIMROD data have flow ratios greater than one, where the cylindrical simulation stays at a ratio of one, as expected.

Greater inboard flow relative to the outboard is not surprising, if an incompressible flow and flat density profile is assumed, since the total volume on the inboard side is less than the outboard. The ratio of cross-sectional area between the inboard and outboard sides of the MST core would give an expected ratio between inboard and outboard velocity of $R_{IN/OUT} = 1.4$. This simple geometrical factor would account for the asymmetry in most of the non-axisymmetric flow in the experiment. The stronger ratio for the axisymmetric flow on the inboard side may be due to a stronger flow drive on the inboard side. The toroidal geometry simulation has an inboard to outboard flow ratio larger than 1.4. The specific mechanisms causing this are unclear and likely complex, but since the only change between the two simulations is the geometry, it clearly stems from the influence of the toroidal geometry.

B. Two-fluid effects in the saturated SHAx state

The NIMROD simulations were run using single-fluid MHD. However, two-fluid effects are likely important in the MST core. and have been measured experimentally during sawtooth crashes in MH plasmas. In single-fluid, resistive MHD (or two-fluid MHD with cold ions), the ion and electron flows associated with a magnetic island form convective cells out-of-phase with the island structure, where there is inflow at the x-point and outflow at the O-point. When ions are warm and the motions of the ions and electrons begin to decouple, the electron flow responsible for advecting flux into the island will remain unchanged, but the global ion flow profile will shift along the helical angle of the island such that the ion convective cells are no longer out-of-phase with the island structure.

The convective cells associated with an island consists of a radial and helical flow. There are no experimental radial flow
measurments available to reconstruct the full global flow profile, but the helical flow \( V_\phi = mV_\theta + nV_\phi \) can be reconstructed at locations where the poloidal and toroidal measurements sufficiently overlap, within \(-0.2 < r/a < 0.45\). The phase of the flow perturbation relative to the toroidal angle can then be extracted. For single-fluid MHD, the helical flow profile measured along a toroidal angle centered at the tearing island axis should be \( \pm \pi/2 \) relative to the axis. A shift away from that peak is indicative of a decoupling of the electron and ion fluid.

Figure 14 shows the phase of the helical flow for the experimental data and the NIMROD simulation with toroidal geometry. The NIMROD simulation data shows good agreement with the expected single-fluid flow pattern; however, the experimental data differs from the single-fluid expectations. This is attributed to the effect of the two-fluid physics in the experimental plasma. This measurement is only an inference, since it is unclear exactly what the SHAx flow pattern in the experimental parameter regime should look like. Future two-fluid, toroidal SHAx simulations would improve the ability to interpret experimental data.

C. Flow shear of in the saturated SHAx state

It has been suggested that the mechanism responsible for producing the QSH and subsequent SHAx states in experiments is a decoupling of the dominant and sub-dominant modes by magnetic or flow shear\(^57,30\). Strong shear generally causes a reduction in nonlinear energy transfer in plasma turbulence by breaking apart eddies before they can transport material away from the inner plasma. For example, strong flow shear in the plasma edge is considered to be the cause of the L to H transition seen in tokamaks\(^59\). Peaks in flow shear have been previously measured in SH simulations as well as reconstructed experimental flow profiles and observed to exist outside the electron internal transport barrier (eITB) of helical states in RFX-mod\(^59,21,40\). The reduced model presented by Terry et al.\(^57\) uses a predator-prey model to describe the interaction between dominant and sub-dominant modes and determine conditions for QSH formation and sustainment. The quantity under consideration is the shear of the helical angular velocity about the dominant mode, with a helicity matching the dominant mode. The proposed effect of this shear is demonstrated in the cartoon in Figure 15, which shows the dominant mode as an island (black lines) with a sub-dominant tearing mode “eddy” (blue lines) projected onto it. Figure 15 a) is the system without shear, and Figure 15 b) is the system with angular shear (black lines) about the tearing mode axis. For angular shear above a critical value, energy transfer from the unstable dominant mode to the sub-dominant tearing modes is suppressed, allowing the dominant mode to grow large. In the case of the SHAx state, it is the helical magnetic axis about which the helical angular shear is measured.

The helical angular velocity shear calculation used is

\[
\Omega_\ell = \frac{\partial}{\partial \phi} \left( \frac{V_\ell}{\rho_{\text{in}}(\phi)} \right),
\]

where \( V_\ell = mV_\theta + nV_\phi \) is the helical velocity and \( \rho_{\text{in}}(\phi) = r - r_\ell \) is the radius measured from the location, \( r_\ell \), of the surface containing the O-point or helical magnetic axis. The radius \( r \) is measured from the Shafranov-shifted axis, which is the point where the neutral beam intersects the mid-plane of the MST. Since no radial velocity information is available, the full angular velocity can only be measured at the helical angle of the tearing O-point, where contributions to the helical angular
Since the tearing mode velocities in these 500 kA SHAx cases are comparable (on the order of a few Gauss) to the Alfvén speed would modify tearing mode stability it- self, not the energy transfer from an unstable tearing mode to a stable one. The calculation of the helical angular shear at $\Phi_{\text{SHAx}} = 0$ rad is given in Figure 16. As can be seen from the figure, the shear measured between the magnetic axis ($r = 0$ surface) and the nearest sub-dominant tearing mode is on the order of $10^5$ (m s)$^{-1}$, with a measurement of shear nearest the magnetic axis close to $10^6$ (m s)$^{-1}$. This puts the shear in the right regime to have an effect on tearing mode energy transfer. However, the uncertainties associated with the helical angular velocity shear measurements are very large and encompass zero shear, thus the strongest statement that can be made is that shear flow cannot be ruled out as a possible effect.

Improving the resolution of the angular shear flow measurement can be accomplished by collecting significantly more flow data, as well as improving the estimate of the helical magnetic axis location, which was obtained from a V3FIT reconstruction of the SHAx state. The uncertainty in $r_{\text{rad}}$ was estimated to be 2 cm, and it contributed about equally to the angular shear uncertainty as the uncertainty in the velocity. The critical shear estimate may also be modified through advancements in the theory itself, such as by including effects of three wave coupling from to the ion Doppler probe frequency. Improving the resolution of the angular shear flow measurement can be accomplished by collecting significantly more flow data, as well as improving the estimate of the helical magnetic axis location, which was obtained from a V3FIT reconstruction of the SHAx state. The uncertainty in $r_{\text{rad}}$ was estimated to be 2 cm, and it contributed about equally to the angular shear uncertainty as the uncertainty in the velocity. The critical shear estimate may also be modified through advancements in the theory itself, such as by including effects of three wave coupling from to the ion Doppler probe frequency. Improving the resolution of the angular shear flow measurement can be accomplished by collecting significantly more flow data, as well as improving the estimate of the helical magnetic axis location, which was obtained from a V3FIT reconstruction of the SHAx state. The uncertainty in $r_{\text{rad}}$ was estimated to be 2 cm, and it contributed about equally to the angular shear uncertainty as the uncertainty in the velocity. The critical shear estimate may also be modified through advancements in the theory itself, such as by including effects of three wave coupling from to the ion Doppler probe frequency. Improving the resolution of the angular shear flow measurement can be accomplished by collecting significantly more flow data, as well as improving the estimate of the helical magnetic axis location, which was obtained from a V3FIT reconstruction of the SHAx state. The uncertainty in $r_{\text{rad}}$ was estimated to be 2 cm, and it contributed about equally to the angular shear uncertainty as the uncertainty in the velocity. The critical shear estimate may also be modified through advancements in the theory itself, such as by including effects of three wave coupling from to the ion Doppler probe frequency. Improving the resolution of the angular shear flow measurement can be accomplished by collecting significantly more flow data, as well as improving the estimate of the helical magnetic axis location, which was obtained from a V3FIT reconstruction of the SHAx state. The uncertainty in $r_{\text{rad}}$ was estimated to be 2 cm, and it contributed about equally to the angular shear uncertainty as the uncertainty in the velocity. The critical shear estimate may also be modified through advancements in the theory itself, such as by including effects of three wave coupling from to the ion Doppler probe frequency. Improving the resolution of the angular shear flow measurement can be accomplished by collecting significantly more flow data, as well as improving the estimate of the helical magnetic axis location, which was obtained from a V3FIT reconstruction of the SHAx state. The uncertainty in $r_{\text{rad}}$ was estimated to be 2 cm, and it contributed about equally to the angular shear uncertainty as the uncertainty in the velocity. The critical shear estimate may also be modified through advancements in the theory itself, such as by including effects of three wave coupling from to the ion Doppler probe frequency.

VII. SUMMARY

We have reported the first localized flow profile measurements of a self-organized non-axisymmetric plasma. This was made possible by implementing previous improvements in the absolute velocity calibration techniques on the CHERS diagnostic and mode locking control systems on MST. The axisymmetric component of the measurements show uniform toroidal flow in the plasma core, and poloidal flow with solid- body rotation on the inboard side, but peaked flow at $r/a = 0.2$.
on the outside side. There is strong non-axisymmetry in the plasma, with \( n = 5 \) flows on the order of the axisymmetric flow in certain regions as well as significant \( n > 5 \) flow.

We have used these measurements in conjunction with new, limited periodicity, single-fluid NIMROD simulations to analyze the effects of the toroidal geometry and infer two-fluid physics in the experimental plasma flow. An inboard/outboard flow asymmetry observed in experimental data is widely attributable to the toroidal geometry of the machine. The flow asymmetry should scale with \( a/R_0 \) and could be verified with flow measurements from RFP experiments with different aspect ratios. The phase shift of the helical flow relative to the helical magnetic equilibrium is indicative of a decoupling of the ion flow from the electron flow that supports the magnetic equilibrium due to two-fluid effects.

Lastly the helical angular shear flow was characterized in the region where helical flow could be calculated and the contribution to the shear from radial velocity was minimal. Comparing the helical angular shear flow to a hypothesized critical shear necessary to disrupt energy transfer between tearing modes led to a result that is suggestive but ultimately inconclusive because of measurement uncertainties. Efforts must be made to reduce the uncertainty of both the velocity measurements and the location of the helical equilibrium relative to the velocity measurements in order to provide stronger conclusions about the importance of shear flow in SHAx formation and sustainment. A similar analysis of magnetic shear using V3FIT reconstructions, possibly requiring the implementation of SIESTA\(^{22,23}\) in V3FIT, could provide valuable information about the magnetic shear in the plasma relative to a similar critical magnetic shear threshold\(^\text{[3]}\).

The data and analysis presented here provide valuable information about the internal structure of SHAx plasmas, as well as a useful benchmark for future simulations. Self-consistent, toroidal, two-fluid, simulations in a parameter regime similar to experiment are likely necessary to fully understand the mechanisms responsible for QSH and eventual SHAx formation. In the future, it would be interesting to perform an analysis of the time dynamics involved in the initiation of QSH, growth, as well as in the saturated SHAx state itself. However, such an analysis would require significantly more data than that used in this paper, due to the shorter timescales of the QSH onset and growth periods.

**SUPPLEMENTARY MATERIAL**

The data that supports the findings of this study are available within the article and its supplementary material\(^\text{[4]}\). The supplementary material contains the digital format of the data shown in the figures in this paper.

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**DATA AVAILABILITY**

The data that supports the findings of this study are available within the article and its supplementary material\(^\text{[4]}\).


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73. See supplementary material at [inset link] for the digital format of the data shown in this paper.
Physics of Plasmas

Toroidal Velocity Harmonics [km/s]

Poloidal Velocity Harmonics [km/s]

Axisymmetric Velocity [km/s]
n=5 $V_\phi$ Phase [rad]

[Diagram showing phase plots for $n=5$ with various frequencies and phases indicated graphically.]
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\[ a \quad b \]

\[ r_{he} = 0 \]

Measured \( \Omega'_v \):